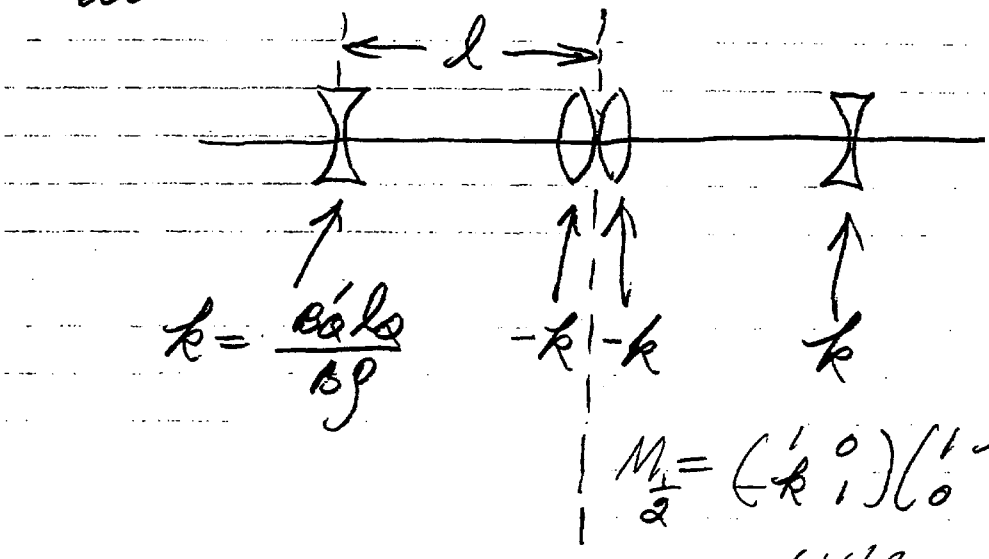
To Get Rid of the Bridge Couplers

Lee 1/25/1989 (1)

Approximate Triplet Calculation

The triplet is approximated by thin lenses as



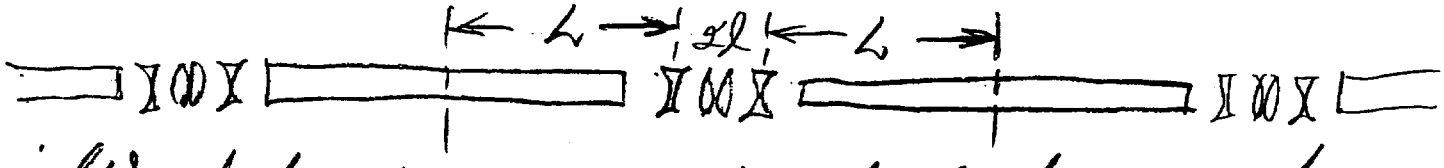
$$M_{\frac{1}{2}} = \begin{pmatrix} 1 & 0 \\ -k & 1 \end{pmatrix}\begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -k & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+kl & l \\ -k^2 l & 1-kl \end{pmatrix}$$

$$M = \begin{pmatrix} 1-kl & l \\ -k^2 l & 1+kl \end{pmatrix}\begin{pmatrix} 1+kl & l \\ -k^2 l & 1-kl \end{pmatrix} = \begin{pmatrix} 1-2k^2l^2 & 2l(1-kl) \\ -2k^2 l(1+kl) & 1-2k^2l^2 \end{pmatrix}$$

$$\equiv \begin{pmatrix} a & b \\ c & a \end{pmatrix}$$

To get β_0 at the middle of a sector we have



We take $L = 4$ m which should be enough. Then from mid-sector to mid-sector we have

$$\mathcal{M} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & a \end{pmatrix}\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a+cL & b+2aL+cL^2 \\ c & a+cL \end{pmatrix}$$

$$\begin{cases} a = 1 - 2l^2 \\ c = -2l(1 \pm l) \end{cases}$$

$$\begin{cases} \cos\mu = a + cL \\ \beta \sin\mu = b + 2aL + cL^2 = \dfrac{(a+cL)^2 - 1}{c} \end{cases}$$

(1) For $k = \dfrac{B'_Q l_Q}{B\rho}$

Using NdFeB permanent magnet we can get $B'_Q \sim 4\,kG/cm$. Thus with $l_Q \cong 4\,cm - 8\,cm$ we can get $B'_Q l_Q \cong 16\,kG - 32\,kG$ or

$$k \cong \frac{B'_Q l_Q}{B\rho} = 1\ m^{-1}$$

over the whole range of $B\rho \cong 16\,kG\,m - 32\,kG\,m$. This make computation easy.

(2) With $L = 4\,m$ we have

l (cm)	β_o (m)		β_e (m)	
14	2.97	4.15	7.09	7.10
16	2.27	3.69	7.66	7.01
18	1.52	3.28	9.58	7.01

(3) With $L = 4.5\,m$ we have

l (cm)	β_o (m)		β_e (m)	
14	2.76	4.12	7.20	7.09
16	1.86	3.59	8.43	7.00
18	0.51	3.11	24.60	7.05

(4) These values indicate that β at the ends of the sector

$$\beta_e = \beta_0 + \frac{(3.5\,m)^2}{\beta_0}$$

could be made $\sim 7\,m$ in both planes. Assuming

$$\epsilon_n(95\%) \sim 7\pi \text{ mm-mrad}$$

$$\epsilon(95\% \text{ at } \beta\gamma = 0.5) \sim 14\pi \text{ mm-mrad}$$

and the largest beam radius (at 116 MeV) is

$$r \sim \sqrt{\beta_e \epsilon/\pi} = \sqrt{7\times 14}\ \text{mm} \cong 10\ \text{mm}$$

which fits in the bore radius of 15 mm. Of course at the top-energy end $\beta\gamma \cong 1$ (at 400 MeV) and

$$\epsilon(95\% \text{ at } \beta\gamma = 1) \sim 7\pi \text{ mm-mrad}$$

$$r = \sqrt{7\times 7}\ \text{mm} \cong 7\ \text{mm}$$

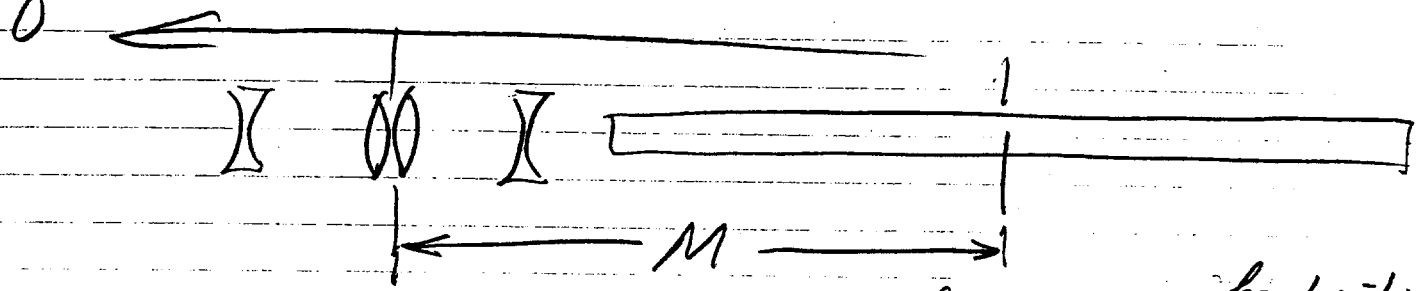
(5) This only shows that the parameters are reasonable. Detailed computer calculation is required. $k = 1\,m^{-1}$ is too strong. Should probably be reduced to ~ 0.7 so that we can use longer l (~ 18 cm) and longer L (4.5 – 5 m).

Table of the Energy Parameters for the 7 Sectors (from table on p. 16)

L_i (m)	T (MeV)	$\beta\gamma$	β	$B\rho$ (kGm)
DTL	116.54	0.5137	0.4569	16.08
5.394	151.4	0.5906	0.5085	18.48
5.938	188.8	0.6655	0.5540	20.83
6.447	228.3	0.7388	0.5942	23.12
6.838	269.4	0.8104	0.6296	25.36
7.210	311.7	0.8802	0.6607	27.55
7.537	355.1	0.9488	0.6883	29.69
7.829	399.5	1.0163	0.7128	31.81

Alignment Sensitivity & Tolerance

The most sensitive is the double F quad in the middle



β_F $\quad \mu = 0.946$ $\quad \sin\mu = 0.811$ $\quad \beta_0 = 3$, $\alpha_0 = 0$, $\gamma_0 = \frac{1}{3}$ $\quad \left(k = 1\,m^{-1},\ l = 0.14\,m,\ L = 4\,m\right)$

$$M = \begin{pmatrix} 1+kl & l \\ -k^2 l & 1-kl \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+kl & l+L(1+kl) \\ -k^2 l & 1-kl-k^2 lL \end{pmatrix}$$

$$\beta_F = (1+kl)^2 \beta_0 + \left[l + L(1+kl)\right]^2 \frac{1}{\beta_0} = 11.3\ m$$

Thus

$$\Delta x_0 = \left(\sqrt{\beta_F \beta_0}\ \sin\mu_{F0}\right) \times 2k \times (\delta x)_F$$

$$= \left(\sqrt{11.3 \times 3}\ \sin 0.811\right) \times 2 \times (\delta x)_F$$

$$= \underline{8.44}\,(\delta x)_F$$

So if $(\delta x)_F = 0.1\ mm$ $\quad \Delta x_0 = 0.8\ mm$