To Get Rid of the Bridge Couplers

To Include the RF Defocusing

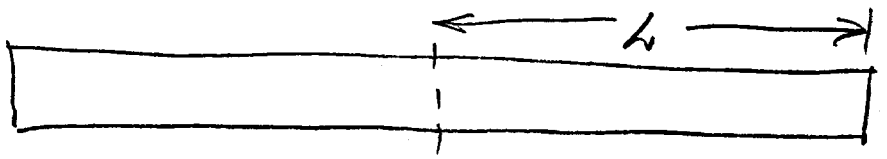
KOT 2/7/89

According to G. Mills

$$k_{rf} = \frac{\pi e T E_0 \cos\phi_s}{\lambda mc^2 (\beta\gamma)^3} \simeq 0.2\ m^{-2} \quad \text{at } 116\,\text{MeV}$$

$(\beta\gamma = 0.572)$

At other energies it scales as $\frac{1}{(\beta\gamma)^3}$. Then inside the accelerating section



$\alpha = 0$ $\qquad$ $\alpha = \alpha_e$

$\beta = \beta_0$ $\qquad$ $\beta = \beta_e$

Transfer matrix across L is

$$M_{\frac{1}{2}} = \begin{pmatrix} \cosh\Phi & \frac{L}{\Phi}\sinh\Phi \\ \frac{\Phi}{L}\sinh\Phi & \cosh\Phi \end{pmatrix} \qquad \Phi \equiv \sqrt{k_{rf}}\, L$$

$$\equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$\beta_e = a^2\beta_0 + \frac{b^2}{\beta_0}$ $\qquad$ min. β_e given by $\frac{d\beta_e}{d\beta_0} = a^2 - \frac{b^2}{\beta_0^2} = 0$

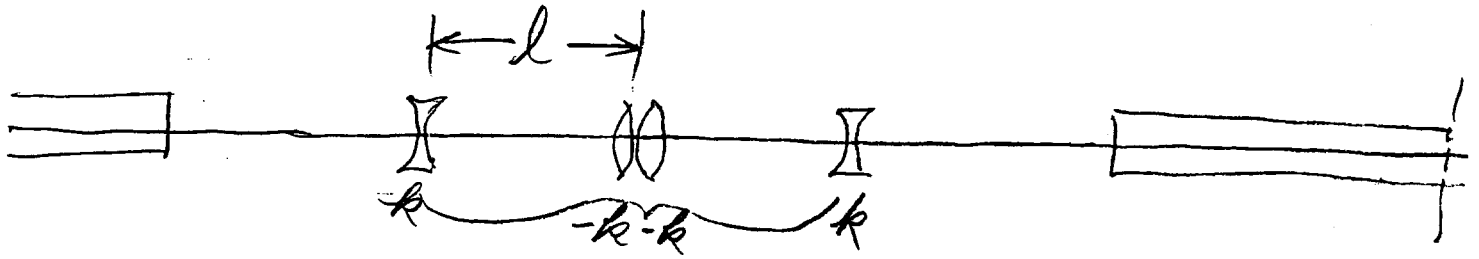
This gives

$$\begin{cases} \beta_0 = \frac{b}{a} = \frac{L}{\Phi}\tanh\Phi \\ \beta_e = 2ab = \frac{L}{\Phi}\sinh 2\Phi \\ -\alpha_e = ac\beta_0 + \frac{bd}{\beta_0} = 2\cosh^2\Phi - 1 = \cosh 2\Phi \end{cases}$$

Use $\langle\sqrt{k_{rf}}\rangle$ to calculate Φ we get the following Table

T (MeV)	L (m)	K_{rf} (m^{-2})	α	β (m)	$\langle\beta_0\rangle$ (m)
116.54		0.199	4.667	10.741	
	2.625				
133.5		0.1603	0	1.9006, 2.0568	1.9787
	2.769				
151.4		0.1309	-4.187	10.668	
151.4		0.1309	3.797	10.591	
	2.905				
169.7		0.1089	0	2.2076, 2.3523	2.2800
	3.033				
188.8		0.0915	-3.478	10.534	
188.8		0.0915	3.207	10.478	
	3.152				
208.2		0.0779	0	2.4907, 2.6234	2.5571
	3.265				
228.3		0.06688	-2.980	10.441	
228.3		0.06688	2.784	10.405	
	3.370				
248.6		0.05801	0	2.7496, 2.8692	2.8094
	3.468				
269.4		0.05068	-2.616	10.374	
269.4		0.05068	2.471	10.356	
	3.561				
290.3		0.04465	0	2.9834, 3.0922	3.0378
	3.649				
311.7		0.03955	-2.345	10.345	
311.7		0.03955	2.233	10.329	
	3.730				
333.2		0.03526	0	3.1946, 3.2922	3.2434
	3.807				
355.1		0.03158	-2.135	10.320	
355.1		0.03158	2.047	10.315	
	3.880				
377.2		0.02842	0	3.3854, 3.4739	3.4297
	3.949				
399.5		0.02567	-1.969	10.313	

Triplet Match between Sectors



$$M = \begin{pmatrix} 1-2k^2l^2 & \frac{2}{k}kl(1-kl) \\ -2k\,kl(1+kl) & 1-2k^2l^2 \end{pmatrix} \equiv \begin{pmatrix} a & b \\ c & a \end{pmatrix}$$

$$\equiv \begin{pmatrix} 1-2x^2 & \frac{2}{k}x(1-x) \\ -2k\,x(1+x) & 1-2x^2 \end{pmatrix} \qquad x \equiv kl$$

$$\begin{cases} \beta = a^2\beta_0 + 2ab(-\alpha_0) + b^2\gamma_0 \\ -\alpha = ac\beta_0 + (a^2+bc)(-\alpha_0) + ba\gamma_0 \\ \gamma = c^2\beta_0 + 2ca(-\alpha_0) + a^2\gamma_0 \end{cases} \quad \text{we want (approx.)} \quad \begin{cases} \beta = \beta_0 \\ \alpha = -\alpha_0 \\ \gamma = \gamma_0 \end{cases}$$

$$\begin{cases} (a^2-1)\beta_0 - 2ab\alpha_0 + b^2\gamma_0 = 0 \\ ac\beta_0 - 2a^2\alpha_0 + ab\gamma_0 = 0 \\ c^2\beta_0 - 2ca\alpha_0 + (a^2-1)\gamma_0 = 0 \end{cases} \qquad \begin{matrix} c\beta_0 - 2a\alpha_0 + b\gamma_0 = 0 \\ c\beta_0 - 2a\alpha_0 + b\gamma_0 = 0 \\ c\beta_0 - 2a\alpha_0 + b\gamma_0 = 0 \end{matrix}$$

Same eqn.

$$\cancel{2}k\beta_0 x(\underline{1}+\underline{x}) + \cancel{2}\alpha_0(1-2\underline{x}^2) - \cancel{2}\frac{\gamma_0}{k}x(\underline{1}-\underline{x}) = 0$$

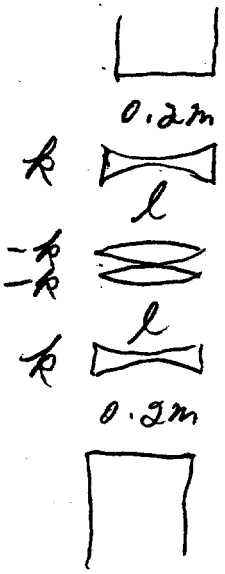
$$\left(k\beta_0 - 2\alpha_0 + \frac{\gamma_0}{k}\right)x^2 + \left(k\beta_0 - \frac{\gamma_0}{k}\right)x + \alpha_0 = 0$$

With all length in units of $\frac{1}{k}$ we can write

$$\boxed{(\beta_0 - 2\alpha_0 + \gamma_0)l^2 + (\beta_0 - \gamma_0)l + \alpha_0 = 0}$$

Solve for l

Take the middle (☆) one



	α	β	γ
	−2.616	10.374	0.75607
	−2.767	11.457	0.75607
	(−2.7)	(11.4)	(0.727)
	(2.7)	(11.4)	(0.727)
	2.608	11.372	0.68616
	2.471	10.356	0.68616

Equ. for l is $(\beta_0 - 2\alpha_0 + \gamma_0)l^2 + (\beta_0 - \gamma_0)l + \alpha_0 = 0$

all lengths in units of $\frac{1}{k}$

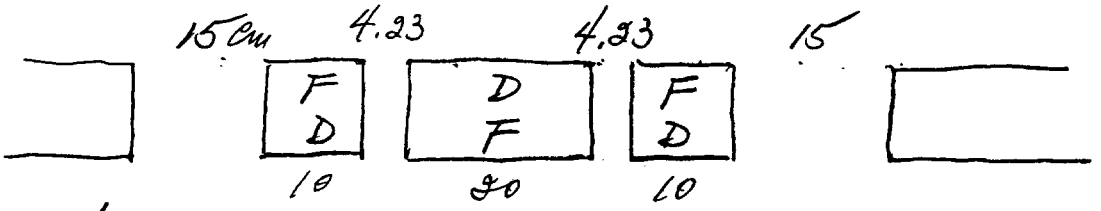
Case 1 With $k = 1\ m^{-1}$

$$(11.4 + 5.4 + 0.727)l^2 + 10.673l - 2.7 = 0$$

$$17.53l^2 + 10.67l - 2.7 = 0$$

$$l = \frac{-10.67 \pm \sqrt{10.67^2 + 4 \times 2.7 \times 17.53}}{2 \times 17.53} = 0.1923\ m$$

So it looks like



And for k we have ($l_Q = 10\ cm = 0.1\ m$)

$$k = \frac{B'l_Q}{B\rho} \qquad B' = \frac{k}{l_Q} B\rho = (10k) B\rho$$

$B\rho = 16.0\ kGm$ — $31.8\ kGm$

for $k = 1\ m^{-1}$

we have $B' = 1.6\ kG/cm$ — $3.2\ kG/cm$ (O.K.)

<u>Case 2</u> with $k = 0.5\ m^{-1}$

$$\left(2\times 11.4 + 5.4 + \frac{0.72}{2}\right)l^2 + \left(2\times 11.4 - \frac{0.72}{2}\right)l - 2.7 = 0$$

$$28.56\,l^2 + 22.44\,l - 2.7 = 0$$

$$l = \frac{-22.44 \pm \sqrt{22.44^2 + 4\times 2.7\times 28.56}}{2\times 28.56} = 0.106\ \frac{1}{k}$$

$$= 21.2\ cm$$

(l did not change much ?)

Now the range of B' required is only

$B' = 0.8\ kG/cm$ — $1.6\ kG/cm$ (easy)

(116 MeV) (400 MeV)