Magnetic Calculations for Transmission-Line Magnet and Current Transformer Test Setup

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1. INTRODUCTION

This note describes 2-D analytic and POISSON calculations for the prototype test of the "Double-C Transmission Line" magnet (see fig. 1). Magnetic and electrical parameters, stray fields, stored energy, and behavior under fault conditions are discussed.

The plan is to drive a 60kA current through a superconducting loop by connecting it as a 1-turn shorted secondary winding of a current transformer. This procedure avoids high current superconducting power leads and limits the energy that can be transferred to or from the magnet under fault conditions. In initial running the maximum current achievable in the secondary is expected to be ~30kA, limited by the size of the iron yoke of the transformer. In the future a larger yoke (or reversing switch on the power supply) will allow a maximum current of 60kA, limited by the number of ampere-turns available from the primary windings and power supply. Calculations are presented here for both the initial (30kA) and the full 60KA design current.

The load magnet is a "double-C" solid iron yoke ~1m (40") long placed on the superconducting secondary winding. The load magnet has two 1/2" gaps and therefore an anticipated transfer function of 1Tesla/20kA. The total load inductance of 10uH includes 4uH from the load magnet plus ~6uH from stray inductance on the secondary winding. These numbers have been calculated (Sect. 10) with POISSON and checked where possible with exact analytic solutions for geometries similar to the prototype setup.

The transformer yoke is the flux return iron from the Main Injector Lambertson Prototype. The transformer core has an iron cross-section of 0.13 m^2 and therefore provides a maximum of 0.3 volt-seconds of excitation to the secondary winding for a 2.2 Tesla flux swing of the core. Thus the maximum current which can be generated in the secondary is (0.3volt-seconds) / 10uH = 30kA. The maximum total stored energy in the initial configuration is $1/2\ LI^2$ = 4.5kJ at 30kA. For running in the final (60kA) configuration the stored energy increases less than quadratically because of iron saturation and is in the range of 10kJ.

The primary windings for the transformer are the same 24 turn x 2.5kA water cooled copper coils originally used for the Lambertson magnet. The current in the primary will be limited to 2.5kA by the DC resistance of the coils and the voltage tap of the power supply.

The superconducting loop secondary consists of 7 turns of SSC cable which are looped through the cryostat, then spliced to itself. The entire secondary is sheathed in a double-shell of grounded stainless steel pipe. The coil is electrically floating except for instrumentation leads. Anticipated voltages during quench (sect. 4) are in the range of a few volts.

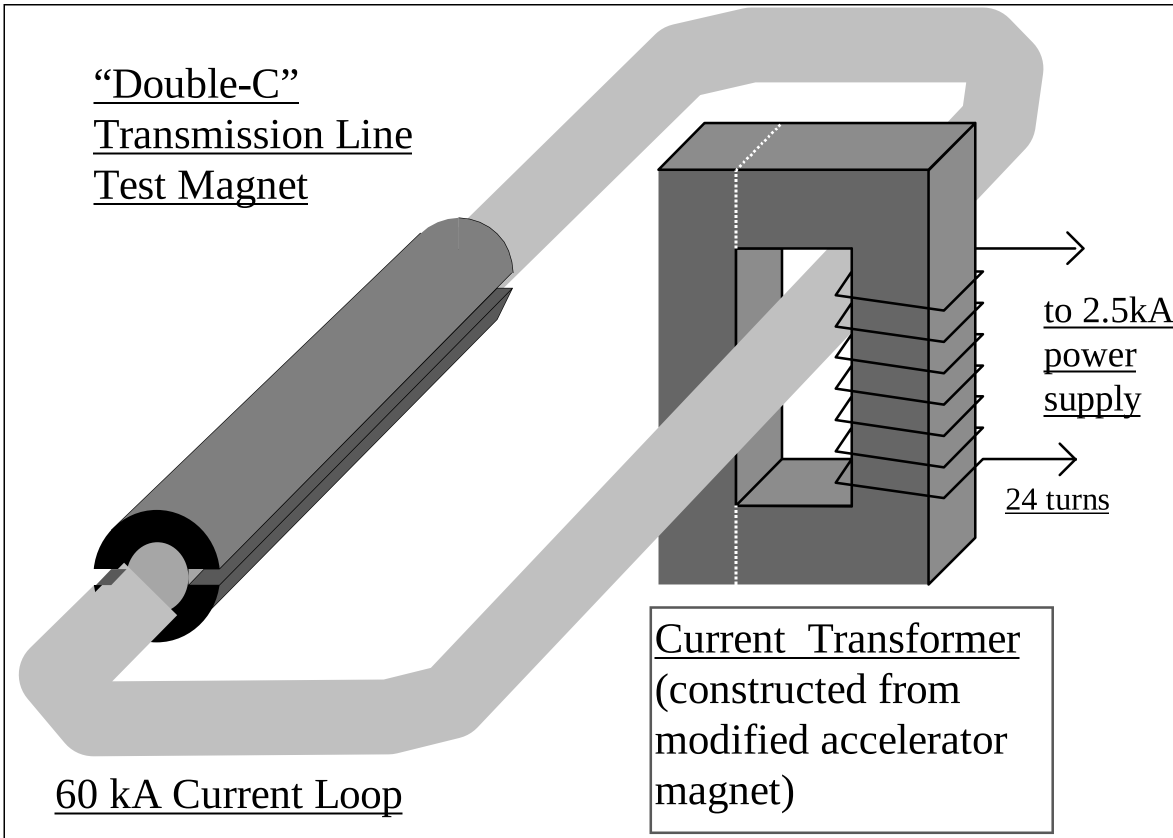


Fig. 1. - Schematic drawing of prototype test setup. A superconducting current loop consisting of 7 turns of SSC cable is placed in a loop cryostat ~5m long. A maximum current of 60kA will be driven through the 1m long “Double-C” test magnet by coupling to a copper-driven transformer yoke. Under ideal circumstances, the number of ampere-turns in the primary and secondary windings are equal, so that 24 turns of a primary winding carrying 2.5kA are sufficient. Also under ideal circumstances, the total flux enclosed in the superconducting current loop is conserved, so that the total flux circulating in the transformer yoke should be equal to the total flux circulating in the “Double-C” test magnet. In reality a significant additional flux circulates in tight loops around the drive conductor, creating stray load inductance which increases the amount of iron (but not the number of ampere-turns) required in the drive transformer. During initial running the amount of iron in the transformer yoke will limit the secondary current to ~30kA.

2. CIRCUIT BEHAVIOR UNDER NORMAL OPERATION

Under normal conditions (i.e. superconductor not quenched, iron yoke not saturated) the power supply will see the DC resistance from the copper primary coil of 0.006Ω and the load inductance through the (square of the turns ratio) of the transformer, for an effective load inductance of $10\mu H*24^2 = 5.7mH$. These parameters are comparable to those of the magnet yoke as it was previously operated at MTF as part of the Main Injector Prototype Lambertson. The PEI power supply has a freewheeling diode which protects against misbehaviors of inductive loads when the power supply trips off, etc.

Table 1: Nominal Operating Parameters

MAGNET PARAMETERS:	ORIGINAL MAIN INJECTOR 30 IN. PROTO LAMBERTSON	INITIAL (30kA) PIPETRON PROTO DRIVE XFORMER	FINAL (60kA) PIPETRON PROTO DRIVE XFORMER
Primary Current	2500 Amps	1250 Amps	2500 Amps
Secondary Current	----	30 kA	60kA
Magnet Resistance	6.01 mΩ	6.01 mΩ	6.01 mΩ
Volts @ terminals	15.03 volts	7.51 volts	15.03 volts
Water Temperature Rise (Based on 4.25 gpm flow rate)	22 °C	5.5 °C	22 °C
Magnet Inductance (as seen from power supply)	1.32 mH	5.7mH	~3m (iron saturates)
Total Stored Energy	4.2 kJoules	4.5 kJoules	~10 kJoules

3. CIRCUIT BEHAVIOR WHEN YOKE SATURATES

The iron yoke will saturate under two conditions. Firstly, the yoke will saturate whenever the magnet quenches or is powered up with the secondary warm (non-superconducting). Secondly, in the initial configuration the yoke will saturate whenever the primary drive current exceeds ~1300A (corresponding to 30kA secondary current). Under either condition the saturation will show up as a drop in the magnet inductance and a stored energy which is smaller than predicted by the low-field inductance (see table above). In all cases the current will be limited by the DC resistance of the primary coil and the power supply output voltage. Thus the result of yoke saturation will be the incomplete transfer of energy from primary to secondary, with no safety implication for personnel or equipment.

When the yoke saturates the stray fields in the vicinity of the drive transformer will increase due to the incomplete shielding of the currents inside the transformer by the saturated iron. The fields should still be smaller than those from the completely unshielded sections of the transmission line. The stray fields under saturated-yoke conditions will be measured prior to cool-down by powering the drive transformer to full current.

4. QUENCH TRANSIENT BEHAVIOR

Sacha Zlobin has carried out a rough calculation of the quench behavior of the conductor in the prototype (included in the safety documentation). His conclusion is that the final temperature of the hot spot on the conductor following a quench is less than 150°K, indicating that the installed quench protection system can fail entirely without any damage to the cryogenic system. This section discusses the electrical transients which might accompany a quench.

If the superconductor quenches at full current, a resistance will "instantaneously" appear in the secondary. This resistance varies with time due to quench propagation but is of order 100μΩ for the 1-turn secondary. Thus the resistive section will develop approximately 60kA*100μΩ = 6v. If nothing else changes then the current in the secondary (and the transient voltage from the quench) will die away with a time constant of L/R=10uH/100μΩ=0.1 seconds.

If the power supply were a perfect current source with large voltage compliance, then the 24:1 turns ratio of the transformer could in principle induce a transient voltage excursion of 6v.*24 = 144v. at power supply leads. In practice this will not happen since the power supply is tapped in such a way that it cannot source more than ~15v, and most of that 15v. will be used up driving the resistance of the primary. From the power supply point of view, the situation is as if an extra resistance suddenly appears in the load while driving a resistive magnet. The value of the resistance which appears is the quench resistance (100 μΩ) times the square of the turns ratio (24^2) or 58mΩ. Since this is larger than the DC resistance of the primary coil (6mΩ) that the power supply can barely drive at full output voltage, the power supply saturates at the full output voltage and the current starts dropping. Thus, when an quench occurs the power supply will immediately go to its maximum voltage of 15V and remain there until the current in the secondary has died away. There are no anomalous voltage or current demands on the power supply during the quench.

5. EDDY CURRENTS IN STAINLESS PIPES DURING QUENCH

The entire superconducting loop is enclosed in a grounded double shell of stainless steel pipe. Therefore this shell also loops the transformer yoke, and flux changes in the core will induce eddy currents in the stainless pipes. A safe upper limit to the voltages and currents which can be induced in the stainless steel transmission line piping is obtained by assuming that the full 0.3 volt-seconds of flux available in the drive transformer collapses in 0.1 seconds during a quench. This generates 3 volts on any conductor looping the transformer core. The shell resistance (assuming a 30uΩ-cm stainless steel cross section of 3cm^2 and a 9m effective length) is 9mΩ. Therefore the maximum current in the stainless shell is 6V/9mΩ = 670A. The total power dissipated is 670A * 3V * 0.1 Sec = 200 J. Thus we conclude that eddy currents in the stainless shell during a quench are unimportant.

6. TRANSIENT BEHAVIOR DURING POWER SUPPLY TRIP

If the power supply trips off at full current, the situation is identical to what happens with a conventional resistive magnet load. The effective load inductance seen by the power supply (5.7mH, see table 1) attempts to keep the current flowing through the supply, which results in a rapid negative excursion of the power supply. However this negative excursion is clamped at -1.5V by freewheeling protection diodes on the PEI supply. Following the supply trip the current will decay with an L/R time constant of 5.7mH / 6mΩ = 1 second. (The actual decay will be slightly faster due to the voltage drop of the freewheeling diode).

The voltage excursions on the secondary during a supply trip will be lower than the voltage transients on the primary side by the 24:1 turns ratio, i.e. the 15v. → -1.5v excursion on the primary winding will correspond to a 0.625v. → -0.06v excursion on the load magnet.

7. INDUCTANCE AND STORED ENERGY

The calculation of the stored energy treats the prototype setup by breaking it down the into a number of 2 dimensional (2-D) POISSON calculations. Each calculation represents a 2-D cross-section of part of the prototype apparatus. Most of these cases can be checked via analytic solutions which approximate the real geometry. This allows us to calculate the conductor forces, total stored energies, stray and mutual inductances, etc. on a per-meter basis for each case. The stored energies and inductances are then summed over the various cases weighted by their lengths in the actual setup. This represents an exact solution in the limit that each section of the prototype setup was infinitely long. This approach tends to overestimate the stored energies and inductances of the actual prototype system, since the actual 3-D fields will use the 3rd dimension to spread out and lower the stored energy for a given circulating current.

The inductance is obtained from the stored energy for a given current via $E_{STORED} = 1/2LI^2$. Since the coupling coefficient for an unsaturated iron-core transformer is essentially 100%, the load inductance can be referred to either the primary or the secondary via the square of the turns ratio of the transformer. When the transformer core and/or the load magnet saturates, the stored energy drops below the low-field extrapolation.

The table below summarizes the 2-D stored energies and effective lengths used to calculate the stored energy and inductance of each section of the prototype. The individual calculations are described in sect. 10. (There are pictures & field maps).

Table 2: Contributions to stored energy and inductance of the prototype.

2-D Cross Section	L(μH/m)	Effective Length	L_{TOT}	E_{STORED} (@30kA)
C-Magnet + Return	4uH/m	1m	4uH	1.8kJ
Drive Transformer + Return Bus	2.6uH/m	0.75m	2uH	0.9 kJ
Two Bare Conductors	1.4uH/m	2.2m	4uH	1.8 kJ
		Total	10uH	4.5kJ

The magnitude of the stray inductance calculated for the prototype is disappointingly large compared to the initial estimate. The main reason for this is that the original estimate was based on a 1" diameter drive conductor with no flux penetration inside the drive conductor. The actual conductor is roughly 0.5" square and significant flux penetrates inside the cable. Thus a large amount of stray flux circulates in tight loops around the drive conductor and current return. This stray inductance increases the amount of iron required in the drive transformer, since the total flux circulating "upwards" through the drive transformer must equal the flux circulating "downwards" through the (double-C load magnet + leakage inductance). As a result, the transformer yoke is expected to drive only about 30kA of current before it saturates rather than the 50-60kA originally planned. Two remedies which would enable the full 60kA to be generated are to double the size of the iron yoke (which is mechanically possible), or to install a reversing switch on the power supply (which would provide a 4Tesla flux swing in the drive transformer rather than a 2T swing). In the mean time it provides an "fail-safe" current limit of ~30kA in the secondary winding.

The leakage inductance does not change the number of ampere-turns required to drive the magnet.

8. STRAY MAGNETIC FIELDS

This stray magnetic fields from the prototype will me measured in situ, at low current then full current, and with the secondary warm then superconducting. The expected fields can be estimated in two approximations.

1) At distances large compared to the dimensions of the prototype setup, the field is only a function of the total magnetic moment of the system (which is dominated by the current loop of the superconducting secondary). The accuracy of the far-field dipole approximation will be limited by the presence of large iron objects, etc. at MTF but provides an order-of magnitude estimate for the stray field on the other side of the building.

2) At distances small enough that the transmission line loop can be considered "infinitely long", the 2-dimensional solutions from POISSON or simple analytic calculations provide good estimates. These should provide good predictions of the Hall probe measurements within a foot or two of the conductors, and a reasonable guess for the stray field standing in the aisle ~1m from the test setup.

8.1. Stray Field Far Away: dipole field far from a 50kA x 3m^2 current loop.

This gives a zero-order estimate for the stray fields far from the test setup, ignoring the effects of nearby steel objects. The magnetic moment **m** of the loop [Lorrain & Corson p.321] is:

$\mathbf{m}$ = (Current)x(Area) = (50E3 Amps)*3m^2 = 1.5E5 Amp m^2.

The nonzero field components $\mathbf{B_r}$ and $\mathbf{B_\theta}$ are:

$\mathbf{B_r} = (\mu_0/4\pi)\, 2\mathbf{m}/r^3 \cos\theta$, and

$\mathbf{B_\theta} = (\mu_0/4\pi)\, \mathbf{m}/r^3 \sin\theta$, where r,θ are the usual polar coordinates.

The field is maximum on axis (perpendicular to the current loop)

$\mathbf{B_{MAX}} = (\mu_0/4\pi)\ 2\mathbf{m}/r^3$ = 1E-7 x 2 x 1.5E5 / = 300Gauss/ r^3

= **2.4 Gauss @5m**, and decreasing as 1/ r^3.

The field at the "equator" is 2x smaller and also decreases as 1/ r^3. As mentioned above, the actual field at remote parts of MTF can be either shielded or concentrated by the presence of nearby iron structures.

8.2. Stray Field Nearby: Two infinite parallel wires carrying opposite currents.

This gives an estimate for the stray field, conductor forces, and stray inductance in the region near and between the wires but away from the iron yokes and only at distances small compared to the length of the setup. It also gives an estimate for the inductive stored energy of the system excluding the magnet gaps and the stored energy in the iron of the transformer yoke.

The magnetic field from one infinitely long conductor in free space is:

B(Tesla) = $\mu_0 I/2\pi R = (4\pi \times 10\text{-}7)\ I/2\pi R$

= 0.01T (100 Gauss) at 50kA at a distance of 1m.

The magnetic field of *two* conductors (carrying opposite currents in the Z-direction) and located at $Y = \pm\Delta$ [Lorrain & Corson P.307] is largest on the midplane (Y=0) and is:

$B_x(\text{midplane}) = (\mu_0 I/\pi)*\Delta/(x^2+\Delta^2)$ for our case Δ=0.3m, I=50kA, so:

= 666 Gauss / $(1 + (x/0.3)^2)$

Thus, standing in the aisle (x=1m, y=0) your credit card will see 55 Gauss with 50kA in the superconducting loop.

9. FORCES ON CONDUCTORS

The transverse forces on the superconductors can be estimated from the same POISSON or analytical 2-D calculations used to obtain the inductance and stored energy. This piecewise 2-D approach does *not* necessarily underestimate the forces on the superconductor, since in some cases (e.g. the superconductor inside the double-C magnet or the transformer yoke) these depend on canceling of forces due to symmetries which may not be present in the full 3-D situation.

Forces on two unshielded conductors:

The simple analytical case of two parallel wires in sect. 8 applies. The second conductor is located at a distance of 0.6m (24in) and feels a force:

F(Newton's/meter) = B(Tesla)*I(Amperes) = 0.01T / (0.6m) * 50kA
= 300 Newtons/m
= 30 kg/m @50kA.

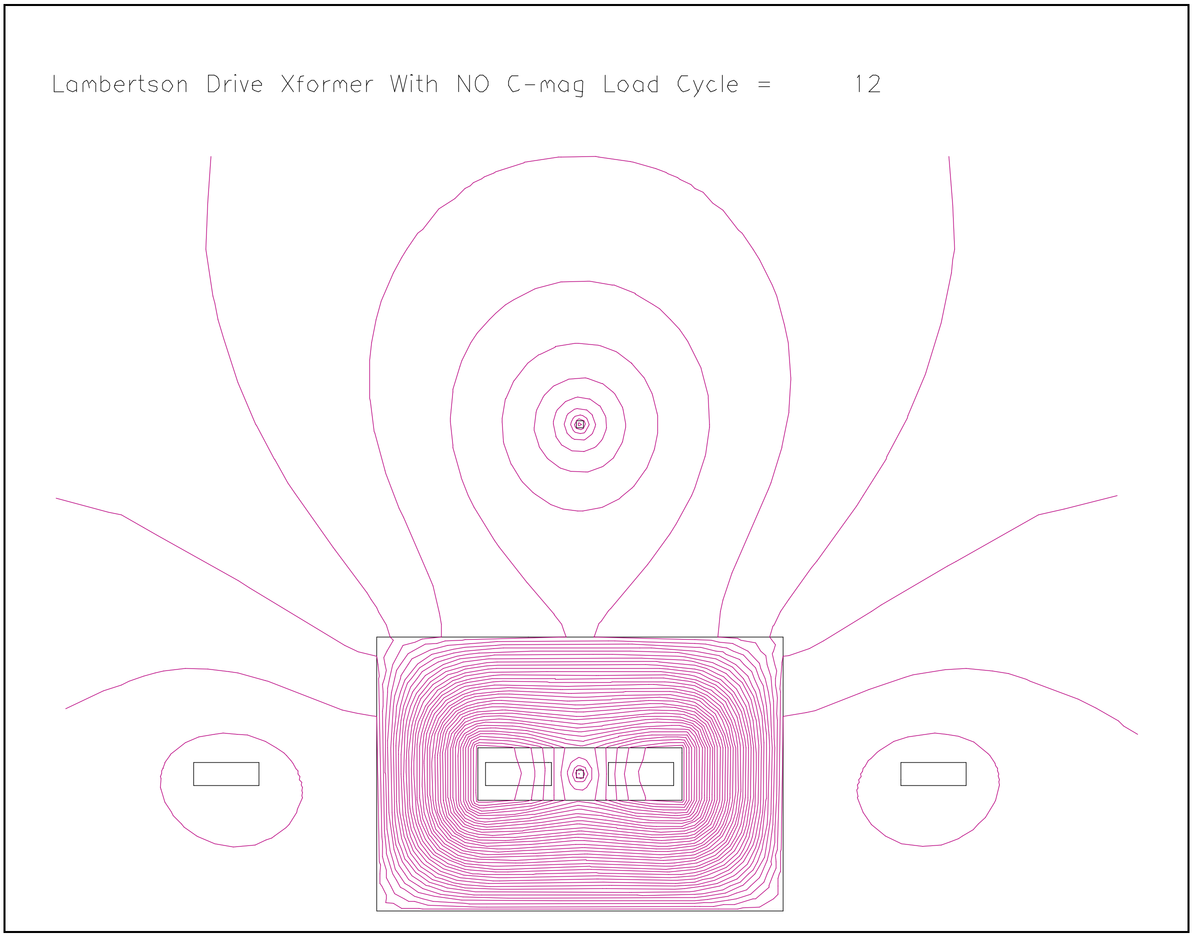
Thus, insulating "spiders" spaced at 0.3m intervals in an unshielded region of the transmission line loop see a side force of ~9kg each. If this is determined to be a problem for the transmission line, it is possible to null this force out by placing a small iron shield plate near the surface of the vacuum jacket of each conductor, so that the attractive force that each conductor feels for the iron plate cancels the repulsive force it feels for the opposite conductor.

Forces on Shielded Conductors:

The superconductors inside the double-C magnet and the drive transformer see less force, due to the shielding effect of the iron and the symmetry of their positions in the iron structures. This force has a large gradient however, and the conductor experiences a large decentering force (a "negative spring constant") which will be investigated in the prototype and discussed in a separate paper. It does not represent a safety issue, since the worst-case behavior of the conductor pipe from these forces is have the cold pipe move ~1" transversely (to get as close as possible to the nearby iron) and rip through the cold mass support spiders. The modulus of the support spiders is believed to be sufficient to resist such behavior.

10. SUMMARY OF POISSON & ANALYTIC CALCULATIONS

10.1. Drive Transformer with Copper Primary, SC secondary & Current return.



Primary Current: 2.5kA*24turns = 60kA-turns
Secondary Current: 60kA
Stored energy = 3.9216E+01 Joules/cm

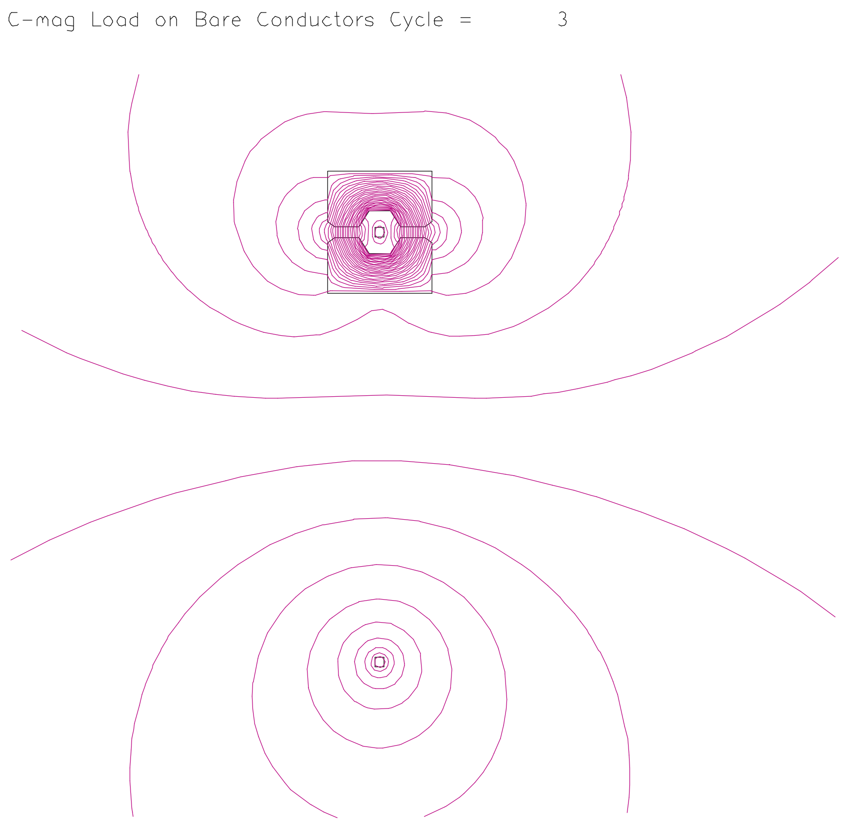
Inductance (referred to primary):

$$L_{PRIMARY} = 2E_{STORED}/I^2_{PRIMRY} = 1.2\text{mH/m}$$

Inductance (referred to secondary):

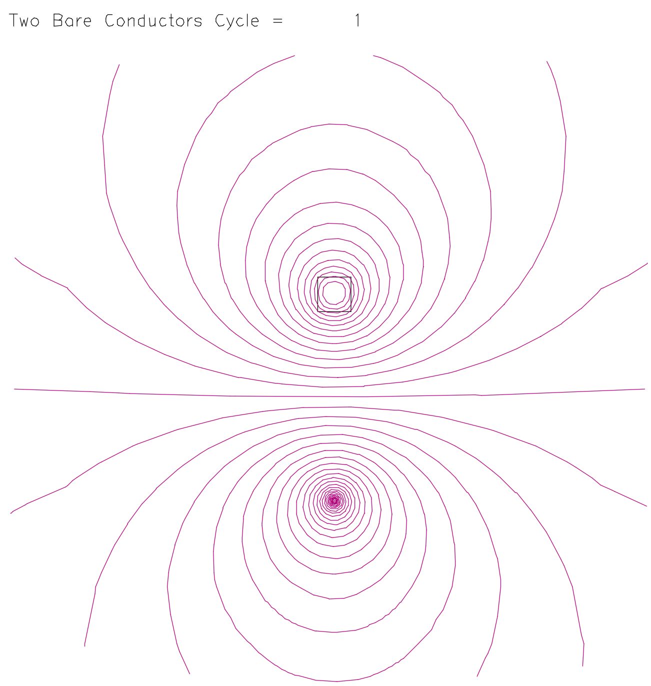
$$L_{SECONDARY} = 2E_{STORED}/I^2_{SECONDARY} = 2.2\mu\text{H/m}$$

10.2. Double-C Magnet with Current Return



Drive Current 10kA
Stored energy = 2.0615E+00 Joules/cm
Inductance = $2E_{STORED}/I^2$ = 4.1µH/m

10.3. Two Unshielded Bare Conductors



Drive Current 60kA
Stored energy = 2.5611E+01 Joules/cm
Inductance = $2E_{STORED}/I^2$ = 1.42μH/m

10.4. Analytic cross-check of the POISSON stored energy calculation

This section describes a cross-check of the "two bare conductors" case to make sure that POISSON is reporting the stored energy reasonably. The inductance of two parallel wires can be obtained from the transmission-line impedance Zo and electrical propagation time Δt of the conductor geometry via the relations:

$L = Z_0 \Delta t$ or
$L/\text{meter} = Z_0/c$.

The characteristic impedance of the two parallel wires (Rad Engr. Handbook Sect. 29-22 solution "T") is:

$Z_0 = 276 \log_{10}(2D/d)$ = 464 Ohms,

where d=1.25cm is the effective wire diameter and D=60cm is the spacing. This yields:

$L/\text{meter} = Z_0/c = 464. / 3E8 = 1.55$uH/m,

which is in reasonable agreement with the POISSON value of 1.42uH/m.