UPC 067

Fermilab

E/D Multipole Field Random Errors and Slow Extraction

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October 20, 1978

For the error multipole fields we shall take those given in Table II-4 of TM-813 (Ohnuma). We shall assume that all the average fields (systematic errors) can be trimmed out if it is necessary* and shall discuss only the effects of the rms errors (random errors). These are given below.

		Normal		Skew	
n	pole	$(\Delta B_n)_{rms}$ at 1"(G)	$(b_n)_{rms}$ (cm^{-n})	$(\Delta B_n)_{rms}$ at 1"(G)	$(a_n)_{rms}$ (cm^{-n})
1	4	2.57	0.51×10^{-4}	4.09	0.81×10^{-4}
2	6	6.83	0.53×10^{-4}	1.12	0.87×10^{-5}
3	8	1.04	0.32×10^{-5}	3.56	1.09×10^{-5}
4	10	2.42	0.29×10^{-5}	0.48	0.58×10^{-6}
5	12	0.66	0.31×10^{-6}	0.91	0.43×10^{-6}
6	14	0.78	0.15×10^{-6}	0.39	0.72×10^{-7}
7	16	1.63	0.12×10^{-6}	1.05	0.77×10^{-7}
8	18	0.88	0.25×10^{-7}	0.53	0.15×10^{-7}
9	20	-	-		
10	22	0.64	0.28×10^{-8}		

The multipole coefficients are defined as

$$b_n = \frac{1}{n!}\frac{B^{(n)}}{B_o} = \frac{\Delta B_n}{B_o}\frac{1}{r^n} \qquad \left(\text{similarly for the skew coefficient } a_n\right)$$

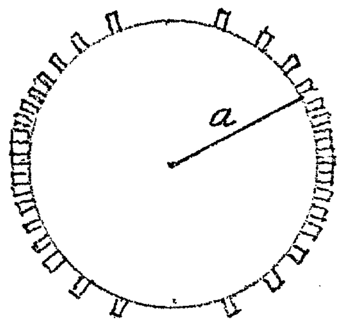
where

B_o = dipole field = 20,000 G in this case

r = radius ΔB_n is measured = 1" = 2.54 cm.

*This assumption is not realistic. The effects of systematic errors will be studied separately.

The first thing we notice is that $(\Delta B_n)_{rms}$ does not seem to go down very fast with increasing n. Errors in a superconducting magnet where the field is determined by coil placement are different in nature than those in a conventional magnet where the field is determined by the contour of the iron poles. The errors persist to very high multipole orders. This can be understood from the following simplified model. Assume that the $\cos\theta$ current for a dipole is supplied by a single layer of conductors distributed in density as $|\cos\theta|$ on a circular aperture of radius a. Most prominent field errors are caused by azimuthal displacements of the conductors which gives azimuthal δ-function error currents.



These error currents produce multipole fields of all orders with equal strength. Specifically we have

$$B^{(n)} = 2\pi I_e \frac{n!}{a^n} \qquad \text{for } 2(n+1)\text{-pole}$$

or

$$\Delta B_n = \frac{1}{n!} B^{(n)} r^n = 2\pi I_e \left(\frac{r}{a}\right)^n \equiv B_e \left(\frac{r}{a}\right)^n$$

which becomes independent of n at r = a. We can get some estimate for the parameters B_e and a from the measured data. We plot $(\Delta B_n)_{rms}$ (both normal and skew) against n on a semi-log scale. All points are fairly well contained within a band a = 3.4 cm and B_e = 2.5 to 10 G. We will use the pessimistic value of B_e = 10 G for error multipole fields to arbitrarily high n values. This gives

$$(b_n)_{rms} = (a_n)_{rms} = \frac{B_e}{B_o}\frac{1}{a^n} = \frac{5\text{x}10^{-4}}{(3.4\text{ cm})^n}\,. \tag{1}$$

During slow resonant extraction the amplitude of oscillation is blown up to at least 2 cm on the resonance in a controlled manner. Two questions must be answered:

(1) Is the tune shift due to the error fields for a 2 cm oscillation small enough that the particles are not shifted off the resonance and their oscillation amplitude will continue to grow?

(2) Are the particles with 2 cm oscillation stable relatively to the stochastic effects of the random field errors. For this we have to look at the coverage of the tune space by resonances, namely the sum of all resonance widths over a unit tune range.

A. Resonance width

The full width $\delta\nu_n$ of the n^{th}-order resonance is given approximately by

$$\delta\nu_n = \frac{(b_{n-1})_{rms}}{\sqrt{N}}\,\beta\, r^{n-2}$$

where

N = number of dipoles = 774

β = average amplitude function = 60 m

r = average excursion of particle.

Taking r = 2 cm, and r = 0.3 cm (normal beam size) the measured $(b_n)_{rms}$ gives

n	$(b_n)_{rms}$ (cm^{-n})	$\delta\nu_{n+1}$ (r=2 cm)	$\delta\nu_{n+1}$ (r=0.3 cm)
1	0.51×10^{-4}	0.011	0.011
2	0.53×10^{-4}	0.023	0.0034
3	0.32×10^{-5}	0.0028	0.62×10^{-4}
4	0.29×10^{-5}	0.0050	0.17×10^{-4}
5	0.31×10^{-6}	0.0011	0.54×10^{-6}
6	0.15×10^{-6}	0.0010	0.79×10^{-7}
7	0.12×10^{-6}	0.0017	0.19×10^{-7}
8	0.25×10^{-7}	0.00069	0.11×10^{-8}
9	-	-	
10	0.28×10^{-8}	0.00031	0.12×10^{-10}

These are all tolerable values. It would be nice to reduce $(b_2)_{rms}$ (sextupole) by a factor of 2 or more.

B. Stochasticity limit (Chirikov)

To investigate the stability against the nonlinear stochastic motion we need the sum of all resonance widths in unit tune range. For this we will use the fitted values for $(b_n)_{rms}$ as given by Eq. (1). The sum of widths is

$$W \equiv \Sigma(n-1)\delta\nu_n = \frac{\beta}{\sqrt{N}}\,\Sigma(b_{n-1})_{rms}(n-1)r^{n-2}$$

$$= \frac{1}{\sqrt{N}}\,\frac{\beta}{a}\,\frac{B_e}{B_o}\sum_{n=2}^{\infty}(n-1)\left(\frac{r}{a}\right)^{n-2}$$

$$= \frac{1}{\sqrt{N}}\,\frac{\beta}{a}\,\frac{B_e}{B_o}\,\frac{1}{(1-\frac{r}{a})^2} = \frac{0.0317}{(1-\frac{r}{3.4\text{ cm}})^2}\,.$$

The absolute limit is $W<1$, but W should be comfortably below 1. For r = 2 cm we get

$$W = 0.19$$

which is uncomfortably large. But remembering that Eq. (1) corresponds to the pessimistic value of B_e = 10 G and that B_e

may optimistically be as small as 2.5 G (W = 0.047) we consider the error fields tolerable.

C. Tune shift

The tune shift caused by even order multipole field is given by

$$|\Delta\nu_n| = \frac{n!}{2^n(\frac{n}{2}!)^2} \frac{\beta}{\sqrt{N}} (b_{n-1})_{rms} \; r^{n-2} \qquad n = \text{even}.$$

The tune shift $\Delta\nu_2$ is caused by quadrupole error and independent of r. This tune shift is irrelevant and will be tuned out in any case. For n = 4, 6 and 8 the measured $(b_{n-1})_{rms}$ give at r = 2 cm

n	$(b_{n-1})_{rms}$ $(cm^{-(n-1)})$	$\lvert\Delta\nu_n\rvert$
4	0.32×10^{-5}	0.00104
6	0.31×10^{-6}	0.00033
8	0.12×10^{-6}	0.00045 .

These are all small values, but again it is annoying to see that $|\Delta\nu_n|$ does not go down very rapidly with increasing n. So we shall again use formula (1) for all n. We will also use Stirling's approximation for factorials to get

$$\frac{n!}{2^n(\frac{n}{2}!)^2} \cong \frac{e^{-n}n^n\sqrt{2\pi n}}{2^n\left[e^{-\frac{n}{2}}(\frac{n}{2})^{\frac{n}{2}}\sqrt{\pi n}\right]^2}$$

$$= \frac{e^{-n}n^n\sqrt{2\pi n}}{2^n e^{-n}(\frac{n}{2})^n \; \pi n} = \sqrt{\frac{2}{\pi n}} \; .$$

Altogether we have

$$|\Delta\nu_n| \cong \sqrt{\frac{2}{\pi n}}\, \frac{\beta}{a}\, \frac{B_e}{B_o}\, \frac{1}{\sqrt{n}}\, \left(\frac{r}{a}\right)^{n-2}.$$

Since $\Delta\nu_n$ for different values of n are uncorrelated, we should add them in guadrature.

$$\Delta\nu \equiv \sqrt{\sum_{n=4} (\Delta\nu_n)^2} = \sqrt{\frac{2}{\pi N}}\, \frac{\beta}{a}\, \frac{B_e}{B_o} \left[\sum_{n=4,6,8}^{\infty} \frac{1}{n} \left(\frac{r}{a}\right)^{2n-4} \right]^{\frac{1}{2}}$$

$$= \frac{1}{\sqrt{\pi N}}\, \frac{\beta}{a}\, \frac{B_e}{B_o} \left[\frac{1}{\left(\frac{r}{a}\right)^4}\, \ell n\, \frac{1}{1-\left(\frac{r}{a}\right)^4} - 1 \right]^{\frac{1}{2}}$$

$$= 0.0179 \left[\frac{1}{\left(\frac{r}{a}\right)^4}\, \ell n\, \frac{1}{1-\left(\frac{r}{4}\right)^4} - 1 \right]^{\frac{1}{2}}.$$

With $\frac{r}{a} = \frac{2\ \text{cm}}{3.4\ \text{cm}} = 0.588$ we get

$$\Delta\nu = 0.0046.$$

This should be smaller than the tune change $\Delta\nu_3$ introduced by the resonance exciting sextupole also at r = 2 cm. The sextupole strength required to give a step-size of 0.5 cm at 2 cm is $\frac{B''\ell}{B\rho}$ = 0.34 m^{-2}. Hence

$$\Delta\nu_3 = \frac{\beta}{16\ \pi}\ \frac{B''\ell}{B\rho}\ r = 0.0081.$$

Although $\Delta\nu$ is not very much smaller than $\Delta\nu_3$, but since $\Delta\nu$ is based on the most pessimistic fitted curve (1) and could optimistically only be $\frac{1}{4}$ x 0.0046 = 0.00115 we consider this value acceptable.

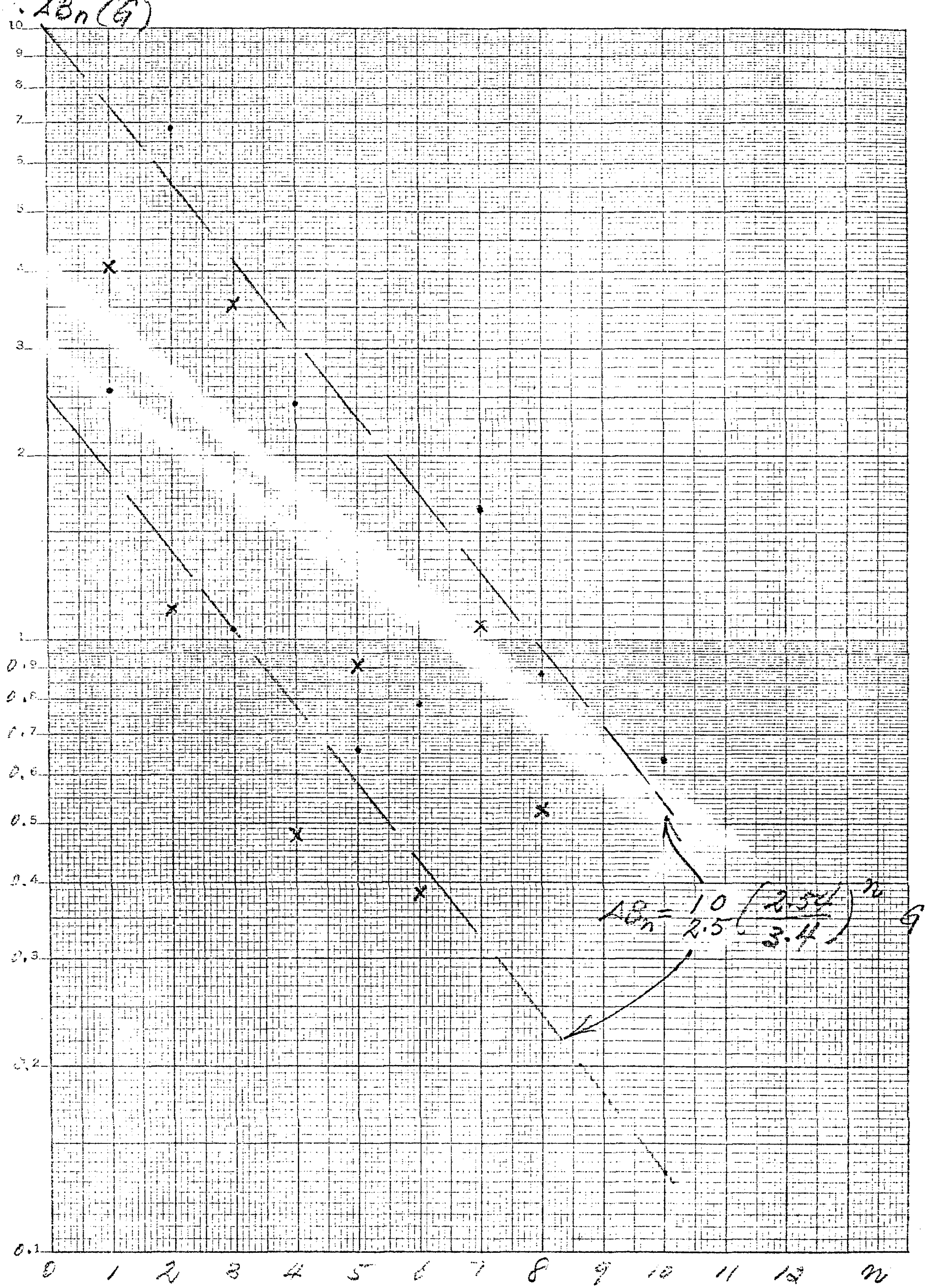
We conclude, therefore, that the measured random multipole field errors are sufficiently small to permit proper resonant slow extraction if the septum is no more than 2 cm from the central orbit and provided that systematic field errors are properly corrected. (See footnote on page 1.) Corrections of systematic field errors so far discussed include:

1. Closed-orbit distortions are eliminated by using correction dipoles.

2. Tune values can be controlled by trim quadrupoles.

3. Tune spread in the beam can be adjusted or eliminated by chromaticity sextupoles.

We shall see in the separate study of the effects of systematic errors that these corrections alone are not adequate and that corrections up to much higher order systematic multipole errors are needed.


ΔBn (G)
10
9
8
7
6
5
4
3
2
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
7
8
9
10
11
12
n
$\Delta B_n = \frac{10}{2.5}\left(\frac{2.54}{3.4}\right)^n$ G