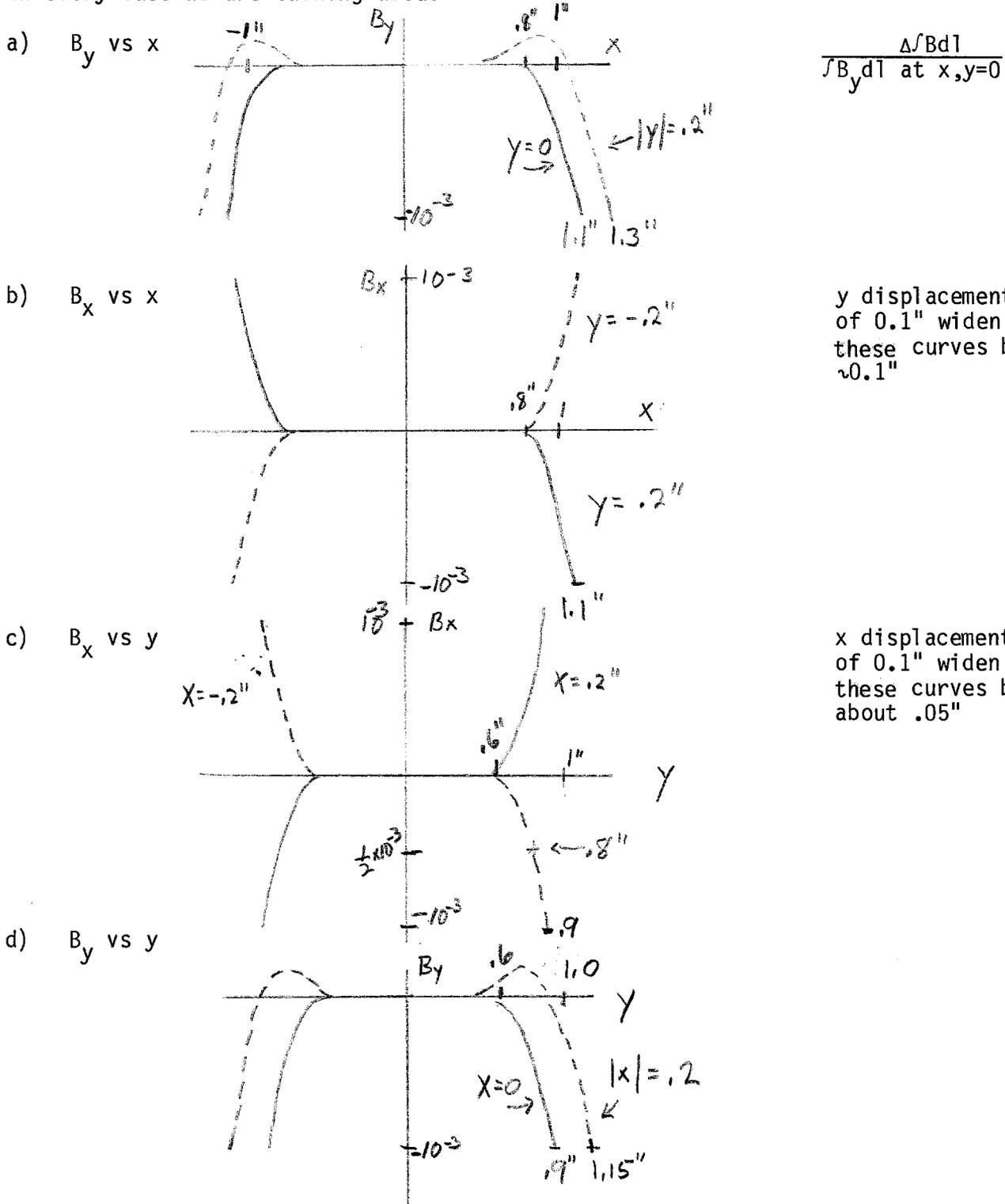


H. T. Edwards/M. Harrison

IX. GOOD FIELD REGION OF THE DESIGN BEND MAGNET
AND EXPECTED BEHAVIOR OF EXTRACTION

The fields in the Snowdon design magnet can be summarized as follows. Where in every case we are talking about

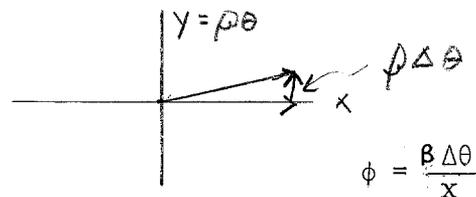


Typically for horizontal extraction the field begins to fall apart at .8" and has an error of 10^{-3} by 1.1". In the vertical plane the field starts to fall at .6" and has an error of 10^{-3} by 0.9".

Effect on Horizontal Extraction

a) To see the effect on horizontal extraction on a perfectly aligned machine one must consider a slightly off-momentum orbit so that field errors in (a) do not cancel. A momentum offset of 0.05 will give a maximum off-momentum excursion of 3 mm and an average offset of about 1.5 mm. It is felt that this sort of momentum error in some sense is representative of the sort of tolerance one could reasonably expect in operation of a machine and implies holding the beam to within a couple of mm of the center of the machine and knowing the position of the bending magnets relative to the beam detectors to accuracies of the order of 1 mm, etc. (i.e. everything must be done with considerable care).

A model for what happens to the extraction separatrix trajectory is as follows. Consider a resonant particle on the separatrix at a maximum in its oscillation amplitude then in $x, \beta\theta = y$ transverse phase space, that particle will receive a kick from the field error which will mostly rotate it in phase without appreciably changing the length of the radius vector.



For a particle at 1.1 inches amplitude passing through one magnet ϕ will be

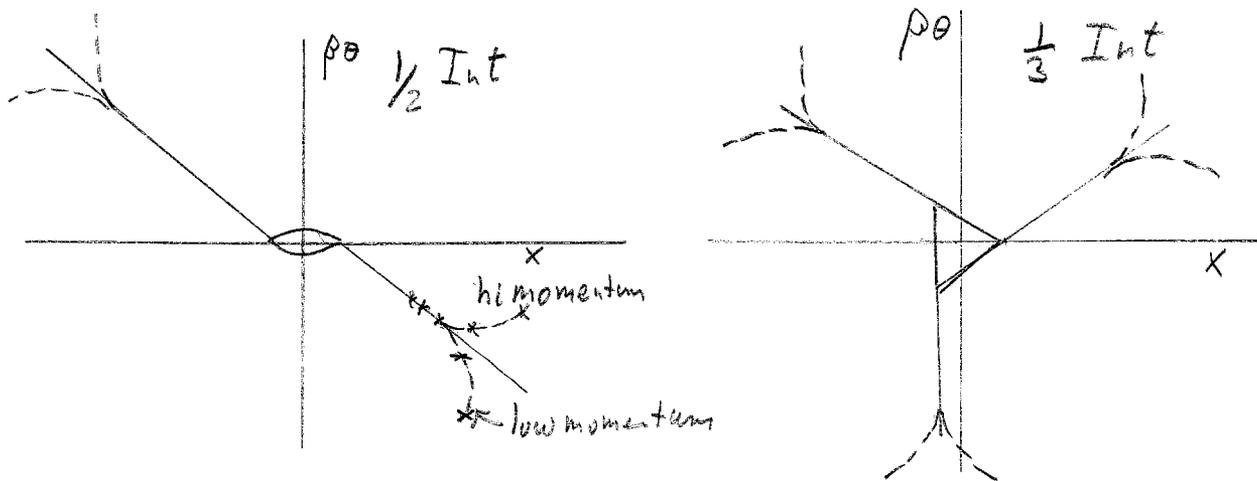
$$\phi = \frac{100 \text{ m} \times 10^{-3} \times 8.11 \text{ mr}}{1.1'' \times .0254} = 29 \text{ mr}$$

$$\beta_{\text{max}} = 100 \text{ m} \quad \theta_{\text{BM}} = 8.11 \text{ mr.}$$

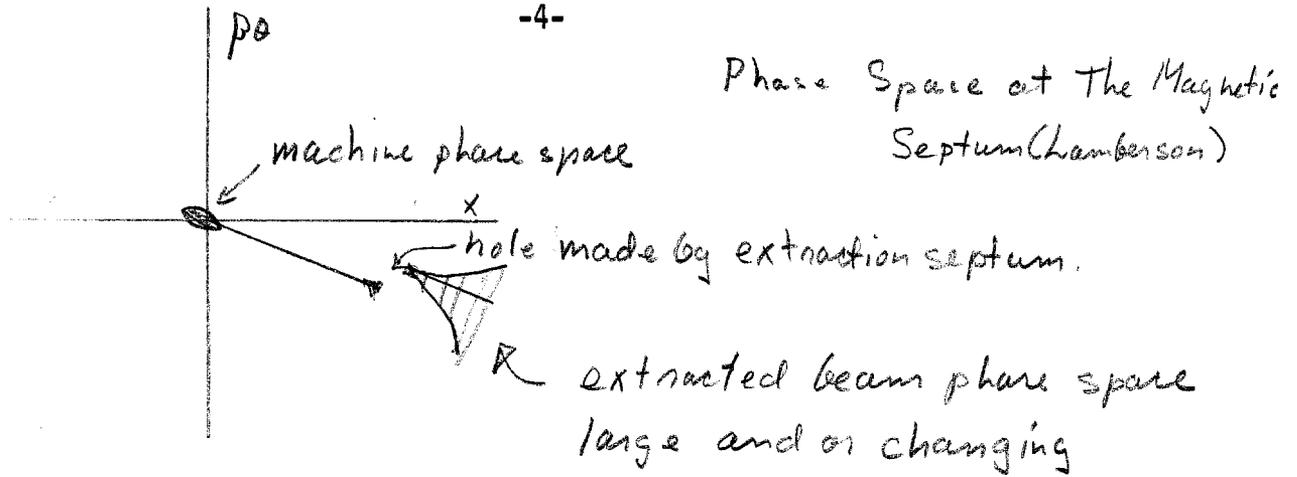
So at this radius the particle is retarded in phase a $\Delta\psi$ of ~ 1.7 degrees for each magnet it goes through at large amplitude. If the offset of the equilibrium orbit

were uniform around the machine ($\alpha_p = \text{const}$) and large enough so that only the positive bad field region of the magnets was explored one could expect a phase change of approximately $19 \times 1.7 \text{ deg.}$ or 32.0 deg. , in one turn which would produce a significant deviation from the ideal separatrix. Actually because the particles are growing rapidly in radius, because α_p does vary in amplitude around the machine on the 18th harmonic thus producing a beat pattern with the oscillation frequency of 19.5 (1/2 integer extraction), and because there will be some cancellation with the bad fields from the inside a smaller phase change by a factor of two or three can be expected, but it still should be substantial.

Proceeding to trace trajectories in turns beyond this point will undoubtedly lead to very distorted trajectories with particles either getting lost from the machine or folding back into smaller radii. From the fields (a) then one would expect the phase space plot of the separatrix to look something like:



The behavior should be insensitive to the type of resonance. The real question is whether one can operate in the region where the extracted beam lies in the distorted field region so that small changes in the machine will change the extracted beam phase space considerably. Can one effectively tolerate a large phase space?



An other question is whether one can place the septum sufficiently close to the center of the machine so that one does not need to use the bad field region. Note as a curiosity that the horizontal good field region is increased slightly if one does not stay in the vertical midplane.

b) The coupling field in the horizontal (x) plane has the worst effect if there is an average y offset to the beam. Here the inphase driving term could add up to a vertical angle of

$$\begin{aligned} \theta_y &= 8.11 \text{ mr} \times 10^{-3} \times (19-1/2) \times 2 \\ &= .316 \text{ mr or } y_{\text{max}} = \sqrt{\beta_1 \beta_2} \theta^1 = \sqrt{29,100} \theta^1 = 17 \text{ mm} \end{aligned}$$

after one turn with a x amplitude ~ 1.1 " and a y offset $\sim .1$ " leading to an error field of 10^{-3} . Here again this is an overestimate but gives an upper bound of what to expect. Because the beam is growing through the last turn and does not hit a maximum amplitude at all 39 possible locations the 17 mm number most likely is an overestimate by at least a factor of 4 even for an average y offset of $1/10$ " (large). We can expect to see horizontal-vertical coupling in the extracted beam even with the ideal design fields.

Effect on Vertical Extraction

c) ΔB_x vs y is zero if the horizontal x position is zero, however, small x displacements such as those generated by off momentum orbits lead to $\Delta \theta_y$ errors which add as the resonant particle oscillates between large + and - y oscillations (whereas they subtracted in the case of horizontal extraction). The good

field region drops to $\sim 10^{-3}$ more rapidly than for horizontal extraction, though vertical extraction may not be obviously that much worse when one takes into account that x offsets should be ^{half} as large at vertical β_{\max} and do not add to the vertical amplitude.

d) The coupling field for vertical extraction leads an outkick for positive or negative values of either x or y so presumably these should cancel unless there is an average offset of the bending magnets relative to the equilibrium orbit. Here an average y offset at $\frac{1''}{10}$ would limit the oscillation amplitude to $\sim .9''$ for the same magnitude of coupling as discussed in(b).

Summary of Design Fields

	Worst Off-Set	Amplitude of Osc. for 10^{-3} in field	Amplitude for 10^{-4} in field	
B_y vs x	.1" \bar{x}	1" x	.8" x	gets better with any y osc. or offset
B_x vs x	.05 \bar{y}	1.3" x	1.1" x	
B_x vs y	.05 \bar{x}	.95" y	.75" y	
B_y vs y	.1" \bar{y}	.8 y	.7 y	gets better with any x osc. or offset

The fields for horizontal extraction give slightly larger aperture. The advantage of which may be overcome by the least desirable horizontal extraction geometry and the sensitivity to momentum. To decide whether it is necessary to go into the field region of 10^{-4} one must explore the extraction process. In any case we are clearly talking about beams centered to within ± 2 mm of the magnet centers and it is crucial that both horizontal and vertical position detectors with precise alignment exist at both horizontal and vertical β_{\max} locations.

Septum Location

There really is no place to put the extraction septum but at F0 straight section in the middle of all the rf with all the radiation personnel problems that will entail. The septum probably should not be downstream of the middle of the straight section so as to lessen the probability of quenching the Doubler magnets at the front of F sector.

Two cases will be considered for the location of the septum: a) the upstream end of the long straight; and b) the center of the long straight. The pertinent information is as follows:

Dimensions (meters)	β_x	α_x	α_{px}	$\phi_{a,b}^x$	β_y	α_y	$\phi_{a,b}^y$
a) F0 Upstream	48.4	-.154	1.5	$\left(\begin{matrix} .0671 \\ (24 \text{ deg.}) \end{matrix} \right)$	110.4	1.139	$\left(\begin{matrix} .0433 \\ (15.6 \text{ deg.}) \end{matrix} \right)$
b) F0 Center	67.5	-.65	2.0		68.1	.65	

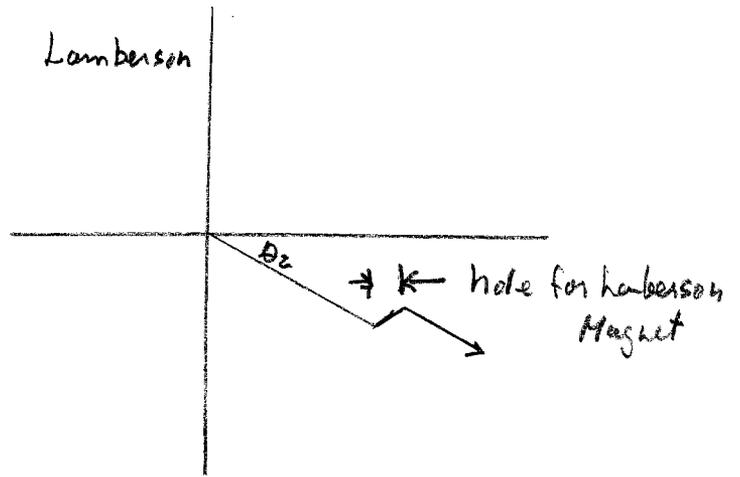
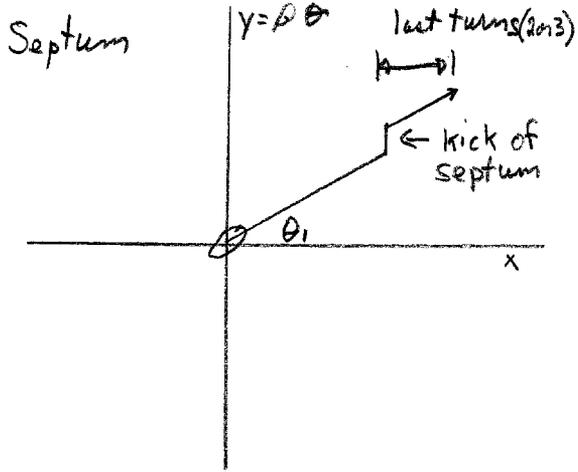
Typically $\beta_{\max} \approx 98.0 \text{ m}$ $\beta_{\min} \approx 29 \text{ m}$ $\alpha_{px \max} \approx 6 \text{ m}$

phase through 1/6 ring; $19.45/6$ (for half integer) = 3.242 (87 deg.).

Considering that the extraction Lambertson magnet is located at the upstream end of the A0 straight section at the identical place to (a) above, it appears that vertical extraction would be the more favorable except that the Doubler vertical aperture is smaller than the horizontal aperture. We will consider vertical extraction with the septum upstream (a) and horizontal extraction with the septum in the center(b).

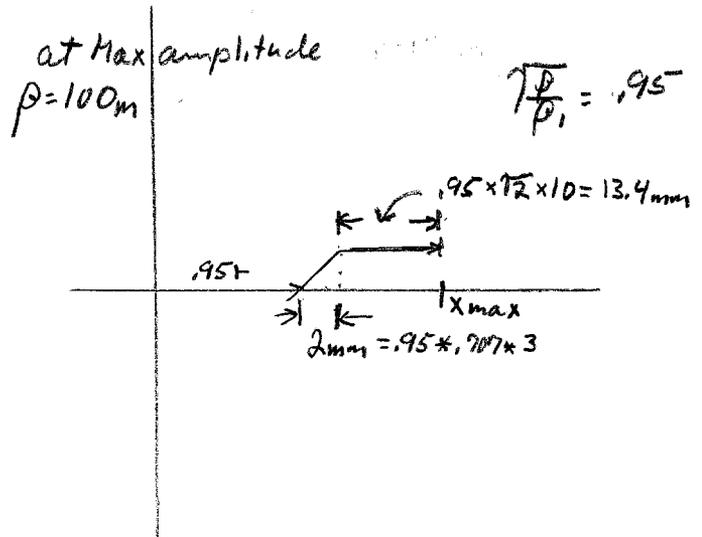
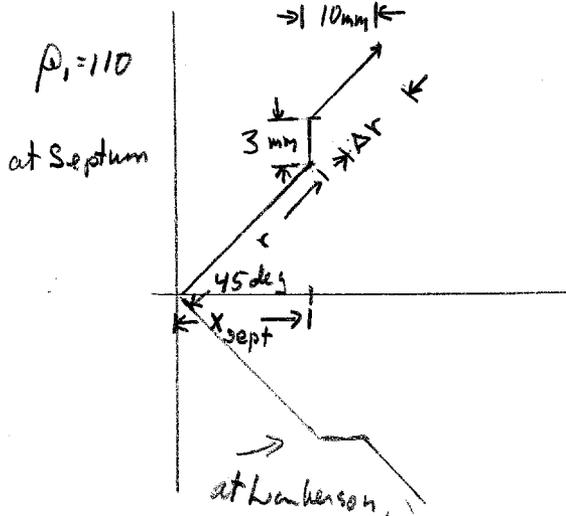
Basic Extraction Geometry

In circular transverse phase space geometry ($x, y = \beta \theta, \alpha$ removed) the pictures of one arm of the separatrix at the septa and Lambertson must look something like this:



Typically $\theta_1 \approx \theta_2 \approx 30$ to 45 deg.

This region is a compromise at minimizing the necessary step size at the septum for extraction efficiency and maximizing the hole made by the septum for the Lambertson. If we require a 1 cm step at the septum consider our vertical geometry (a) - and what happens when the separatrix reaches a phase of maximum amplitude at a β of 100 m.



$$x_{\max} = .95r + 2 + 13.4$$

$$= .95r + 15.4$$

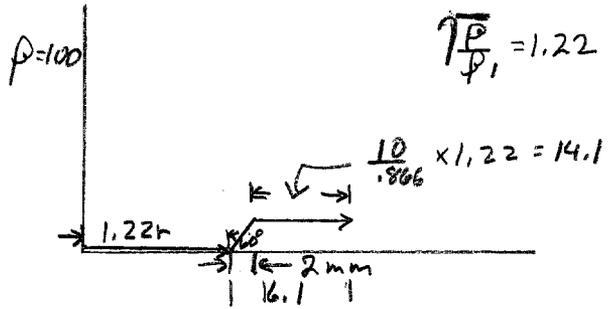
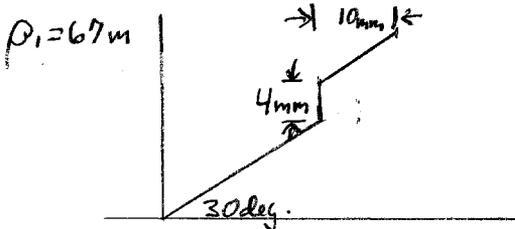
if $x_{\max} = 25.4\text{mm}$

$$r = 10.5\text{mm}$$

and $x_{\text{sept}} = 7.4\text{mm}$.

So for one inch maximum amplitude oscillation the septum is 7 mm from beam center and $\frac{\Delta r}{r} = \frac{14}{10}$. Unfortunately 1" good field region is somewhat optimistic.

For the horizontal geometry (b)



$$x_{\text{max}} = 1.22r + 16.1$$

$$\text{if } x_{\text{max}} = 25.4 \text{ mm}$$

$$r = 7.6 \text{ mm}$$

$$x_{\text{sept}} = 6.6 \text{ mm.}$$

$$\frac{\Delta r}{r} = \frac{11.5}{7.6}$$

This really looks bad indeed.

Phase Stable Area for 1/2 and 1/3 Extraction

Assume + and - elements distributed

at 60 deg. phase advance for 1/3

at 90 deg. phase advance for 1/2

Assume an emittance of $.05\pi \text{ mm mr}$ (400 GeV MR)

then the radius of a circle at $\beta = 100$ for normal phase space $\sqrt{5}$ or $2.24 \text{ mm} = r_0$.

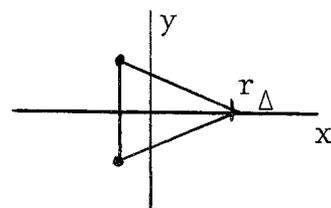
1/3 Extraction

Stable area is a triangle with

$$r_{\Delta}^2 = \frac{\pi}{2}(1.5)^2 r_0^2$$

$$r_{\Delta} = 1.67 r_0$$

$$= 3.75 \text{ mm}$$



The tune shift δ away from $\mu = 2\pi\nu = \frac{2\pi}{3} + \delta$ necessary for this fixed point is given by $\delta = 1/2 A r_{\Delta}$ (radians)

Where the sextupole strength of the sum of the + or - sextupoles is $\Delta y = \beta_0 = Ax^2$ (A units ℓ^{-1}) and the approximate step size in 3 turns is $\Delta r = 3/2Ar^2$

for $r = 15\text{mm}$ and $\Delta r = 10\text{mm}$

$$A = 10 \times \frac{2}{3} \left(\frac{1}{15}\right)^2 \text{ mm}^{-1} = .0296 \text{ mm}^{-1}$$

or about 250 kG-in at 1" for the sum of the positive elements and -250kG for the sum of the negative elements

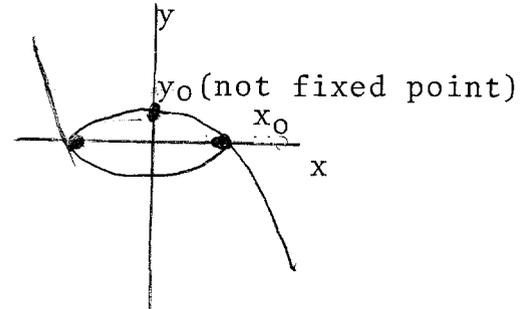
Now

$$\begin{aligned} \delta &= \frac{1}{2} .0296 \text{ mm}^{-1} \times 3.75 \text{ mm} \\ &= .055 \text{ rad. } (.009\Delta\nu) \end{aligned}$$

1/2 Extraction

The fixed points are given by

$$x_0^2 = \frac{\delta - A}{C}$$



where A and C are the sum of the positive quad and oct strengths defined by $\Delta y_0 = \beta_0 = Ax$ A (no dimensions)

$$\Delta y_0 = \beta_0 = Cx^3 \quad C \text{ (dimensions } \ell^{-2}\text{)}$$

$$\delta = 2\pi\left(\frac{1}{2} - \nu\right) \text{ radians)}$$

The height of the phase stable region is given by

$$x_0^2 \approx \left(2\left(\frac{\delta + A}{C}\right)\right)^{\frac{1}{2}} y_0 \approx \left(\frac{4\delta}{C}\right)^{\frac{1}{2}} y_0 \quad \text{for } \delta \gg C$$

Where A is typically 0.31, C is typically $.45 \times 10^{-3} (\text{mm}^{-2})$

$$\nu \approx 19.45$$

$$x_0 y_0 = \frac{x_0^3 \left(\frac{C}{4\delta}\right)^{\frac{1}{2}}}{(4\delta)} = 5 \text{ mm}^2 \rightarrow x_0 = 6.4 \text{ mm.}$$

$$\delta - A = x_0^2 C = .018 \text{ rad } (.003 \Delta\nu).$$

Obviously there may be trouble with 1/2 interger extraction fixed point coming to close to the septum, if the phase space is as large as chosen. Probably some time-dependent control of the beam position and angle at the septa will be necessary to optimize the step size during the slow extraction process.

Longitudinal Emittance and Extraction Process

If we use a momentum spread in the beam of $\pm .02\% \frac{\Delta p}{p}$ (MR 400 GeV) this would give a radial offset of the beam of $\pm .4\text{mm}$ (for $\alpha_p = 2\text{m}$) and a tune spread of $\Delta\nu = 22 \times .2 \times 10^{-3} = \pm .0044$ for the natural chromaticity.

Summarizing, $\epsilon = .05\pi\text{mm}^2$, $\frac{\Delta p}{p} = \pm .02\%$, and $\Delta\nu = -22\frac{\Delta p}{p}$

	1/3 resonance	1/2	$\Delta p/p$
x_0	3.75mm	6.4mm	$\pm .4\text{mm}$
$\Delta\nu$.009	.003	$\pm .004$

Because of the rather large required stable phase area compared to the septum offset from beam center line, careful investigation must be made of step size, and particle density at the septum as a function of transverse phase space area (max to zero) and beam motion due to momentum for both the 1/2 and 1/3 resonance. It seems likely, however, that we may well want to make the chromaticity zero so that the beam can be moved closer to the septum as the phase space shrinks.

From the aforementioned basic geometric considerations, the following seem to be indicated though continuing calculations are necessary.

1. We must use the 10^{-3} error field region of the magnets.
2. It is unlikely that a 1-cm stepsize can be obtained.
3. The density of particles at the septa wires will be considerably higher than $1/\text{stepsize}$.
4. Thus, the extracted beam efficiency will be worse than that for the Main Ring (like 2 or 3 times).
5. The extracted beam phase space will be distorted in shape and probably very sensitive.
6. It would be nice if the vertical magnet aperture were larger. Could this be achieved by minor modifications to the magnet design; or
7. As Tom Collins has suggested, a high horizontal β at the septum would help horizontal extraction; perhaps it could be done with special quads at the straight section. This should definitely be investigated.

Devices Necessary for Extraction Manipulations

Slow spill and 1-ms fast spill are required. Multiple pulse, fast extraction would be nice (so far when trying this in the Main Ring, the losses have been a factor of two higher than for normal running.) For slow-spill extraction devices like bump magnets, quads, sextupoles, octupoles, etc. could be superconducting and incorporated in the correction coil package (though a tune-regulating quad should be conventional). Fast extraction is presently done on

half-integer by pulsing quadrupoles and bump magnets. If $1/2$ integer were used in the Energy Doubler, the pulsed quads could be put in the warm 17 locations. Fast bumps of the double-dogleg sort may be possible across the long straight. If a third integer must be used because of the possible problem with the stable phase-space shape of half integer, proper locations for pulsed sextupoles may be difficult to find, also pulsed sextupoles have the disadvantage that relatively stronger magnets are necessary to get the required spill time, (step size goes like r^2 instead of r). Here again, calculations are necessary.

Another possibility for fast extraction is to kick the beam or part of it into the stable region with a single-turn kicker and let it coherently extract. This would work for either $1/3$ or $1/2$ and would obviate the necessity for pulsed quads or sextupoles, but has not been a very successful mode of operation in the Main Ring. If used for multi-pulse fast extraction as suggested by Jack McCarthy, a damper would be required to take out the coherent oscillations before slow extraction could be established.

Calculations of Particle Trajectories

A computer calculation has been used to trace the extracting particles through the accelerator magnets in a step-by-step fashion. The appendix describes the program. The figures show the phase space at both the septum and the Lambertson positions in x, β_0 space where shear of the diagrams due to α has not been removed, (i.e., we are not in a circular system but rather real angles at the location of the devices are indicated). Figure 1 shows the stable area for the design emittance. Figure 2 gives the typical resonant picture and indicates that the trajectory is essentially the same for design

fields as for perfect fields as it should be for $\Delta p = 0$. Figure 3 indicates the change in step size for 0 stable phase space is about 5%. Figure 4 indicates the trajectories for $\pm .05\% \frac{\Delta p}{p}$ using the Energy Doubler design fields. Figure 5 indicates the effect of a vertical free oscillation on the horizontal and Figure 6 indicates the effect of an average vertical offset of the bending magnets. No effect was found from a 19th harmonic vertical equilibrium orbit distortion. Figure 7 shows the case of vertical extraction. No real surprises or undue strange behaviors have been discovered. Note the typical step size is ~ 5 to 6 mm at extraction.

Numbers indicated on the drawings are first, the β and α at the Lambertson (the septa plot does use its own proper α and β); then the extraction elements, their position from the start of the indicated sector in meters, their strength in terms of a constant K giving $\theta_{\text{rad}} = K(m^{-n}) x^n(m)$. SPT1 (SPT2) is the septum, its angle in mrad is given in the last column. SPO1 (SPO2) gives the offset of the wires in meters from centerline.

Figure 8 gives a multiturn particle trajectory on the separatrix during the extraction process showing the turn-to-turn radial amplitude growth.

Figures 9 - 12 are computerized dipole field plots for various initial conditions corresponding to the text figures a-d.

In order to facilitate calculations on beam extraction we have developed a ray tracing program which is capable of making repetitive orbit calculations. The program, which runs semi-interactively on the PDP-10, differs primarily from existing routines by treating each element of the machine lattice individually and not relying on any form of lattice periodicity. Generally speaking lattice symmetry is broken when dealing with extraction systems.

The basic machine structure is stored on a disk file containing element names and positions sequentially around the ring. From this file the machine lattice can be calculated for given horizontal and vertical tunes. This lattice information can also be stored on a disk file for subsequent access if desired. Besides the basic machine elements of dipoles and quadrupoles the program also recognizes "non-standard" elements used for extraction such as kick and bump magnets, quadrupoles, sextupoles (horizontal and vertical), octupoles, and septa (horizontal and vertical). An interactive editing feature of the program will insert these elements into the basic machine at any given position with a given strength. A new lattice can then be calculated from this "updated" element file.

From an initial point in transverse phase space (both horizontal and vertical) a single particle orbit can then be traced for as many revolutions as desired. If no initial orbit information is given then the program will find (analytically where possible, interactively where not) a closed orbit solution to the given lattice. Momentum dispersion is included in the matrix elements to allow off-momentum orbits to be calculated. Chromaticity correction to these orbits is an available option and is accomplished by correcting sextupoles at each quadrupole location in the standard half-cell. Other options in the program include a forced orbit oscillation of a certain harmonic, phase and amplitude in either the horizontal and vertical plane, and also the ability to impose a zeroth harmonic octupole moment around the machine.

In an attempt to make realistic calculations on the ED/S lattice the full multipole field expansion for the Doubler magnets is available as a selectable option which provides an integrated trajectory through the magnets. The integration step size is a function of the particle position so that number of steps increases as the higher order multipoles becomes significant. As more ED/S magnets become available and are measured then a further option is available to select between the theoretical multipole coefficients and the experimentally measured ones distributed with the appropriate standard deviation and mean.

BETA-THETA (MMS)

4-AUG-78 15:17

0.50 CMS PER DIV

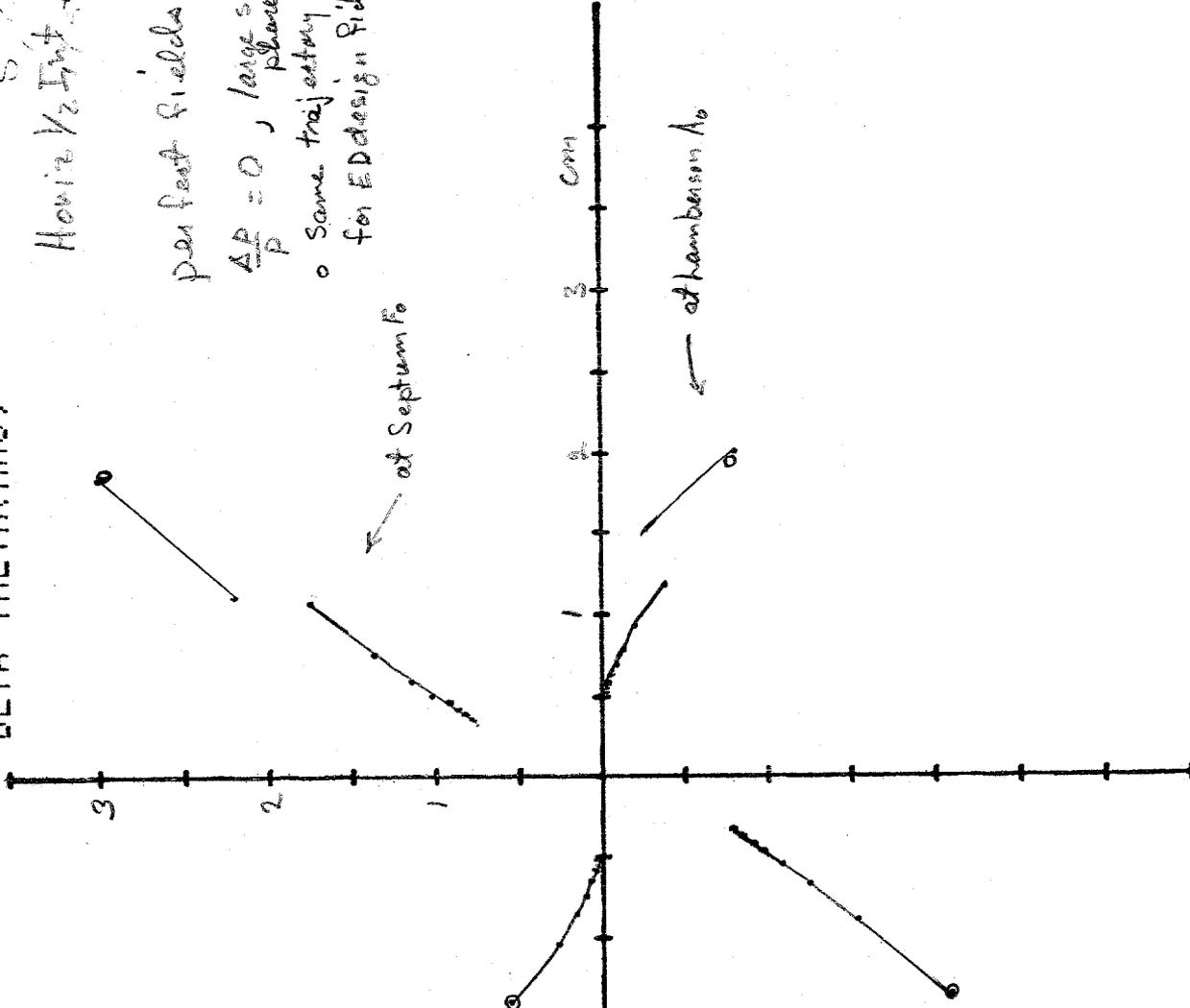
TUNE 19.44999
 BETA 48.69667
 ALPHA -0.15690

8
 Horiz 1/2 Int Ext

per foot fields

$\frac{\Delta P}{P} = 0$, large stable phase space

o Same trajectory for ED design fields



POSITION (MMS)

QUD1	B	214.47	0.00	110
QUD1	D	214.47	0.00	110
QUD1	F	214.47	0.00	110
QUD2	A	214.47	-0.00	110
QUD2	C	214.47	-0.00	110
QUD2	E	214.47	-0.00	110
OCT1	B	215.00	1.50	000
OCT1	D	215.00	1.50	000
OCT1	F	215.00	1.50	000
OCT2	A	215.00	-1.50	000
OCT2	C	215.00	-1.50	000
OCT2	E	215.00	-1.50	000
SPT1	F	0.10	0.00	000
SPO1		01100		
SURV	F	0.11		
SURV	F	1023.70		

Fig 2

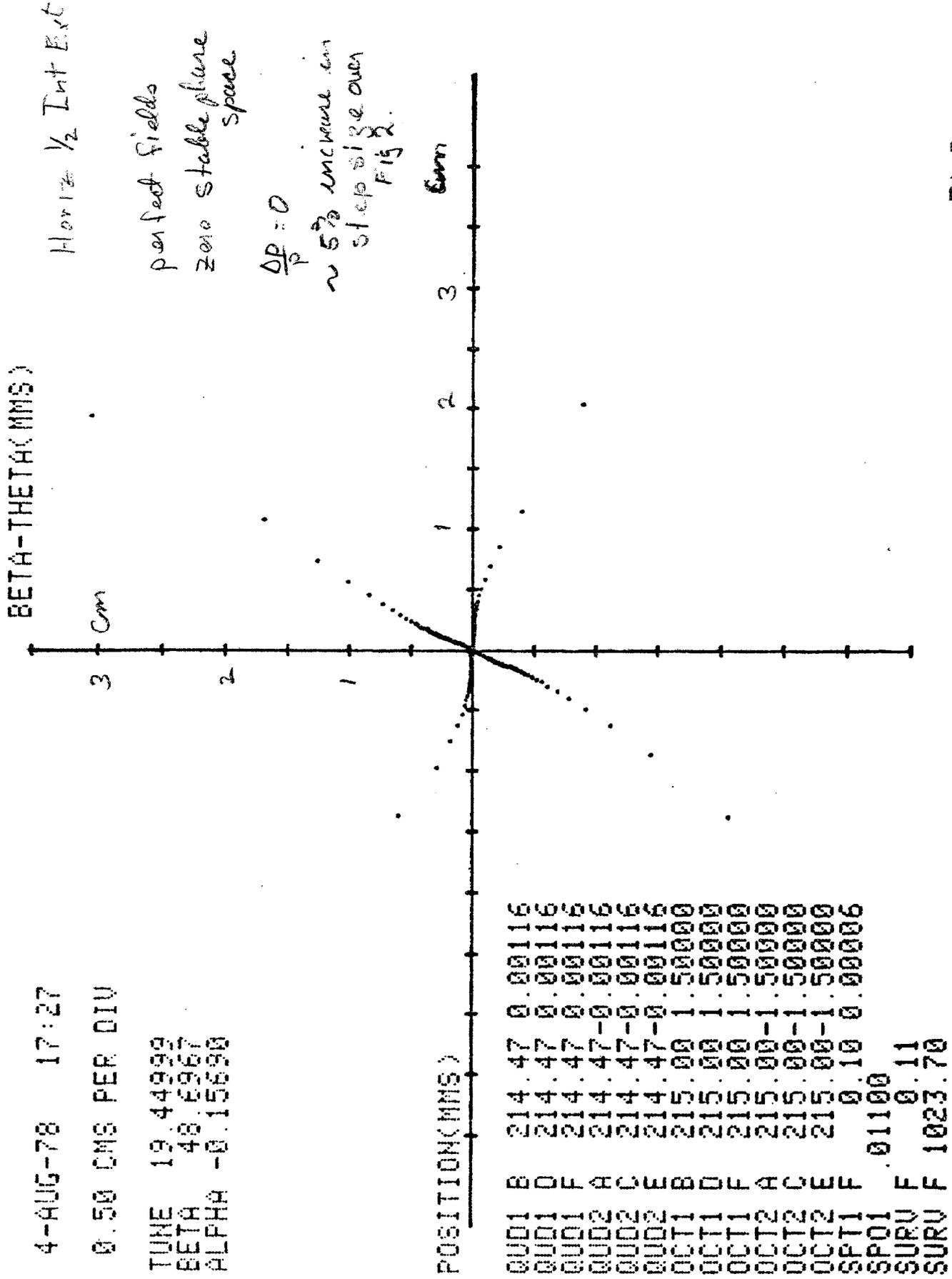


Fig 3

23-SEP-78 17:14

0.50 CMS PER DIV
HORIZONTAL PHASE SPACE
TUNE 19.44929
BETA 148.6824
ALPHA -0.15690

BETA-THETA (MMS)

3 CM
2
1

Horiz V_2 Int Ext
ED Design fields

-1570 AP
displaced
vertically by 4mm
All Dipoles

$X_{max} = 2.4$ cm

Vert oscillation amplitude
grew to 1.4 mm.

POSITION (MMS)

3 CM
2
1

-21-

QUDI	B	214.47	0.001116
QUDI	D	214.47	0.001116
QUDI	F	214.47	0.001116
QUDI	A	214.47	0.001116
QUDI	C	214.47	0.001116
QUDI	E	214.47	0.001116
OCTI	B	215.00	1.500000
OCTI	D	215.00	1.500000
OCTI	F	215.00	1.500000
OCTI	A	215.00	1.500000
OCTI	C	215.00	1.500000
OCTI	E	215.00	1.500000
SPDI	F	0.10	0.00800
SURV	F	0.11	0.00800
SURV	F	1023.70	0.00000
SURV	F	1023.70	0.00000

XINIT	0.240
XPINIT	.0800
YINIT	0.0000
YPINIT	.0000

⊙ trajectory if no
vertical off set of magnets

Fig 6

BETA-THETA (MMS)

16-OCT-78 06:41
 1.00 CMS PER DIV
 VERTICAL PHASE SPACE
 TUNE 19.45000
 BETA 110.83394
 ALPHA 1.14630

Vertical 1/2 Int Ext
 percent and ED design fields
 +1.70 $\frac{AP}{p}$

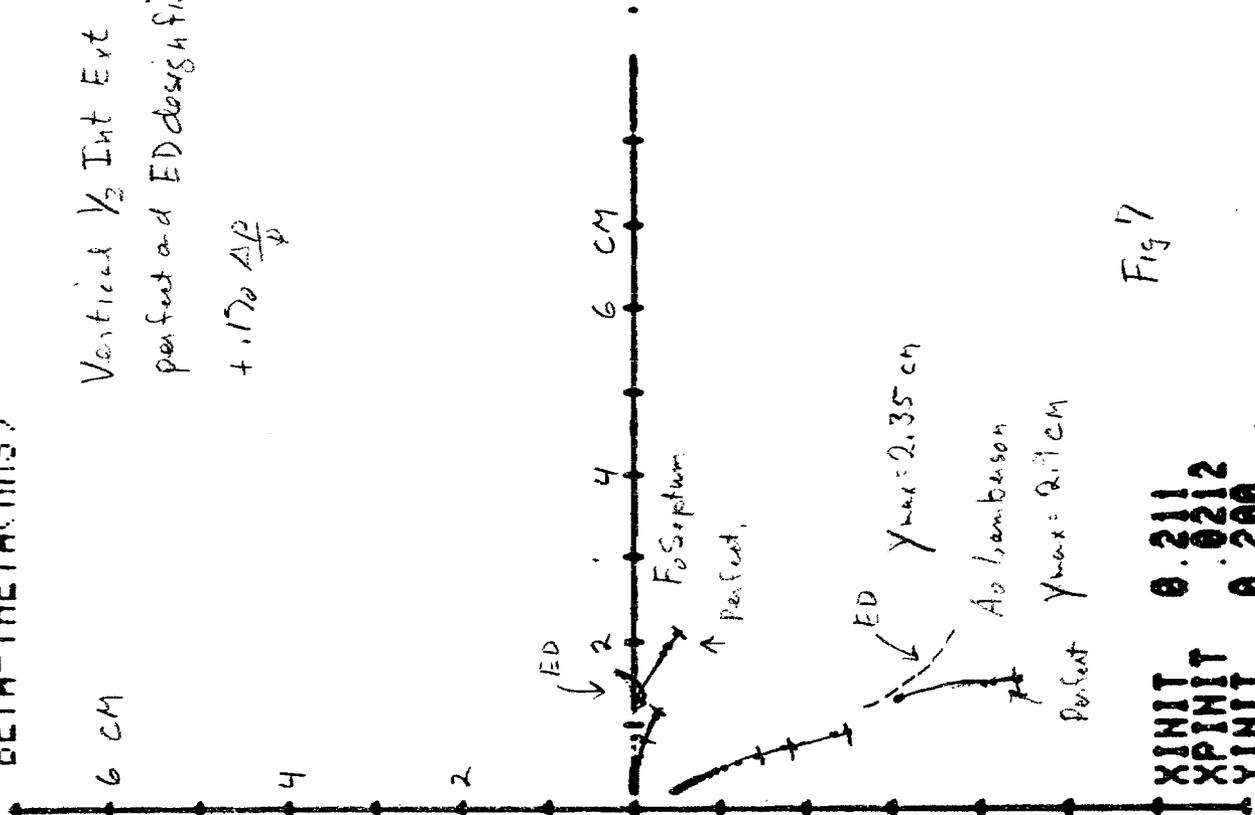


Fig 7

POSITION (MMS)

QU01	BOE	363.17	0.00118
QU01	F	363.17	0.00118
QU02	A	363.17	0.00118
QU02	C	363.17	0.00118
QU02	E	363.17	0.00118
OC11	BOE	363.70	0.00000
OC11	F	363.70	0.00000
OC12	A	363.70	0.00000
OC12	C	363.70	0.00000
OC12	E	363.70	0.00000
SP02	E	1023.70	0.00004
SURV	E	1023.80	0.00000
SURV	F	1023.80	0.00000
SURV	F	1023.70	0.00000

13mm septum

XINIT 0.2111
 XPINIT 0.2212
 YINIT 0.2000
 YPINIT 0.0000

TUNE 19.450
MAX OFFSET(CMS) 3.48
HORIZONTAL PROJECTION

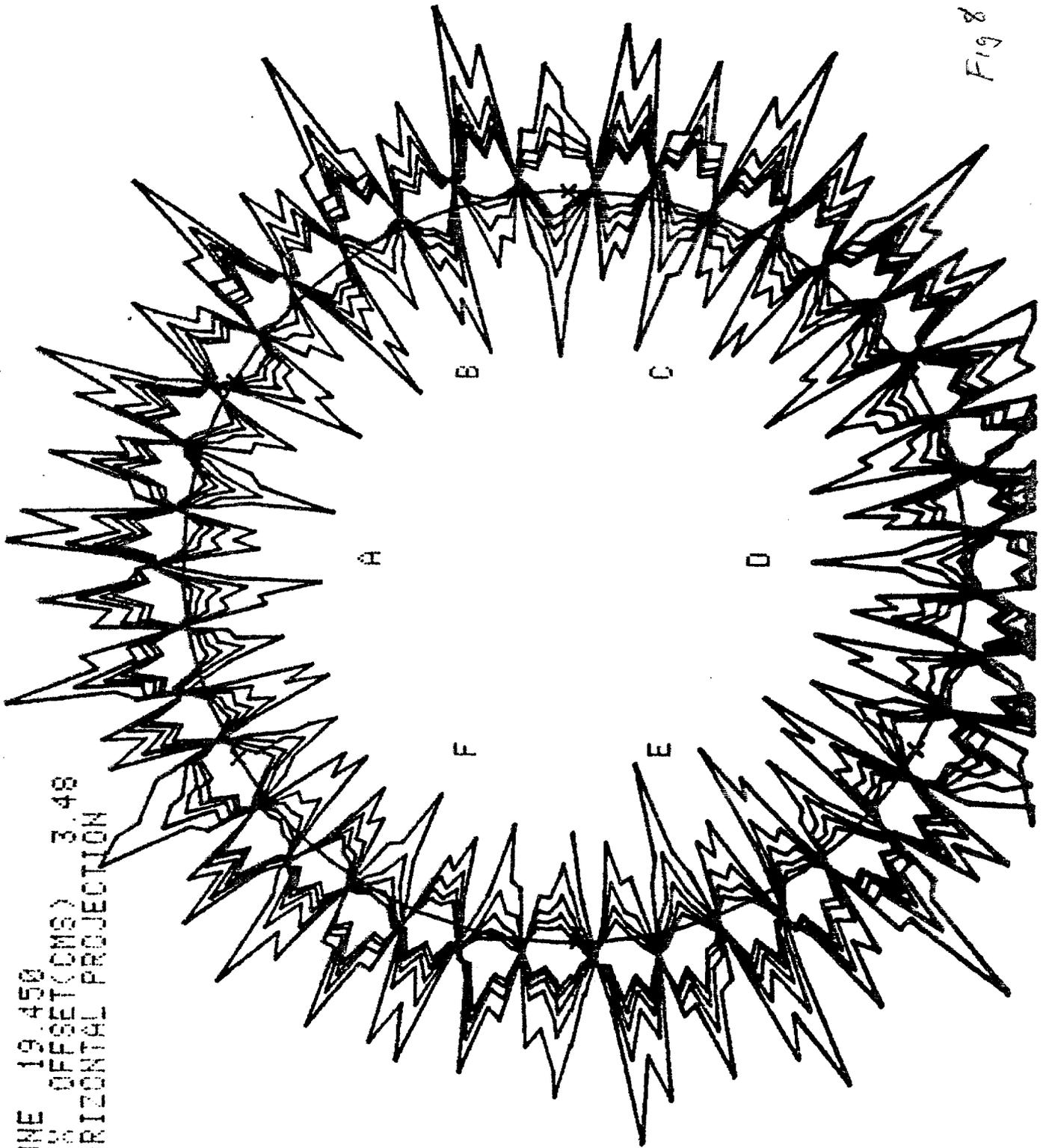
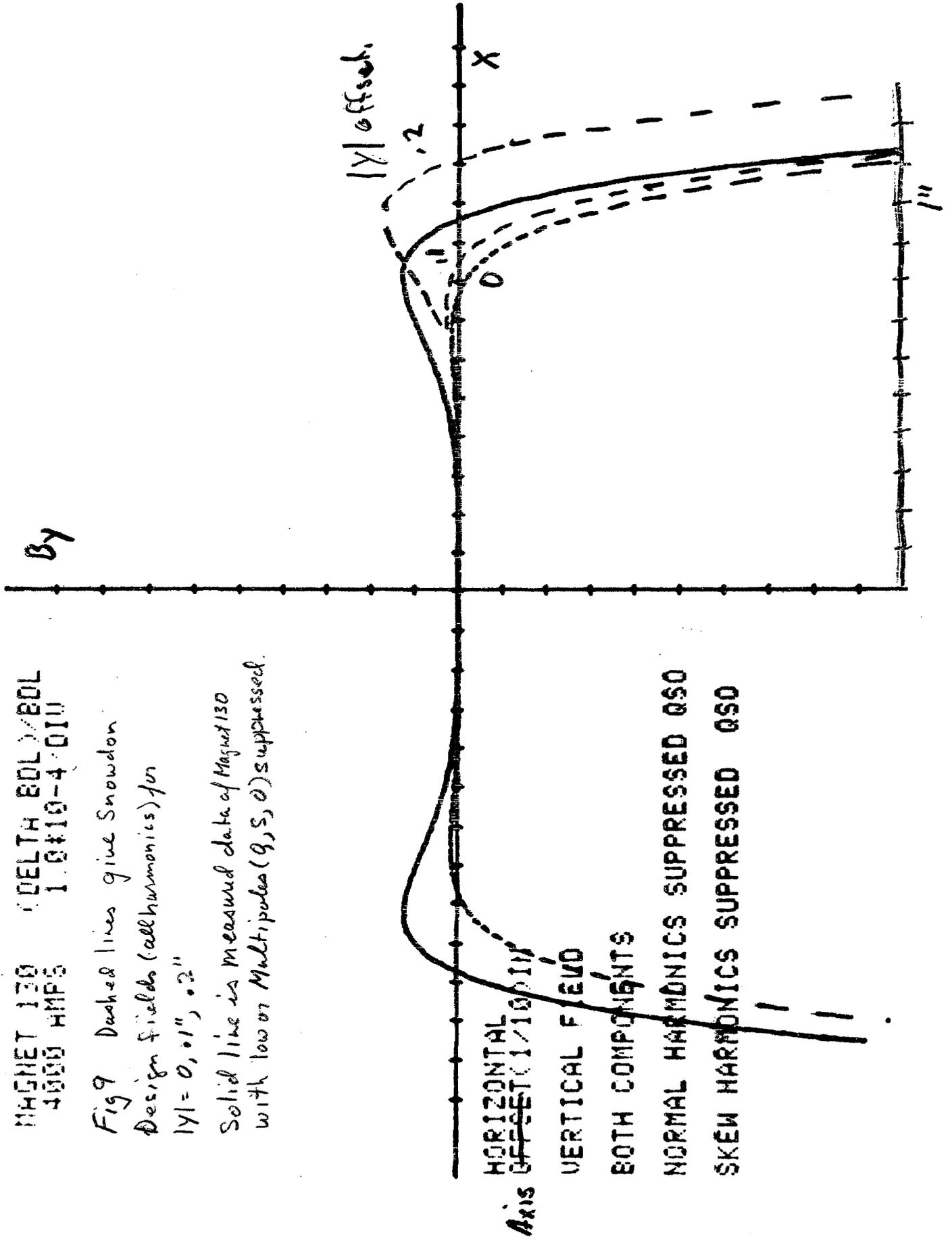


Fig 8

MAGNET 130 (DELTA BOLD) 20L
4000 AMPS 1.0x10-4 OIU

Fig 9 Dashed lines give Snowdon
Design fields (all harmonics) for
 $|Y| = 0, .1", .2"$

Solid line is measured data of Magnet 130
with lower Multipoles (9, S, 0) suppressed.

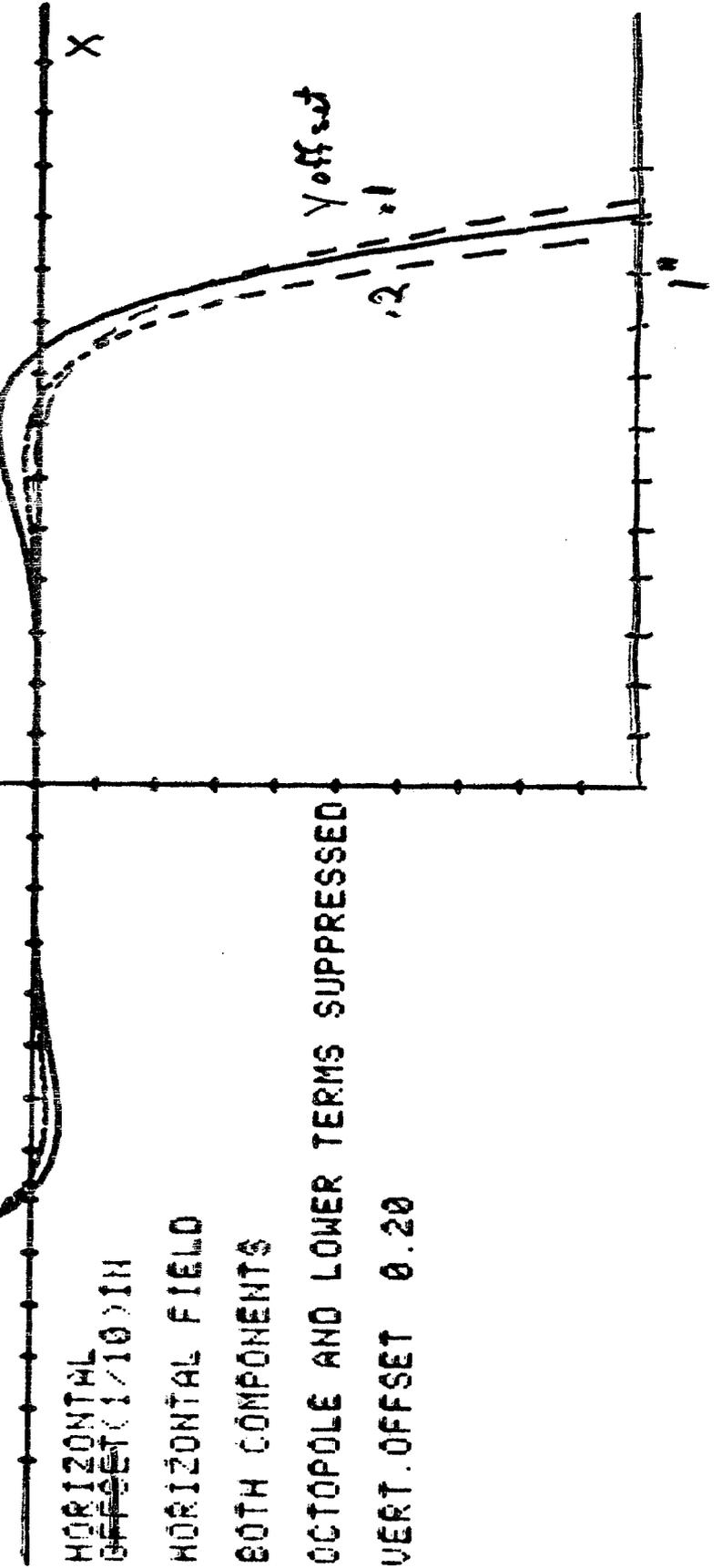


2.0

MAGNET 20 (DELTA BOL) BOL
3001 GAPS 1.0*10-4 DIV

Bx

Fig 10. Dashed lines give design field (all harmonics) for $\gamma = +1$ "cd+1.2"
Solid line is measured data for magnet 20
vert offset 0.2" low harmonics suppressed.



Axis
HORIZONTAL
OFFSET (1/10) IN

HORIZONTAL FIELD

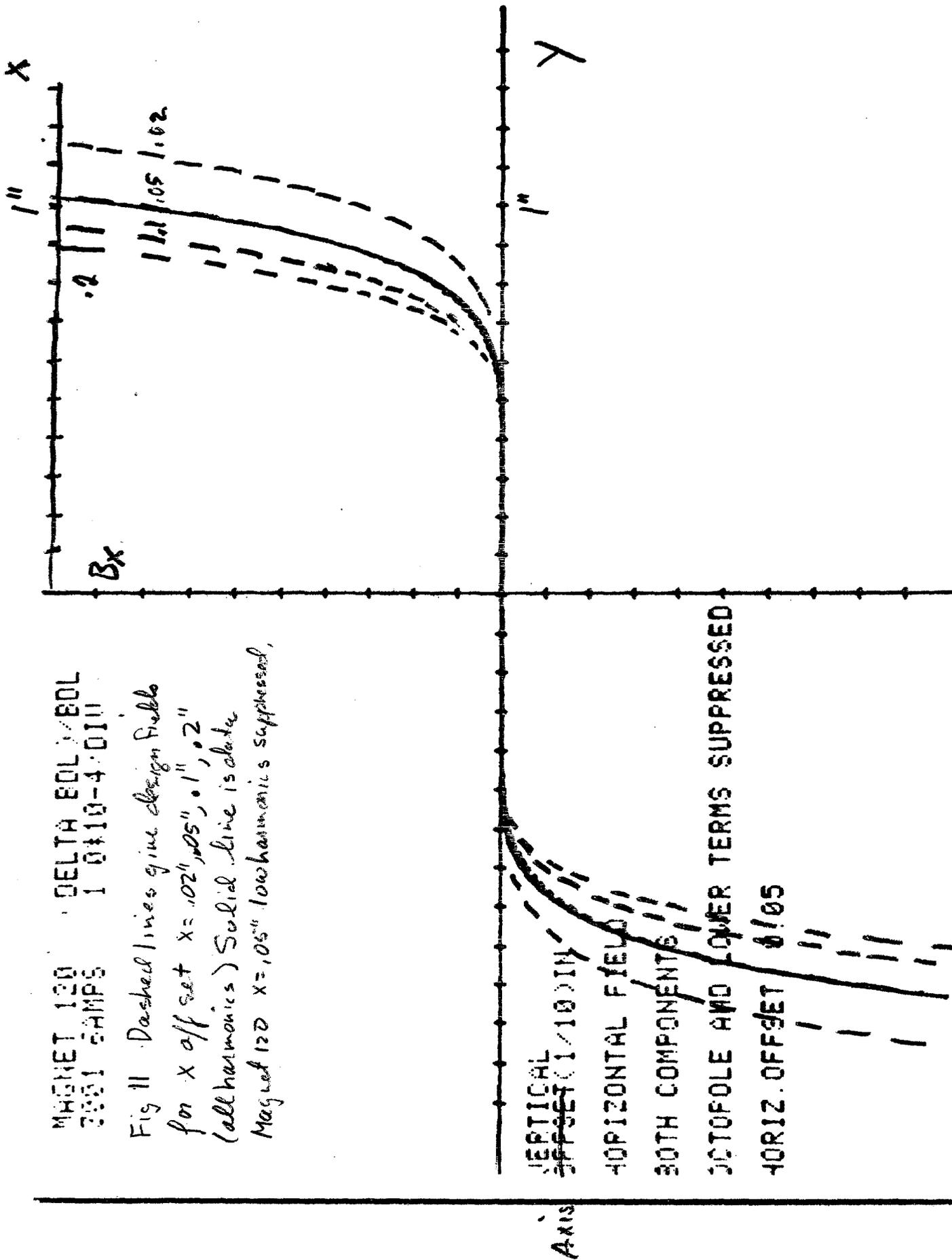
BOTH COMPONENTS

OCTOPOLE AND LOWER TERMS SUPPRESSED

VERT. OFFSET 0.20

MAGNET 120 DELTA EOL 1/80L
3001 CAMPS 1 0x10-4 010

Fig 11 Dashed lines give design fields
for X offset X = .02", .05", .1", .2"
(all harmonics) Solid line is data
Magnet 120 X = .05" low harmonics suppressed,



Axis

MAGNET 120 DELTA EDL . EDL
 3001 EMPS 1.0+10-4 0111

Fig 12 Dashed lines give design fields
 for $|x| = 0, 0.1'', 0.2''$ (all harmonics)
 Solid line is data Magnet 120
 with low harmonics suppressed.

