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Compensation of Dogleg Effect in Fermilab Booster

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1 Introduction

The edge focusing of dogleg magnets in Fermilab Booster has been causing severe distortion to the horizontal linear optics. The doglegs are vertical rectangular bends, therefore the vertical edge focusing is canceled by body focusing and the overall effect is focusing in the horizontal plane. The maximum horizontal beta function is changed from 33.7m to 46.9m and maximum dispersion from 3.19m to 6.14m. Beam size increases accordingly. This is believed to be one of the major reasons of beam loss. In this technote we demonstrate that this effect can be effectively corrected with Booster's quadrupole correctors in short straight sections (QS). There are 24 QS correctors which can alter horizontal linear optics with negligible perturbation to the vertical plane.

The currents of correctors are determined by harmonic compensation, i.e., cancellation of dogleg's harmonics that are responsible for the distortion with that of QS correctors. By considering a few leading harmonics, the ideal lattice can be partly restored. For the current dogleg layout, maximum β_x is reduced to 40.6m and maximum D_x is reduced to 4.19m.

This scheme can be useful after the dogleg in section #3 is repositioned. In this case it can bring β_x from 40.9m down to 37.7m, D_x from 4.57m to 4.01m.

2 Harmonic Compensation

The edge focusing effect is a quadrupole effect. The integrated quadrupole strength is [1]

$$k\Delta l = \frac{\tan\delta}{\rho} = \frac{\delta\tan\delta}{L} \tag{2.1}$$

where δ is the entrance or exit angle with respect to the normal direction of the edge, L is the length of the magnet and $\rho = L/\delta$ the bending radius.

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The p'th harmonic half-integer stopband integral is defined as [1]

$$J_p = \frac{1}{2\pi} \oint \beta k(s) e^{-jp\phi}\, \mathrm{d}s = \frac{1}{2\pi} \sum_i \beta_i [k\Delta l]_i e^{-jp\phi_i} \tag{2.2}$$

where $[k\Delta l]_i$ is the integrated quadrupole strength at location i. The perturbation to beta function is [1]

$$\frac{\Delta\beta(s)}{\beta(s)} = -\frac{\nu_0}{2} \sum_{p=-\infty}^{\infty} \frac{J_p e^{jp\phi}}{\nu_0^2 - (p/2)^2} \tag{2.3}$$

For off-momentum particles, there is also a dipole effect because of dispersion. The equivalent dipole strength is

$$\frac{\Delta B}{B\rho} = -[k\Delta l] D \frac{\Delta p}{p_0} \tag{2.4}$$

where $\frac{\Delta p}{p_0}$ is momentum deviation. The minus sign indicates a weaker bending strength for positive momentum deviation. For a unit value of $\frac{\Delta p}{p_0}$, the integer stopband integral is [1]

$$\begin{aligned} f_n &= \frac{1}{2\pi\nu} \oint \sqrt{\beta} \frac{\Delta B}{B\rho} e^{-jn\phi} \mathrm{d}s \\ &= -\frac{1}{2\pi\nu} \sum_i \sqrt{\beta_i} [k\Delta l]_i D_i e^{-jn\phi_i} \end{aligned} \tag{2.5}$$

The change to dispersion function is then [1]

$$\Delta D = \frac{\Delta x_{\mathrm{co}}}{\Delta p/p_0} = \sqrt{\beta(s)} \sum_{n=-\infty}^{\infty} \frac{\nu^2 f_n}{\nu^2 - n^2} e^{jn\phi} \tag{2.6}$$

There are 4 magnets for each dogleg but each magnet has focusing effect at only one edge. Table (1) lists related parameters of doglegs.

dogleg #	δ	L (m)	$k\Delta l$ (m^{-1})
03 old	0.062510	0.24722	0.015826
03 new	0.023440	0.24722	0.002223
13	0.059876	0.24722	0.014519

Table 1: dogleg parameters. '03 old' and '13' are current layout. '03 new' is dogleg 03 after repositioning.

According to equations (2.3) and (2.6), the contribution of a harmonic to the changes of beta function or dispersion is weighted by a factor. Figure (1) shows magnitude of the weighted harmonics $J_p/(\nu^2 - p^2/4)$ and $f_n/(\nu^2 - n^2)$

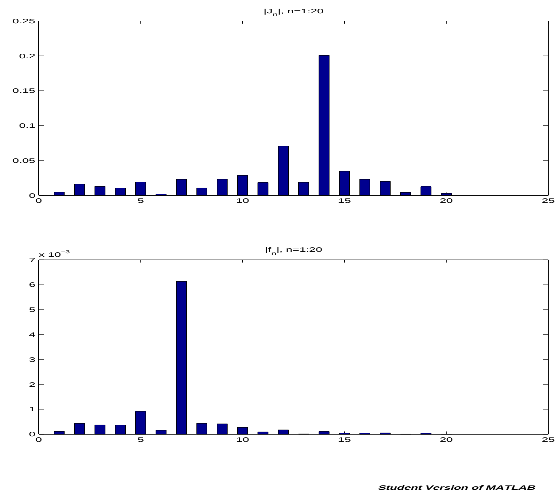

Figure 1: The weighted harmonics for current dogleg layout. Top, $|J_p|/(\nu^2 - p^2/4)$. Bottom $|f_n|/(\nu^2 - n^2)$.

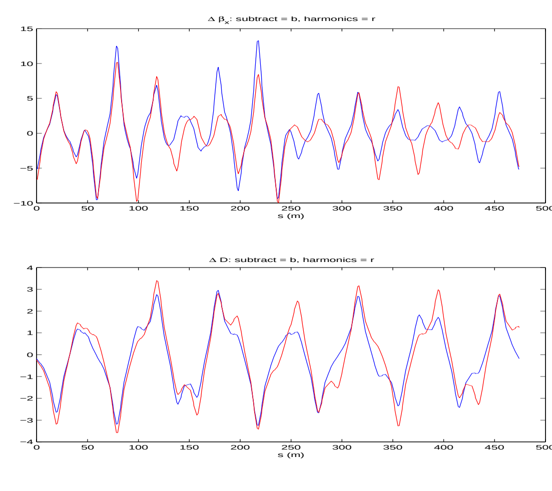

Figure 2: Changes to beta amplitude β_x and dispersion D_x due to current dogleg layout. Blue curve is by subtracting MAD output with and without doglegs. Red one is obtained using equations (2.3) and (2.6). Top, β_x. Bottom, D_x.

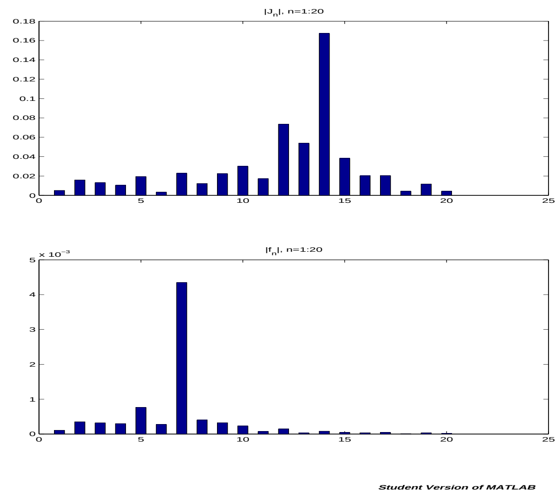

Figure 3: The weighted harmonics for repositioned dogleg layout. Top, $|J_p|/(\nu^2 - p^2/4)$. Bottom $|f_n|/(\nu^2 - n^2)$.

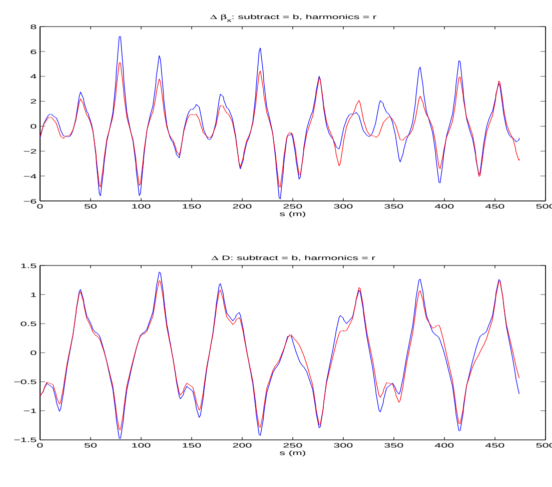

Figure 4: Changes to beta amplitude β_x and dispersion D_x due to repositioned dogleg layout. Blue curve is by subtracting MAD output with and without doglegs. Red one is obtained using equations (2.3) and (2.6).

for the current dogleg layout. Figure (2) shows changes to β_x and dispersion D_x according to equations (2.2), (2.3), (2.5) and (2.6) with comparison to the same quantities obtained from subtraction of MAD output. We can see the 7th of f_n and 14th of J_p dominate the two spectra, respectively. Figure (3) and (4) are for the new dogleg layout.

For the i'th QS quadrupole, given a current of +1A, we can also get its stopband harmonics spectrum J_p^i and f_n^i. To cancel the p'th half-integer harmonic J_p of doglegs, we want to pick up those quadrupoles whose J_p term is large and points along the direction of J_p of doglegs in the complex plane. The same is true for f_n. Define efficiency

$$A_p^i = \frac{|J_p^i|}{|J_p|}\cos(\Delta\phi_a^i) = \mathrm{Re}\left(\frac{J_p^i}{J_p}\right) \tag{2.7}$$

$$B_n^i = \frac{|f_n^i|}{|f_n|}\cos(\Delta\phi_b^i) = \mathrm{Re}\left(\frac{f_n^i}{f_n}\right) \tag{2.8}$$

where $\Delta\phi_a^i$ ang $\Delta\phi_b^i$ are angles between J_p^i and J_p, f_n^i and f_n respectively.

Now our goal is to minimize the total harmonics of doglegs and quadrupoles for both J_p and f_n. In choosing a set of quadrupole current to achieve this, we study the efficiency table to determine which quadrupoles are to be used and calculate the needed current values. Then we update the total weighted harmonics spectrum and repeat. Quadrupoles having large efficiency for the leading harmonic of one spectrum (J_p or f_n) and efficiency of the same sign for the leading harmonic of the other spectrum are of our interest. This process involves a lot of "eyeball inspection" and thus is not very efficient. To automate the process, we found that a bit-by-bit approach is needed. The procedure is

Step 1 sort the magnitude of the weighted overall harmonics of J_p and f_n to descending order and locate the most important harmonics, the 7th of f_n and the 14th of J_p, for example.

Step 2 sort the 24 A_{14}^i to descending order

Step 3 for each of the first 6 sorted A_{14}^i, if it has the same sign as B_7^i, change the current of the i'th QS by adding $0.1/A_{14}^i$. (i.e. cancel 10% of J_{14} with this quadrupole)

Step 4 sort the 24 B_7^i to descending order

Step 5 for each of the first 6 sorted B_7^i, if it has the same sign as A_{14}^i, change the current of the i'th QS by adding $0.1/B_7^i$. (i.e. cancel 10% of f_7 with this quadrupole)

Step 6 go to step 1

It is important for us to make only a small change to the leading harmonic and recalculate the total harmonics in each iteration. Otherwise the solution would be incorrect because of the interference among quadrupoles. When the leading harmonics get closer to the next biggest one, we may consider more than 6 QS's and require the efficiencies have the same sign with that of the next biggest one, too. The process stops when no quadrupoles can be found to reduce the leading harmonic without increase another hamonic to same level.

It can quickly bring the weighted harmonics down to a lower level. Also maximum beta function and dispersion become smaller. Figure (5) and (6) shows the total weighted harmonics after compesation for the current layout and the new layout.

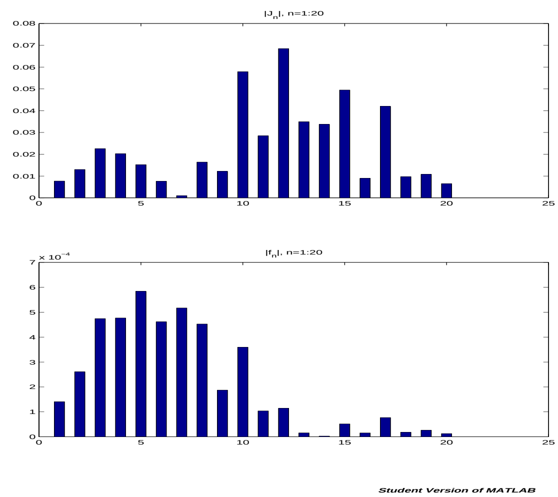

Figure 5: The weighted harmonics after compsensation, $|J_p|/(\nu^2 - p^2/4)$ and $|f_n|/(\nu^2 - n^2)$ for current layout

The solution obtained by harmonic compensation might not be the best one possible. A local maximum search in the 24-dimension space is then necessary. The search starts from the solution obtained by harmonic compensation, then change each quadrupole until it won't bring the objective function down. The obejective is the maximum horizontal radius of the beam defined as

$$R_x = \sqrt{\frac{\beta_x \epsilon_x}{\pi}} + D_x \frac{\Delta p}{p_0} \tag{2.9}$$

where ϵ_x is the 95% horizontal emittance. Typical values $\epsilon_x = 15\pi\ \mathrm{m}\cdot\mathrm{mrad}$ and $\frac{\Delta p}{p_0} = \pm 0.3\%$ are used. This can often improve the solution slightly.

3 Result

For both the current lattice and the one with respositioned dogleg 03, we can improve the linear optics properties with the scheme described in the last section. We show the results by making comparsions between 5 cases.

case 1 Ideal lattice, no doglegs

case 2 Current lattice, with two doglegs, no compensation

case 3 New lattice, dogleg 03 repositioned, no compensation

case 4 Current lattice, with two doglegs, with compensation

case 5 New lattice, dogleg 03 repositioned, with compensation

For each case, we compare the maxima of β_x, β_y, D_x and R_x as defined in equation (2.9). The quatities are listed in Table (2).

case	max(β_x)	max(β_y)	max(D_x)	R_x (mm)
1	33.68	20.46	3.19	32.04
2	46.95	24.16	6.14	43.86
3	40.88	23.00	4.58	38.02
4	40.61	23.16	4.19	34.53
5	37.66	22.60	4.01	34.23

Table 2: compare maxima of β_x, β_y, D_x (in meters) and R_x.

The original and compensated horizontal beta function and dispersion are shown in Figure (7) and (8) for the two lattice layouts. For both cases, we can see that the regularity of the ideal lattice function is partly restored by compensation.

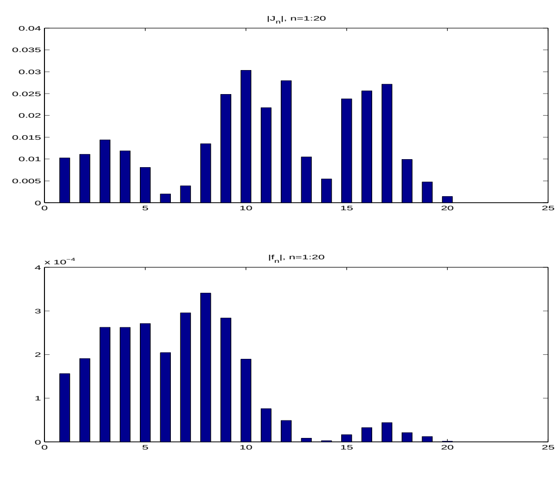

Figure 6: The weighted harmonics after compsensation, $|J_p|/(\nu^2 - p^2/4)$ and $|f_n|/(\nu^2 - n^2)$ for new layout

The quadrupole current values for case 4 and case 5 are well below the limit 2A. The tune shifts are small. $Q_y = 6.78111$ and $Q_x = 6.71748$ for case 4. $Q_y = 6.77953$ and $Q_x = 6.74996$ for case 5. Tunes can be nearly independently adjusted by modifying quadrupole corrector AC current *itunel* and *itunes*.

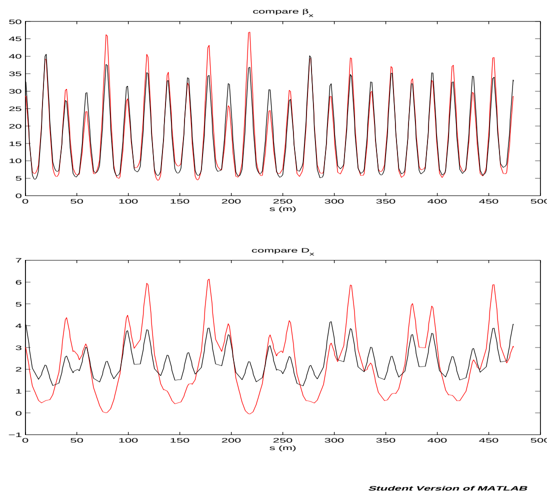

Figure 7: Comparison of horizontal beta function and dispersion for the current dogleg layout. Red curve is original and black one is compensated. Top, β_x. Bottom, D_x.

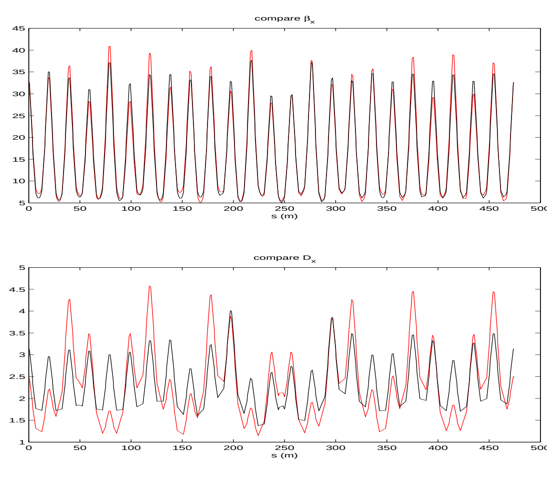

Figure 8: Comparison of horizontal beta function and dispersion for the new dogleg layout with repositioned dogleg 03. Red curve is original and black one is compensated. Top, β_x. Bottom, D_x.

APPENDIX Local Beta Bump and Its Application in the Booster

Under some circumstances, we may want to suppress (or increase) the beta function at one location keeping it intact elsewhere. Here we show how this can be done with three quadrupoles. One is placed at the location and the other two beside it, one upstream and one downstream, respectively.

For the normalized betatron phase-space coordinates Y, $\mathcal{P}$, which are related to the usual coordinates y, y' by

$$\begin{pmatrix} Y \\ \mathcal{P} \end{pmatrix} = \begin{pmatrix} 1/\sqrt{\beta} & 0 \\ \alpha/\sqrt{\beta} & \sqrt{\beta} \end{pmatrix} \begin{pmatrix} y \\ y' \end{pmatrix}$$

the transfer matrix is

$$M(s_2|s_1) = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} \tag{A-1}$$

where ψ is the phase advance from s_1 to s_2. It is easy to see that the transfer matrix for a thin quadrupole is

$$M = \begin{pmatrix} 1 & 0 \\ \beta[k\Delta l] & 1 \end{pmatrix} \tag{A-2}$$

where $k = \pm\frac{B'}{B\rho}$ with '$-$' for horizontal plane and '$+$' for vertical plane.

Recalling that the linear optics functions β, α and γ transfer according to [1]

$$\begin{pmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{pmatrix} = \mathbf{M} \begin{pmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{pmatrix} \tag{A-3}$$

with

$$\mathbf{M} = \begin{pmatrix} M_{11}^2 & -2M_{11}M_{12} & M_{12}^2 \\ -M_{11}M_{21} & M_{11}M_{22} + M_{12}M_{21} & -M_{12}M_{22} \\ M_{21}^2 & -2M_{21}M_{22} & M_{22}^2 \end{pmatrix} \tag{A-4}$$

we get

$$\begin{pmatrix} \beta_2^N \\ \alpha_2^N \\ \gamma_2^N \end{pmatrix} = \begin{pmatrix} c^2 & -2cs & s^2 \\ cs & c^2 - s^2 & -cs \\ s^2 & 2cs & c^2 \end{pmatrix} \begin{pmatrix} \beta_1^N \\ \alpha_1^N \\ \gamma_1^N \end{pmatrix} \tag{A-5}$$

for a section with $c = \cos\psi$ and $s = \sin\psi$, and

$$\begin{pmatrix} \beta^{N+} \\ \alpha^{N+} \\ \gamma^{N+} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -\beta[k\Delta l] & 1 & 0 \\ (\beta[k\Delta l])^2 & -2\beta[k\Delta l] & 1 \end{pmatrix} \begin{pmatrix} \beta^{N-} \\ \alpha^{N-} \\ \gamma^{N-} \end{pmatrix} \tag{A-6}$$

for a thin quadrupole. Superscript N indicates properties in the normalized coordinates (normalized to the unperturbed lattice), $+$ indicates the downstream side and $-$ indicates the upstream side.

In normalized coordinates, the unperturbed lattice has $(\beta^N, \alpha^N, \gamma^N) = (1, 0, 1)$ everywhere. Let's label parameters at the three quadrupoles with subscript 1,2,3 with 1 for the one at upstream, 3 for the one at downstream and 2 in between. By applying the quadrupole correctors, we want to meet three conditions

1 $\beta_2^N = r$

2 $\beta_3^N = 1$

3 $\alpha_3^{N+} = 0$

In condition 1, r is a pre-set value to specify how much change we want to make at location 2 (e.g. 0.95, to suppress β_2 to 95% of its original value) and the other two conditions are required so that the perturbation is confined in the range between quadrupole 1 and 3.

According to equation (A-6), a thin quadrupole will not change β^N at its location. But it will change α^N to $\alpha^N - \beta[k\Delta l]\beta^N$ and γ^N will be changed according to $1 + \alpha^2 = \beta\gamma$. Thus at the downstream side of quadrupole 1, we have

$$\begin{pmatrix} \beta_1^N \\ \alpha_1^{N+} \\ \gamma_1^{N+} \end{pmatrix} = \begin{pmatrix} 1 \\ -x_1 \\ 1 + x_1^2 \end{pmatrix} \tag{A-7}$$

with $x_1 = \beta_1[k\Delta l]_1$. Combining with equation (A-5), the condition $\beta_2^N = r$ becomes

$$s_1^2 x_1^2 + 2c_1 s_1 x_1 + (1 - r) = 0 \tag{A-8}$$

in which $c_1 = \cos\psi_{12}$,$s_1 = \sin\psi_{12}$. Also we let $c_2 = \cos\psi_{23}$ and $s_2 = \sin\psi_{23}$. Equation (A-8) has 2 roots of x_1 in which we pick up the one with smaller absolute value. Quadrupole 2 turns $(\beta_2^N, \alpha_2^{N-}, \gamma_2^{N-})$ to $(\beta_2^N, X_2, 1 + X_2^2)$ with $X_2 = \alpha_2^{N+} = \alpha_2^{N-} - x_2\beta_2^N$. The latter is then transfered to quadrupole 3 to meet condition $\beta_3^N = 1$, which yields

$$\frac{s_2^2}{r} X_2^2 - 2c_2 s_2 X_2 + (\frac{s_2^2}{r} + c_2^2 r - 1) = 0 \tag{A-9}$$

Again we choose the solution with smaller absolute value,

$$x_2 = \frac{\alpha_2^{N-} - X_2}{\beta_2^N} \tag{A-10}$$

Now we can calculate α_3^{N-} with equation (A-5)

$$\alpha_3^{N-} = c_2 s_2(r - 1 - X_2^2) + (c_2^2 - s_2^2)X_2 \tag{A-11}$$

Condition $\beta_3^N = 1$ and condition $\alpha_3^{N+} = \alpha_3^{N-} - x_3\beta_3^N = 0$ together lead to

$$x_3 = \alpha_3^{N-} \tag{A-12}$$

The integrated strengths of quadrupole 1, 2, 3 can thus be determined with x_1,x_2 and x_3 known.

As an example, we apply this local beta bump scheme to Fermilab Booster. To change linear optics of the horizontal plane effectively, we use QS quadrupoles. Since the solution to equation (A-8) is of the form

$$x_1 = \frac{1}{s_1}(-c_1 \pm \sqrt{r - s_1^2}) \tag{A-13}$$

and to equation(A-9)

$$X_2 = \frac{r}{s_2}(c_2 \pm \frac{1}{r}\sqrt{r - s_2^2}) \tag{A-14}$$

$r > s_{1,2}^2$ must be ensured for a meaningful solution to exist. For the Booster, the phase advance over one section is about 100 degrees. So if we want to suppress β_x at section n, we have to use QS at section n-2, n and n+2. For the current dogleg layout, with all quadrupole correctors turned off, the maximum of β_x is 46.95m at S11. To suppress 10% of it, the procedure described above requires

- iqs09=0.6490A
- iqs11=-0.7202A
- iqs13=0.5674A

β_x at S11 becomes 42.47m, which is 90.5% of the unperturbed value. Figure (9) shows the change of β_x and dispersion throughout the ring. There is little change to β_x out of the range from QS09 to QS13. However, inside it, at QS10 and QS12, β_x is increased. And dispersion is perturbed globally. We can apply this method together with harmonic compensation to constrain dispersion. The method can still be useful despite the two side effects.

Since our knowledge about the unperturbed lattice is not perfect, we are interested in the sensitivity of the locality to errors in beta functions and phase advances. A Monte Carlo simulation is carried out for it. For the above example the result is shown in Figure (10) and Figure (11). When the error of phase advance is in the level of 2 degrees, the global perturbation is roughly 30% of the desired local bump.

References

[1] S. Y. Lee: *Acclerator Physics*, World Scientific, Chapter 2

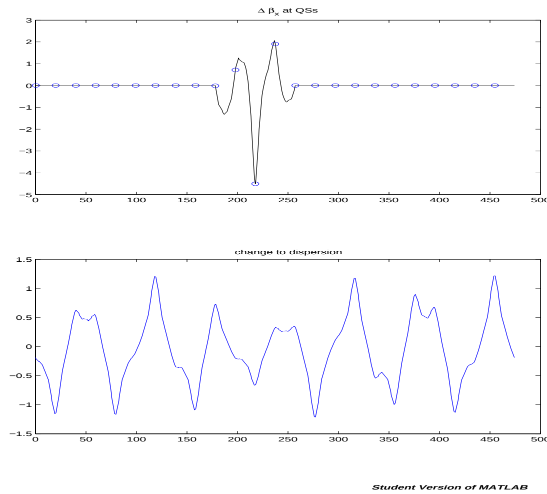

Figure 9: Changes to β_x and dispersion due to QS09, QS11, QS13 set up for local beta bump. Top, β_x. Bottom, D_x.

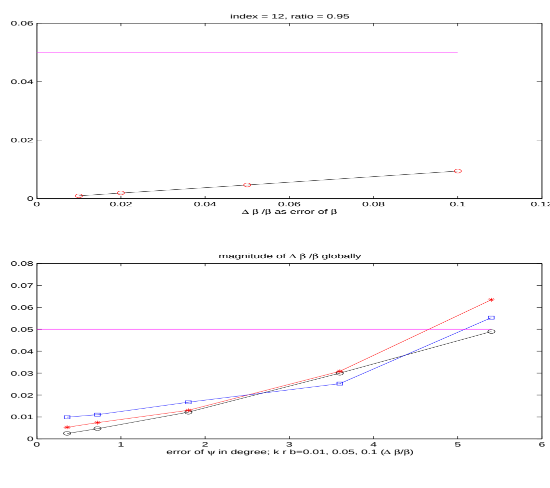

Figure 10: Magnitude of global $\frac{\Delta\beta}{\beta}$ indicating the locality error. To suppress 5% of β_x at S11 of the current layout. Top, consider β error only. Bottom, both phase and β considered, with $\frac{\Delta\beta}{\beta} = 0.01, 0.05, 0.1$ for black, red and blue curves, respectively

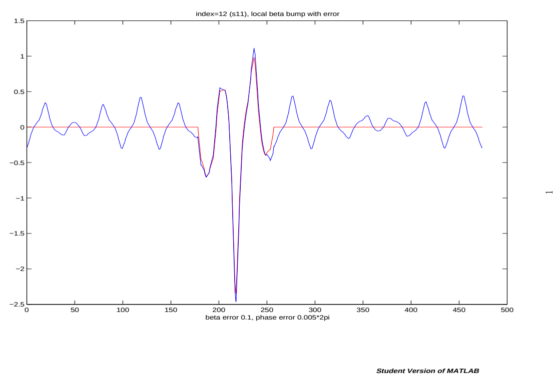

Figure 11: Changes to β with (blue) or without (red) error in the case of suppressing 5% of β at S11 of the current layout. Phase error is 2 degrees and $\frac{\Delta\beta}{\beta} = 0.1$