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BEAM STORAGE STUDIES IN THE FERMILAB MAIN RING

J. A. MacLachlan

May 6, 1982

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Abstract

Bunched beams of 100 and 150 GeV have been stored in the Fermilab Main Ring for periods of up to one hour. The observations of beam current and beam profiles are analyzed for the effects of gas scattering, chromaticity and non-linear magnetic field.

Talk given at the Workshop on Accelerator Orbit and Tracking Programs, May 3-6, 1982, at Brookhaven National Laboratory, Upton, Long Island, NY.

The first studies of beam life time in the Main Ring directed toward colliding beam application were carried out in 1976 and 1977[1]. The work up to December 1976 was reported at the 1977 Particle Accelerator Conference[2]. There are several excellent articles in the Proceedings of the 1977 Fermilab Summer Study[3] on the work up to August 1977. Beginning in 1978 the character of the studies changed somewhat as the idea of p-p collisions between beams in Main Ring and the Energy Doubler/Saver was dropped in favor of p-$\bar{p}$ operation in the ED/S[4]. Thus, such consideration as developing a low-β insertion and measuring backgrounds in the proposed collision straight section were dropped; attention was focused on the basic problems of long term storage and technical studies of the beam manipulations required for $\bar{p}$ production and $\bar{p}$ acceleration. In this talk I will concentrate on measurements taken in early 1980 which provide a good basis for a discussion of the dynamic aperture of the Main Ring, chromaticity effects, effects of high order resonances, and non-linear coupling[5].

In Fig. 1 one sees a typical beam survival curve taken during a 100 GeV store. The qualitative explanation for this behavior is straightforward. There is a very slow loss during the early part of the store from nuclear scattering and an exponential loss during the late part of the store as the diffusion begins to bring a significant fraction of the beam out to amplitudes for which the betatron oscillation is unstable. One of the questions we will want to address is whether the source of that diffusion is also scattering, i.e., multiple coulomb scattering, or involves some other mechanism such as multiple crossing of non-linear resonances as well. To that end we compare the data to a basic model which incorporates the effects of gas scattering only. In this model, then, we will expect a slow early loss dependent on pressure, dependent on composition of the residual gas ($\sim\overline{A^{2/3}}$), but independent of beam energy. We also expect an exponential loss late in the store with a time constant dependent on pressure, dependent on gas composition ($\sim\overline{Z^2}$), and a momentum dependence of p^{-2}. Another consequence of multiple coulomb scattering is a growth of the rms beam width proportional to $t^{1/2}$. Diffusion due to the presence of high order resonances on the other hand should be very sensitive to the choice of operating point (ν_x, ν_y).

The early loss is then assumed to be the result of nuclear collisions leading to a slow exponential loss of beam current

$$I = I_o \, e^{-\Lambda t} \tag{1}$$

where

$$\Lambda = \frac{1}{I_o} \frac{dI}{dt}\Big|_{t=0} = \frac{\beta c}{760} \sum_i \rho_i \, P_i/\lambda_i . \tag{2}$$

In these equations t is the time in seconds, βc is the proton velocity, ρ_i is density at standard temperature and pressure of the i^{th} component which has a partial pressure P_i. The composition of the gas is not really well known. We use in the model a composition similar to that measured in regions of good vacuum and typical of what can be expected from an unbaked stainless steel vacuum chamber, viz., 40% H_2, 40% CO, and 20% H_2O. For this composition Λ=430P where P is the absolute pressure in Torr. Pressures are measured in the ring by reading back the ion pump currents. Therefore to establish the true pressure one must correct for conductance between pumps (a factor $\sim$1.5), gas composition, and pump leakage current. In consequence, the average gas pressure is not a well measured quantity and the value is inferred by combining a number of observations including the beam loss rate into a self-consistent set.

One can find from the diffusion equation in the four dimensional transverse phase space an expression for the beam current as a function of time[6]. In this expression, the correction for the loss arising from single scattering has also been included:

$$\frac{I(t)}{I_0} e^{\Lambda t} = F(y) = -\sum_{m=1}^{\infty} e^{-yx_m^2}/J_0(x_m) \tag{3}$$

where

$$y = \frac{1}{2}\left(\frac{a_0}{a}\right)^2 + \frac{D\beta}{a^2} t \tag{4}$$

and x_m is the mth zero of the Bessel function $J_1(x)$, a_0 is the rms width of an initial gaussian beam distribution, a is the critical betatron amplitude or "dynamic aperture", D is the diffusion constant in m/s, and β is the average Courant-Snyder beta. Although the precise coefficients in the series of eqn. 3 do depend on the choice of initial distribution, the asymptotic decay constant is the same for all distributions and the assymptotic solution becomes established rather promptly because the x_1 is substantially smaller than the higher eigenvalues. Therefore, the time constant for the late loss is

$$\tau^{-1} = x_1^2 \, D\beta / a^2 . \tag{5}$$

If the source of the diffusion is multiple coulomb scattering only, the diffusion constant is

$$D = (\beta/4)(15/p\beta)^2 \; c/x_0 \tag{6}$$

where p is the proton momentum, c is the velocity of light, and the β in the denominator is the relativistic velocity factor v/c. The quantity x_0 is the radiation length in meters for the residual gas

$$x_0 = X_0 \,|_{STP} (\frac{P}{760}) \; . \tag{7}$$

Either from the solution of the diffusion equation or directly by averaging small random kicks over many beam turns one finds linear growth of the spatial second moments of the beam distribution

$$\sigma^2(t) = a^2/4\ G(y) = -\frac{1}{F(y)} \sum_m \frac{(x_m - 8/x_m)}{x_m J_0(x_m)} e^{-x_m^2 y}$$

$$\underset{y\to 0}{\rightarrow} 2a^2 y = a_0^2 + 2\beta\ Dt \qquad (8)$$

Thus, from the growth of the beam width one can infer the diffusion at early time t so long as $a_0 \ll a$, i.e. so long as intensity loss is insignificant. Equation 5 allows one to infer the diffusion constant from the long time beam loss; the two evaluations are independent but both based on the MCS diffusion model.

In Fig. 2 are plotted the second moments determined by fitting the vertical and horizontal profiles with a gaussian at various times in a 100 GeV store from early 1980. These profiles, taken by collecting electrons produced in the ionization of the residual gas, are statistically inferior to data of a similar kind gathered during earlier studies simply because of improved accelerator vacuum. Despite the spread of data points, both profiles indicate a convincing linear time dependence. Note, however, that the slopes are not the same. This can be understood as a consequence of longitudinal diffusion because the profile monitor is in a region with dispersion η=2m. It is known that the main ring transverse emittances are approximately equal so that in the expression for the horizontal beam width including dispersion we can represent the betatron amplitude contribution with the measured vertical beam spread[7].

$$\sigma_x{}^2 = (\eta\frac{\delta p}{p})^2 + \frac{\beta_x}{\beta_y}\sigma_y{}^2 \tag{9}$$

where both momentum and betatron amplitude distributions are assumed to be gaussian. Thus,

$$(\frac{\delta p}{p})^2 = \sigma_p(t)^2 = \frac{1}{\eta}\,(\sigma_x{}^2(t) - \frac{\beta_x}{\beta_y}\sigma_y{}^2(t)). \tag{10}$$

The data in Fig. 2 give $\sigma_p(0)\sim .5\times10^{-3}$ and $\sigma_p(1000)\sim10^{-3}$, numbers which are consistent with bunch width measurements taken during the studies.

Table I summarizes the observations on 10 stores made in January 1980, including eight at 100 GeV and two at 150 GeV. The table includes measured average pressure inferred by correcting the value measured at the ion pumps by a calculated chamber conductance correction factor of 1.5 and the three independent determinations made from the data according to eqn. 2 for nuclear scattering and according to eqs. 5 and 8 for multiple coulomb scattering. Because each of these determinations has its own scaling in such quantities as average atomic number, average atomic weight, momentum, etc., inconsistencies of the four numbers may be interpreted as a measure the of the diffusion not driven by multiple scattering. Unfortunately, because of uncertainties arising from imprecisely known parameters like gas pressure and composition, the pressure numbers must be used as well to evaluate the consistency of the basic model. The data in Table I are not sufficient in themselves to make these simultaneous determinations convincingly. Rather the tabulated data, which are drawn only from Ref. 5, are a good sample to illustrate assertions most of which can be justified only by drawing as well upon a larger body of data which are reported in the other references. They are a special sample in that they include the stores with the longest survival times. We conclude in this broader context that the agreement of the pressure determinations is reasonable, especially if we choose to attribute the consistently higher values obtained from $\sigma_y^2(t)$ to plausible deficiencies of the ion beam scanner profile monitor.

Table I records two 100 GeV stores (No. 4&5 of 1/3/80) made with the beam stored significantly off the central closed orbit. The 100 GeV chromaticity was corrected rather well as shown in Fig. 3. The dynamic aperture inferred from eqs. 3&4 is smaller for these than for the other 100 GeV stores although certainly not dramatically so. More evidence from earlier studies on the effects of chromaticity correction is give in Fig. 4 and Table II. Figure 4 shows beam half life plotted against the $\Delta p/p$ offset of the orbits. The curve joins points taken with the horizontal chromaticity corrected as shown in Fig. 5 but vertical chromaticity nearly at its uncorrected value[8]. The points plotted with triangles apply to lifetime with the chromaticity correcting sextupoles turned off but with the tunes set to the same values applying for the measurements with corrected chromaticity. Table II shows a comparison of lifetimes for relative height of the loss peaks one can say in general that the fifth order resonances can reduce the beam lifetime by one to two orders of magnitude. The experiment was not made in fine enough steps nor the actual crossing of the resonance sufficiently carefully executed to justify quantitative conclusions about the relative strength of the lines.

It is conventional wisdom that, in the absence of tune sweeping, the beam can sit on a resonance of fifth or higher order without loss. Thus, we attribute the loss to diffusion driven by multiple crossing, an interpretation roughly consistent with the claimed tune spread $\Delta\nu \sim .005$ full width and the valleys between loss peaks. Although the peaks do seem on the broad side, some loss is probably attributable to somewhat less stable conditions obtained during the actual tune change. That is, some sudden as well as gradual losses were noted during the tune changes.

Resonances lower than 5th order are a qualitatively different case. Store No. 6 on 1/3/80 was intentionally set very near a fourth order vertical resonance by taking ν_y=19.245. Besides the markedly shorter beam survival one can see from the profile growth rates plotted in Fig. 11 that the linear growth in second moments σ^2 expected from the diffusion model is not observed even though the beam survival curve has a typical form. The results of applying the diffusion model analysis to this fourth order resonance case is indicated by the figures in parentheses in Table I. The inappropriate model gives an absurd result that dynamic aperture is, if anything, a little larger but the pressure has increased by a factor of $\sim$6.

A byproduct of the studies described above is an indication of the order of zeroth harmonic non-linear fields in the main ring arising from observations of the horizontal-vertical coupling as a function of momentum offset from the central orbit noticed during chromaticity meaurements[10]. The chromaticity was measured by recording the horizontal and vertical tunes of coherent oscillations excited about various off momentum orbits. It was noticed that when the tunes came close together they did not vary smoothly with momentum and that the fast fourier transform of the position detector signal used to evaluate the tunes would show two lines appearing with comparable strength in both radial and vertical planes of oscillation regardless of the plane of the exciting kick. An example of the position detector data and its 256 point fast transform is given in Fig. 11 showing very strong coupling between radial and vertical oscillations. It was also noted that the effect was stronger for off-momentum orbits toward the outside of the vacuum chamber.

The strength of the coupling was explored in a systematic way by measuring the horizontal and vertical tunes while changing the quadrupole currents in such a manner that the horizontal tune should remain constant. Starting from well separated tunes, the vertical tune was swept toward and through the horizontal tune value. Examples of such data applying to different momentum values are given in Figs. 12 and 13. As the two tunes approach each other, the results of the FFT of position detector signals becomes like that shown in Fig. 11 and it becomes impossible to say what is vertical and what is horizontal tune. As the tune sweep is continued the normal modes return to the uncoupled radial and vertical modes. The value $\delta\nu_{12}$ giving the mminimum value of separation for the normal mode tunes is a direct measure of the strength of the coupling. Although the linear coupling coefficient is a complex quantity its modulus is simply

$$|K| = \frac{1}{2}\,\delta\nu_{12} \tag{11}$$

Fig. 14 gives the result of plotting this quantity against $\Delta p/p$ of the closed orbit including a check point taken at 150 GeV and one taken a year later than the rest of the 100 GeV data. The coupling is manifestly non-linear. Its greater strength toward the outside of the vacuum chamber is likely related to the same cause as the assymmetry of the beam lifetime with respect to momentum displacement shown, for example, in Fig. 4. It is also part of the machine operators' folklore that the main ring "runs better" with the beam toward the inside. This non-linearity results from the lattice as a whole and not, for example, from extraction devices because the results can be closely reproduced with these devices off and well removed from the beam.

If we pretend that the betatron amplitudes induced to measure the tunes are infitesimal compared to the radial offset between the several off-momentum orbits, we can squeeze some crude estimates of zeroth harmonic multipole strength from these observations. Converting the $\Delta p/p$ offset into an average radial offset

$$\bar{x} = \eta_{avg}\,\Delta p/p \tag{12}$$

one can fit $|K|$ vs $\bar{x}$ as

$$|K| = .0075 + 4.38\bar{x} + 2.32\times10^{3}\ \bar{x}^{2} \tag{13}$$

The linear coupling is the constant term in this expression which has about the value of ~.006 according to earlier analysis of high field coupling at $\Delta p/p\equiv 0$[11]. Ignoring the imaginary part of K one can write

$$K = \mathrm{Re}\ \{K\} = \frac{1}{4\pi} \oint \frac{(\beta_x\beta_y)^{1/2}}{B\rho} \frac{\partial B_y}{\partial y} ds \tag{14}$$

The multipoles contributing the linear and quadratic terms are respectively the skew sextupole for which

$$B_y = \frac{1}{2} S_{xy}\ xy \tag{15}$$

and the normal octupole for which

$$B_y = \frac{1}{6} O_x\ (y^3 - 3x^2 y) \tag{16}$$

The fit using just these terms is not very good because there appears to be a higher curvature for $\bar{x}>0$, i.e., something like skew decapole which would give a curvature odd with respect to $\bar{x}=0$, but the quality of the data scarcely supports such refined analysis. Using just the three terms from eqn. 13 one calculates

$$\frac{1}{B\rho}\oint\frac{\partial B_y}{\partial y}ds = 1.7\times10^{-3}\ m^{-1} \tag{17}$$

$$\frac{1}{B\rho}\oint\frac{\partial^2 B_y}{\partial x\partial y}ds = 1.1\ m^{-2} \tag{18}$$

$$\frac{1}{B\rho}\oint\frac{\partial^3 B_y}{\partial^2 x\partial y}ds = 5.7\times10^{2}\ m^{-3} \tag{19}$$

One certainly should not take these numbers very seriously, but the value in eqn. 17 is about the value reported after some effort had been made to eliminate it by rolling selected quadrupoles[12]. The values in 18 and 19 are in order of magnitude agreement with measured properties of some main ring magnets and observed Landau damping times for coherent betatron oscillations.

Acknowledgements

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References

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11. S. Ohnuma, "Linear Coupling of Betatron Oscillations in the Main Ring", Fermilab Internal Note TM-766, February 1978.

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Figure Captions

Figure 1. Beam Intensiy vs. Time for 100 GeV Beam Stored at $\Delta p/p=-0.25\%$ Off Central Orbit.

Figure 2. Horizontal and Vertical Beam Profile Second Moments During 100 GeV Storage.

Quantity	units	January 3, 1980						January 27, 1980				
		1	2	3	4	5	6	1	2	3	4	
Energy	[GeV]	100	100	100	100	100	100	150	150	100	100	INITIAL
I_o (initial Int.)	[10^{12}p]	12.2	13.2	13.1	12.5	12.5	12.6	10.6	2.9	10.3	2.9	
ν_x		19.443	19.443	19.444	19.447	19.448	19.443	19.424	19.424	19.447	19.447	
ν_y		19.432	19.432	19.428	19.424	19.428	19.245	19.448	19.448	19.432	19.432	
$\Delta p/p$ (closed orbit)	[%]	-.051	-0.47	-.202	+.119	-.319	-.100	+.065	+.065	-.004	-.004	
$\bar{P}$ (avg at ion pumps)	[10^{-8}T]	5.0	5.0	5.0	4.8	4.8	4.8	3.8	3.8	3.8	3.8	
$\sigma_x^2(0)$	[mm^2]		3.30		2.80		2.55	2.50	1.72	3.60		MEASURED
$\sigma_y^2(0)$	[mm^2]		2.10		1.99	1.75	1.90	1.46	1.22	1.85		
$d\sigma_x^2/dt$	[10^{-3}mm^2/s]		6.6		5.01		(4.07)	2.14	~2	3.12		
$d\sigma_y^2/dt$	[10^{-3}mm^2/s]		3.03		3.34	4.31	(5.1)	1.14	~2	2.25		
$1/I_o(dI/dt)_{t=0}$	[10^5p/s]	4.1	4.1	4.1	3.1	3.7	(5.0)	2.2	1.9	2.6	2.7	
P_{NS} (fr nucl scat.)	[10^{-8}T]	9.5	9.5	9.5	7.2	8.6	(12)	5.1	4.4	6.0	6.3	DERIVED
P_{MCS} (fr $d^2\sigma_y/dt$)	[10^{-8}T]		10		11	14	(17)	8.5	~15	7.5		
P_{MCS} (fr I(t))	[10^{-8}T]		6.3	6.0	5.2	4.8	(33)	5.1	4.8	4.8		
a ("dynamic aper.")	[mm]		6.0	6.2	5.5	5.6	(6.6)	5.3	4.8	7.4		
$(\delta p/p)^2_{t=0}$	[10^{-6}]		.30		.20		.16	.28	~.12	.44		
$d(\delta p/p^2)/dt$	[10^{-3}/s]		.76		.42			.25		.22		
τ (long time decay)	[h]		1.04	1.12	1.05	1.12	(.24)	2.17	1.97	1.11		
$T_{½}$ (half life from I(t))	[h]		1.2	1.3	1.2	1.3	(.28)	2.6	2.3	1.5		

TABLE II

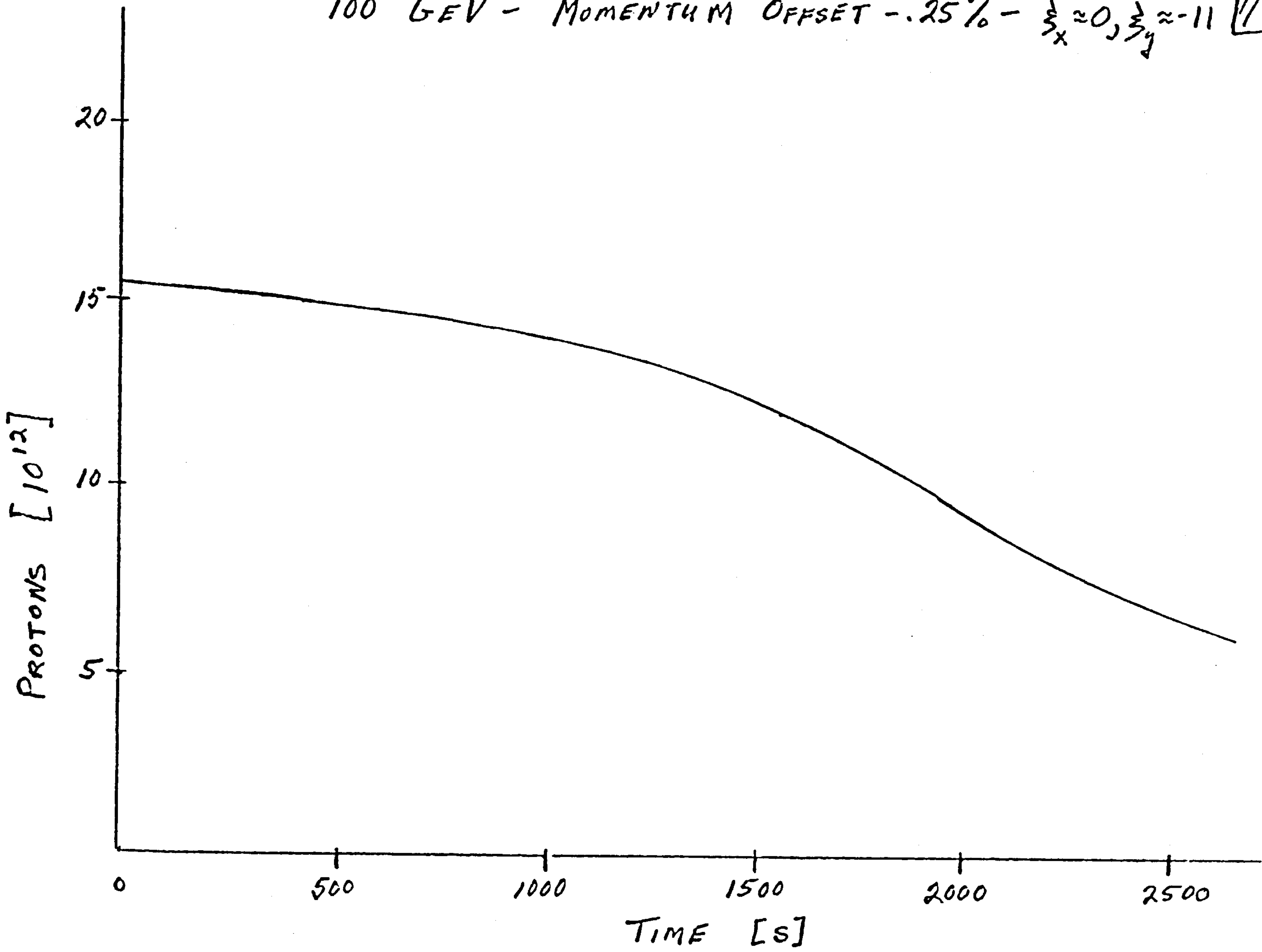
100 GeV BEAM SURVIVAL

(CLOSED ORBIT $\Delta p = 0$)

DATE	ξ_x	ξ_y	$T_{1/2}$ [s]	$\frac{1}{I_0}\left(\frac{dI}{dt}\right)_{t=0}$ [10^{-5} s]	P_{pump} [10^{-8} T]
1/79	-25	-16	2000	11.4	4.5
			1646	16	4.5
	0	-16	2070	8.8	4.5
			1985	5.9	4.5
			2951	6.5	4.5
1/80	0	0	4320	4.1	5.0
			5400	2.6	3.8

100 GEV - MOMENTUM OFFSET -.25% - $\xi_x \approx 0, \xi_y \approx -11$ 1/5/79

#3



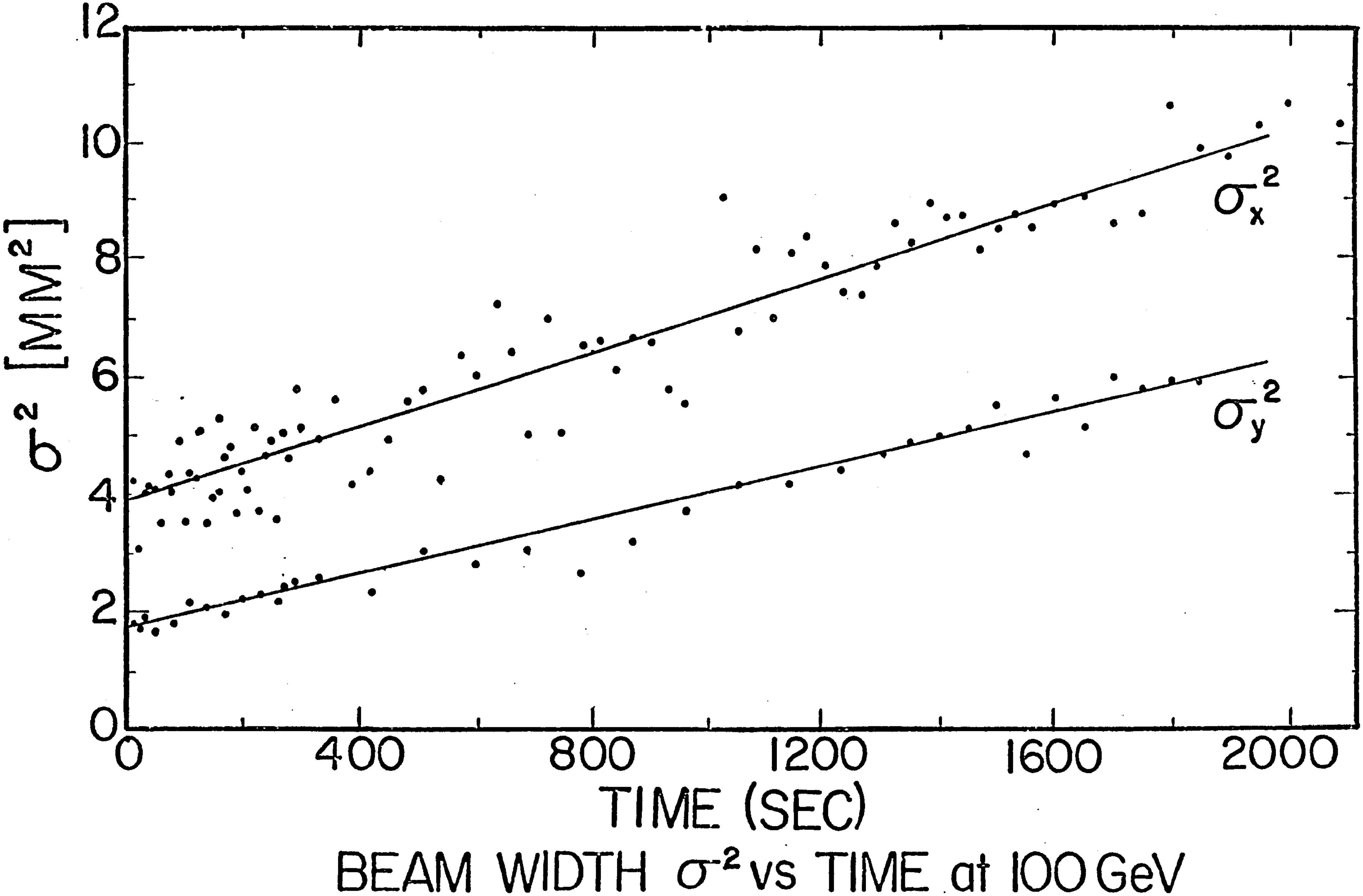
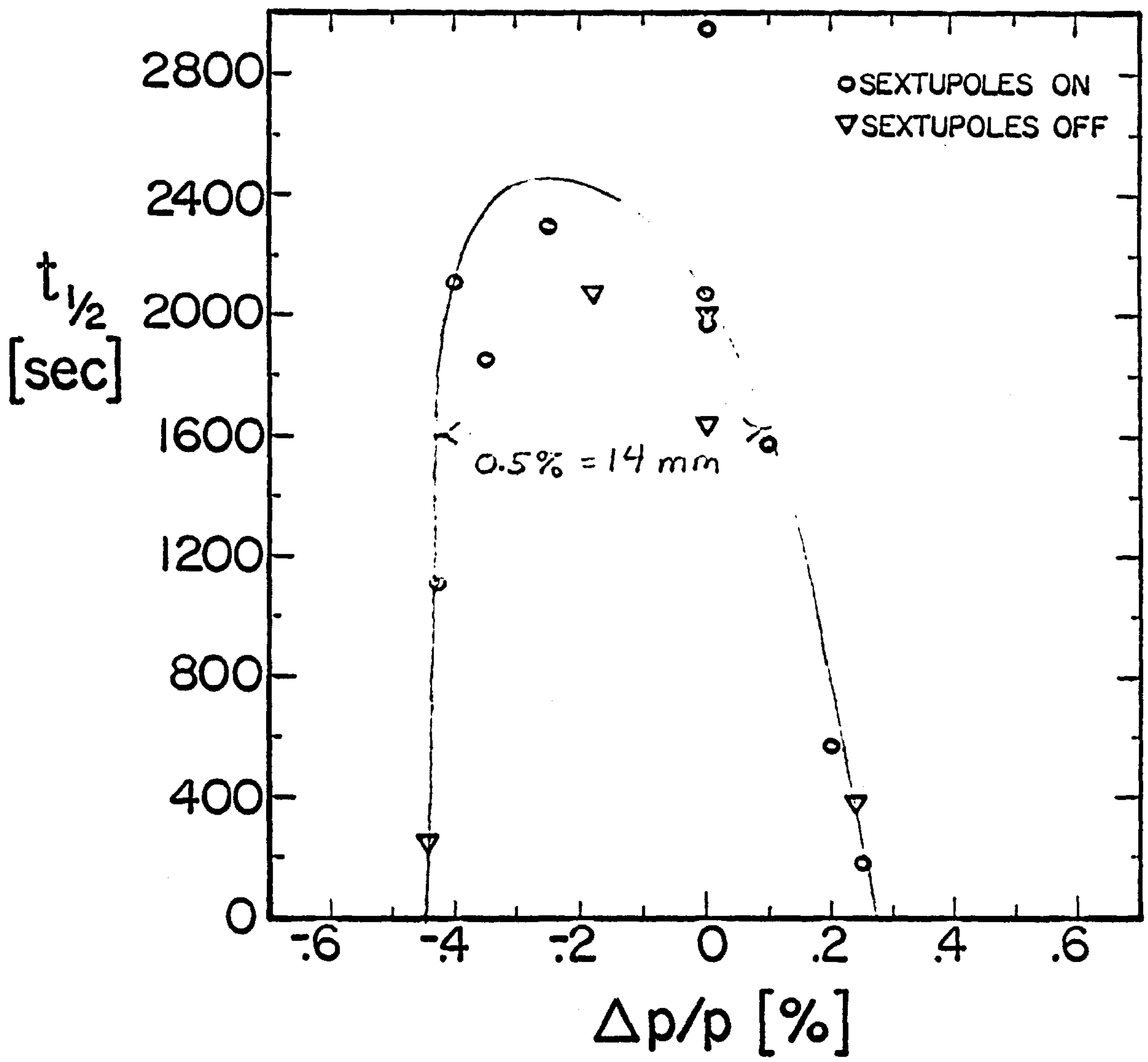
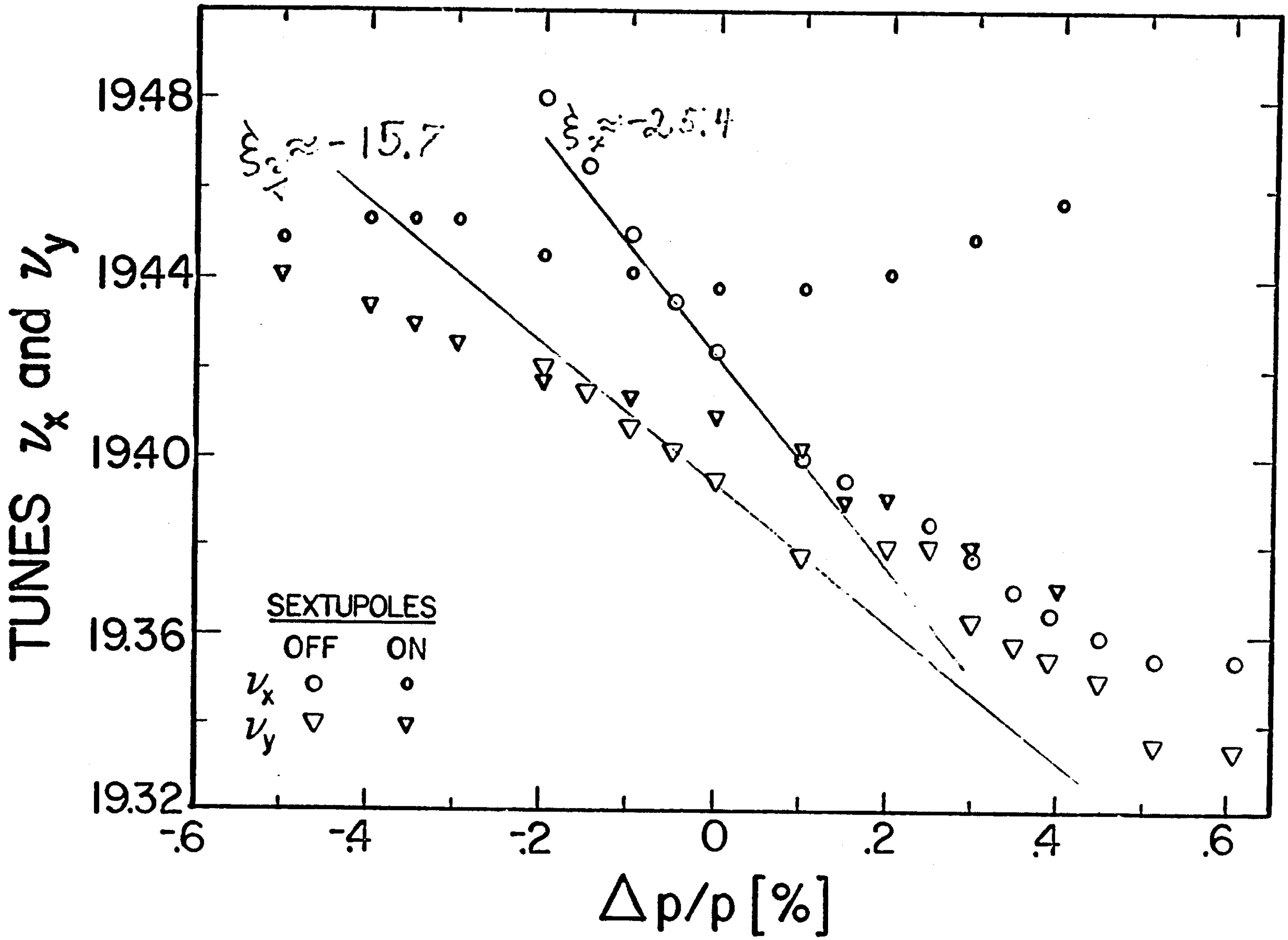

12
10
8
6
4
2
0
σ^2 [MM^2]
0
400
800
1200
1600
2000
TIME (SEC)
σ_x^2
σ_y^2
BEAM WIDTH σ^2 vs TIME at 100 GeV

Figure 2



SEXTUPOLES ON
SEXTUPOLES OFF
$t_{1/2}$ [sec]
2800
2400
2000
1600
1200
800
400
0
-.6
-.4
-.2
0
.2
.4
.6
Δp/p [%]
0.5% = 14 mm

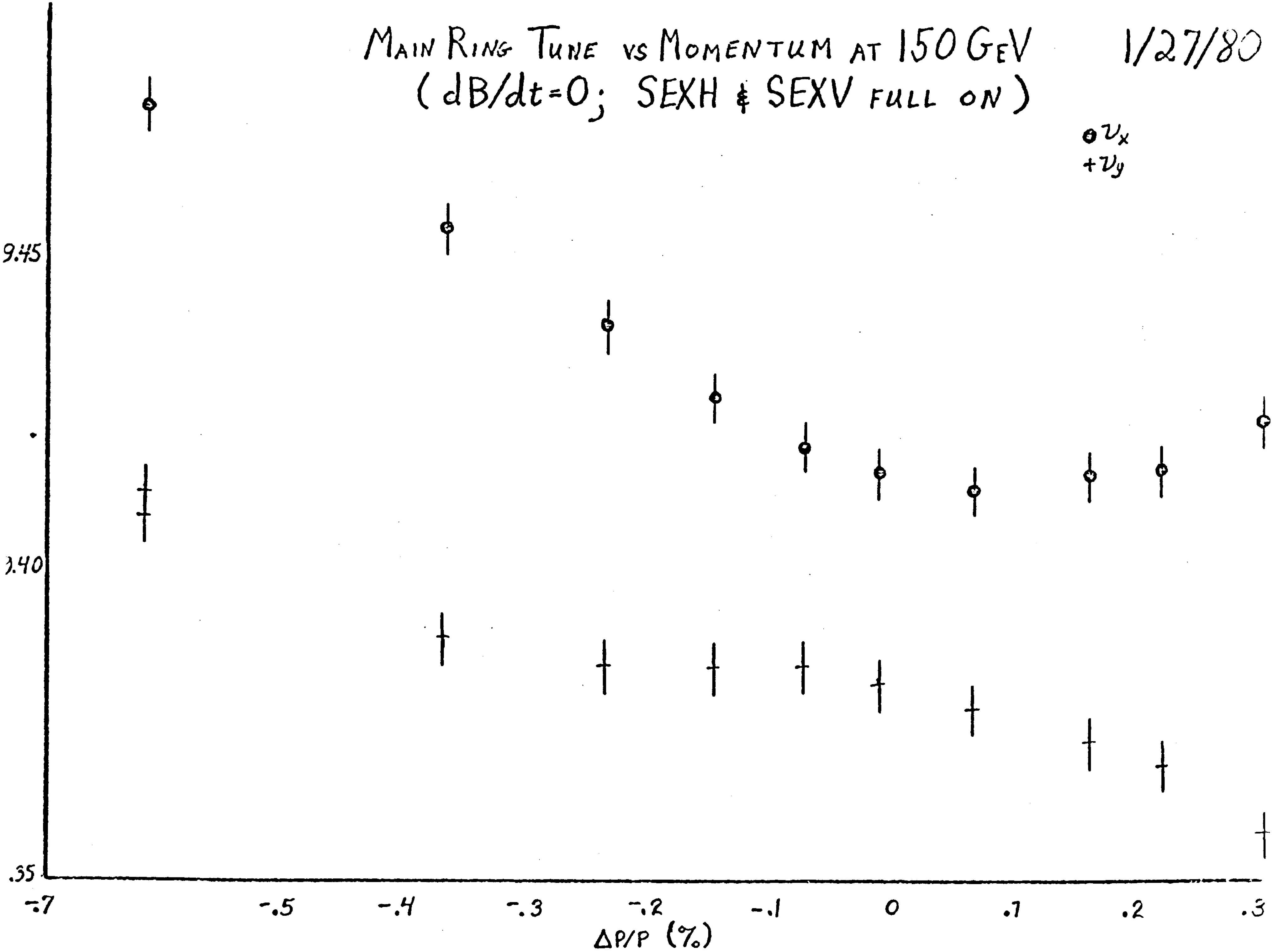
Figure 4

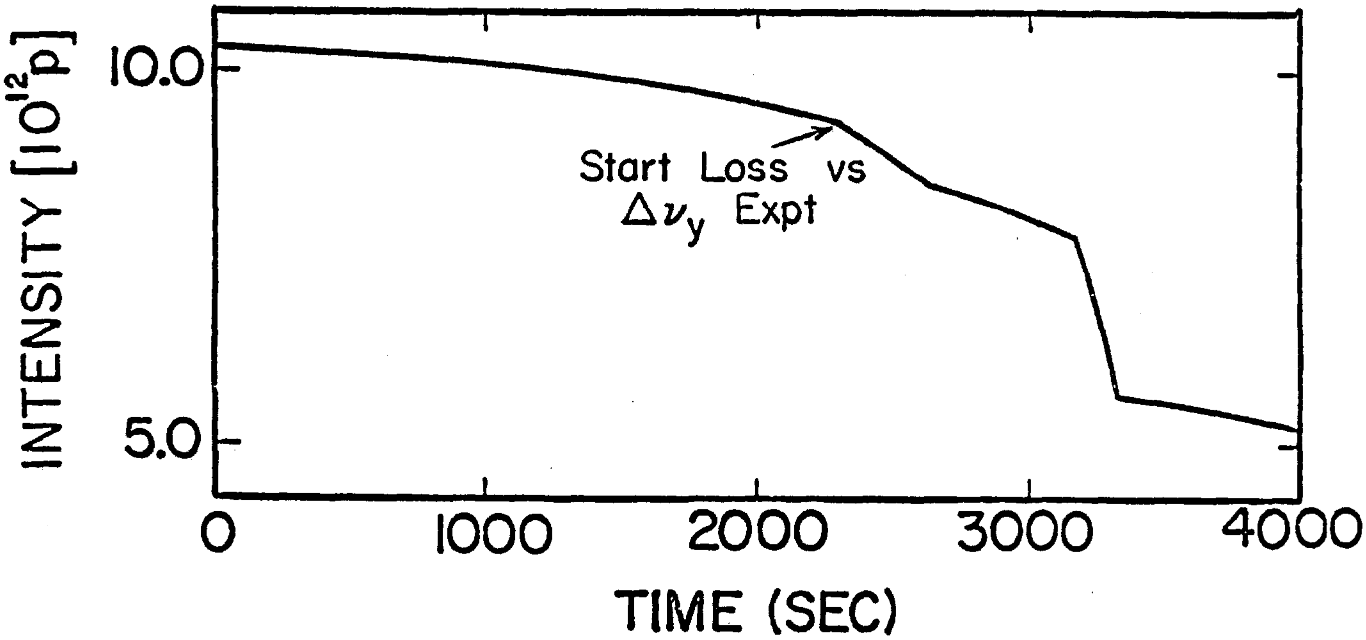


TUNES ν_x and ν_y
19.48
19.44
19.40
19.36
19.32
-.6
-.4
-.2
0
.2
.4
.6
$\Delta p/p$ [%]
$\xi_y \approx -15.7$
$\xi_x \approx -25.4$
SEXTUPOLES
OFF ON
ν_x
ν_y

Figure 5

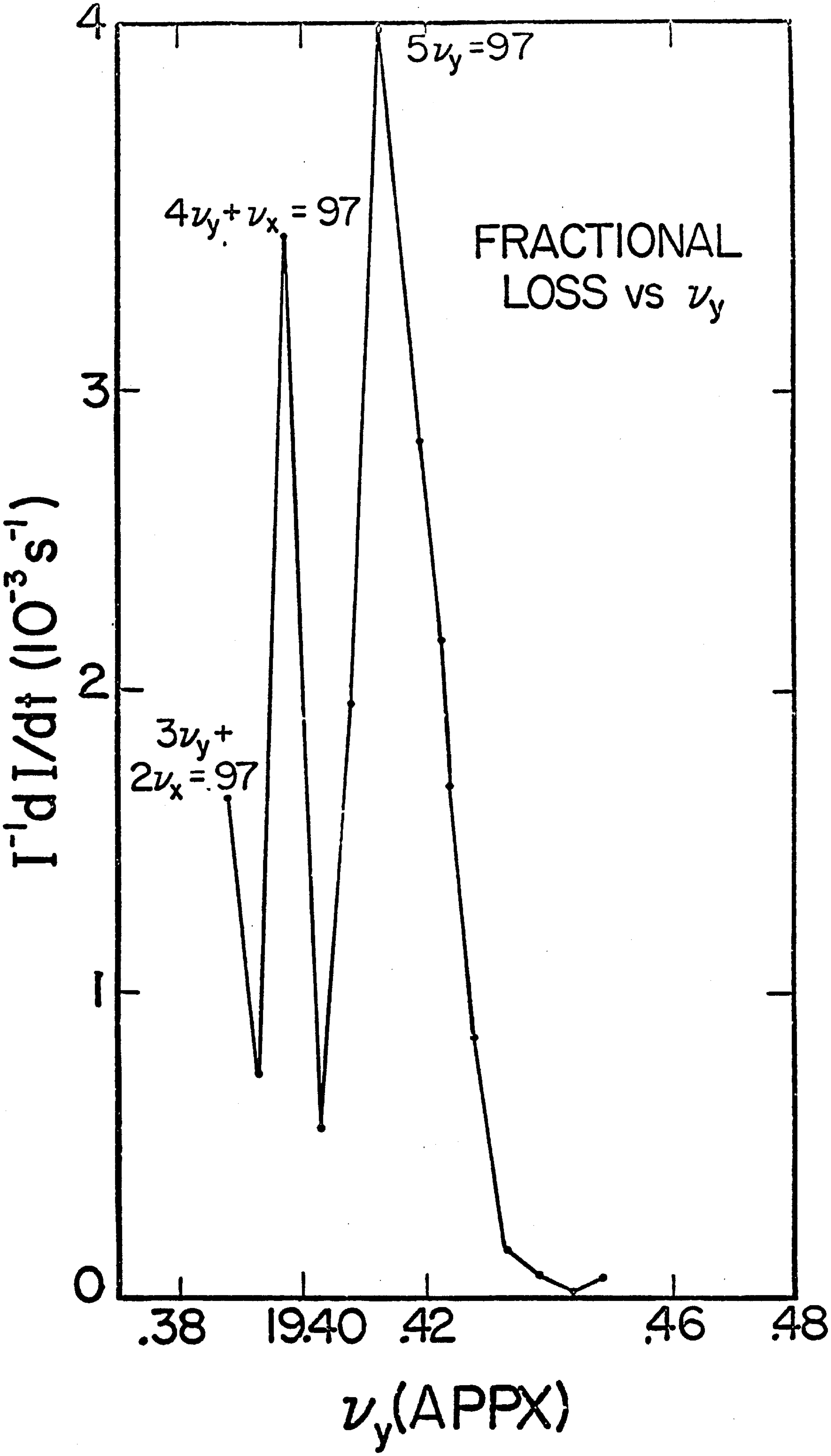
Main Ring Tune vs Momentum at 150 GeV 1/27/80
(dB/dt=0; SEXH & SEXV full on)



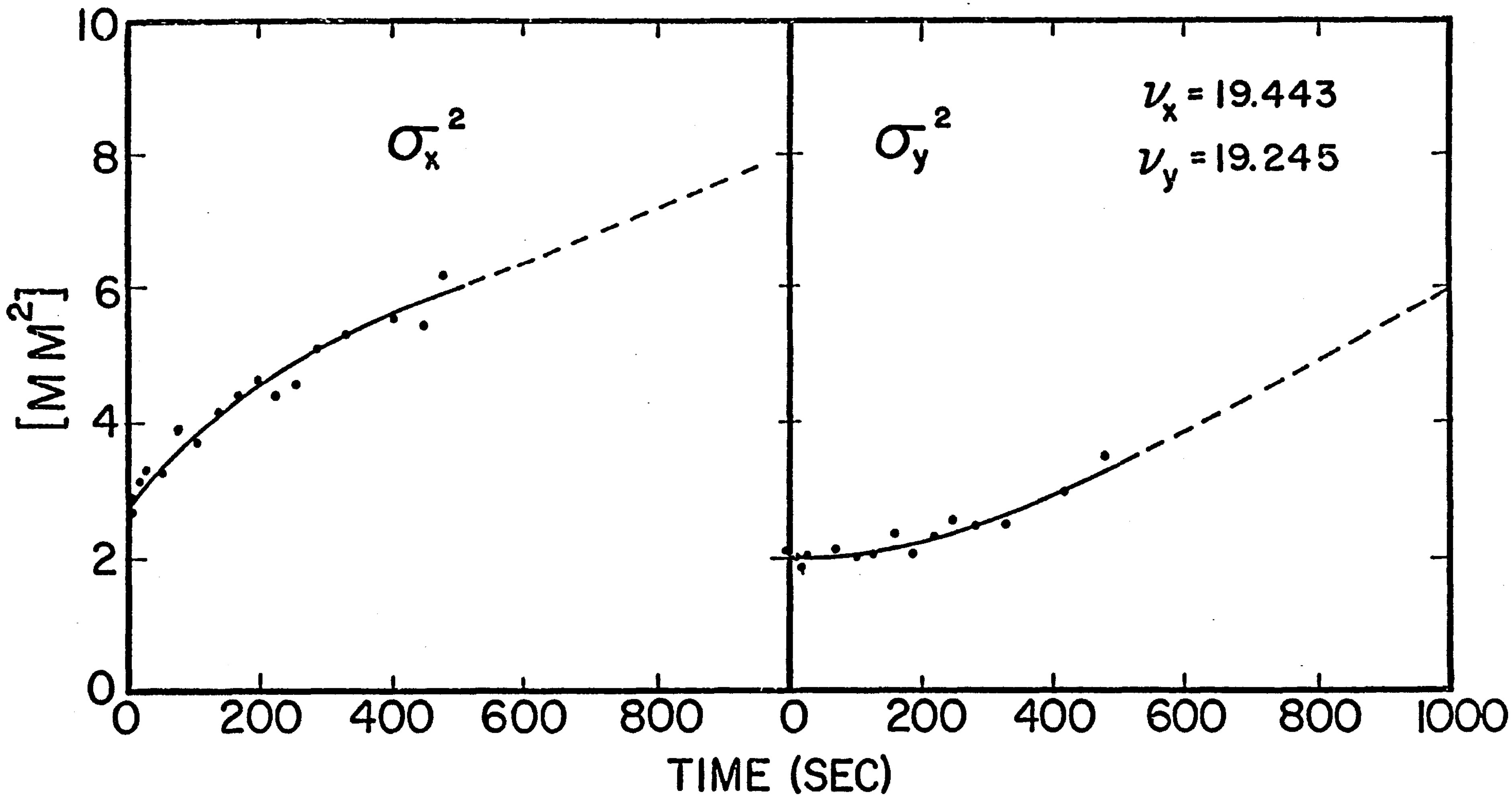


BEAM INTENSITY vs TIME 100 GeV

Figure 8



FRACTIONAL LOSS vs ν_y
$5\nu_y = 97$
$4\nu_y + \nu_x = 97$
$3\nu_y + 2\nu_x = 97$
$I^{-1} dI/dt\ (10^{-3} s^{-1})$
0
1
2
3
4
.38
19.40
.42
.46
.48
ν_y(APPX)



BEAM WIDTH σ^2 vs TIME at 100 GeV

Figure 10

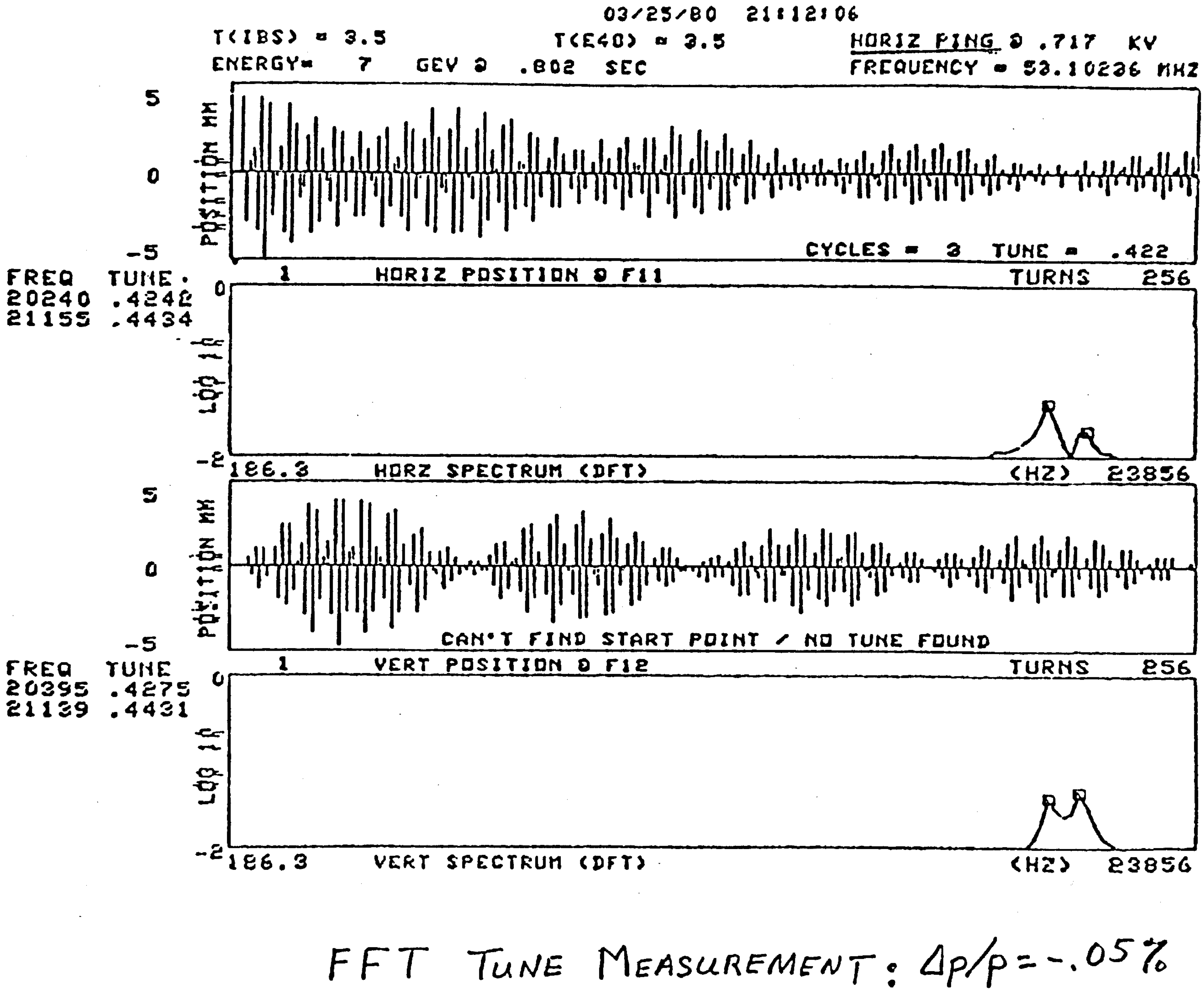
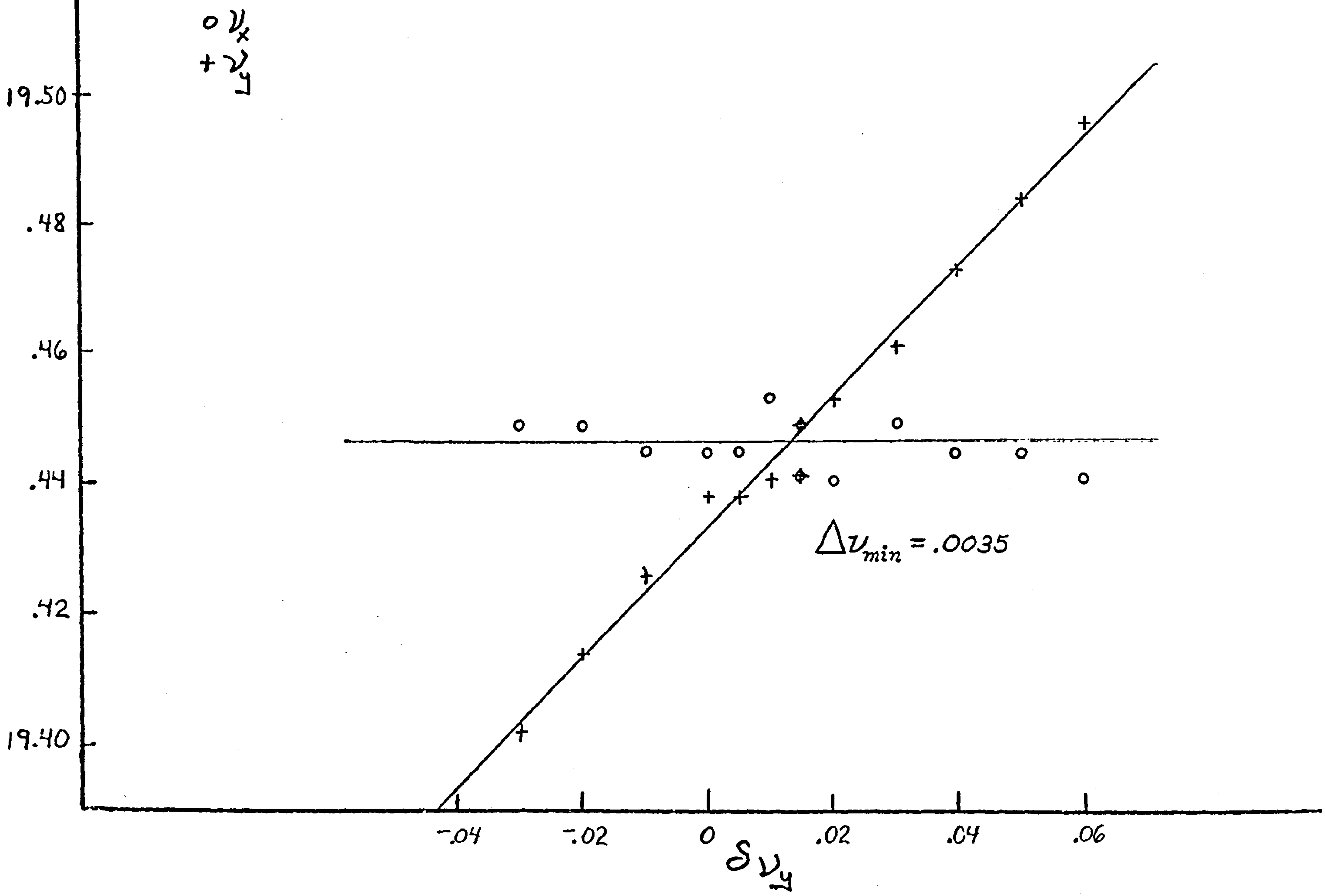
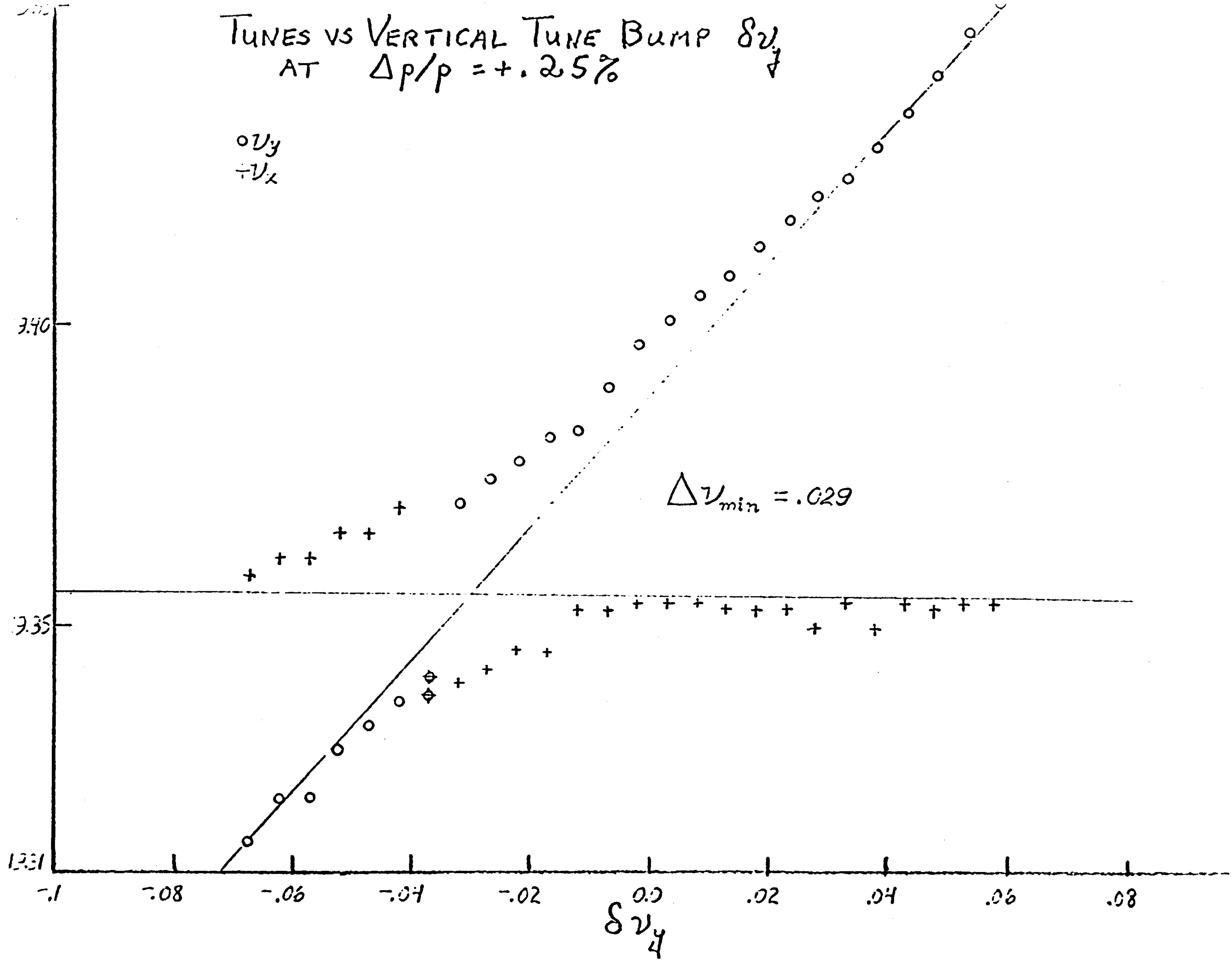

03/25/80 21:12:06
T(IBS) = 3.5
T(E40) = 3.5
HORIZ PING @ .717 KV
ENERGY= 7 GEV @ .802 SEC
FREQUENCY = 53.10236 MHZ
POSITION MM
5
0
-5
CYCLES = 3 TUNE = .422
1
HORIZ POSITION @ F11
TURNS 256
FREQ TUNE
20240 .4242
21155 .4434
LOG 10
0
-2
186.3
HORZ SPECTRUM (DFT)
(HZ) 23856
CAN'T FIND START POINT / NO TUNE FOUND
VERT POSITION @ F12
FREQ TUNE
20395 .4275
21139 .4431
VERT SPECTRUM (DFT)
FFT TUNE MEASUREMENT: Δp/p = -.05%

Figure 11

IUNES VS VERTICAL TUNE BUMP $\delta\nu_y$ AT $\Delta p/p = -.4\%$
1/6/79





TUNES VS VERTICAL TUNE BUMP $\delta\nu_y$
AT $\Delta p/p = +.25\%$
$\circ\ \nu_y$
$+\ \nu_x$
$\Delta\nu_{min} = .029$
3.40
3.35
-.1
-.08
-.06
-.04
-.02
0.0
.02
.04
.06
.08
$\delta\nu_y$

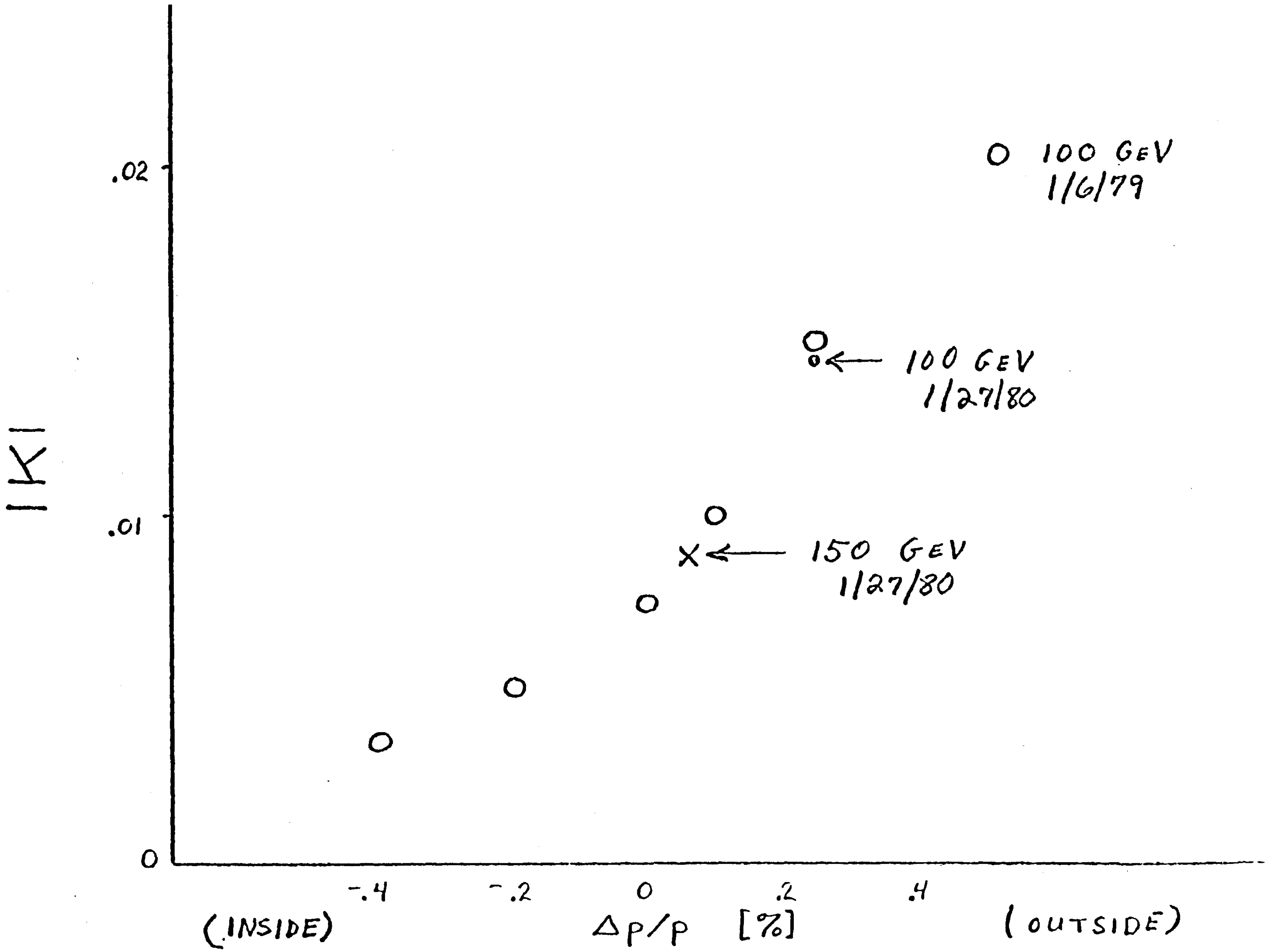


FIGURE 1: MAGNITUDE OF COUPLING VS MOMENTUM