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ERROR ANALYSIS FOR THE LOW-β QUADRUPOLES OF THE TEVATRON COLLIDER

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Because of the rather high β values in some of the low-β quadrupoles, the sensitivity of the beam to errors in these quads is enhanced. In the following we 1) exhibit the formulas giving the dependencies of beam characteristics on field errors, 2) discuss the magnitudes and tolerances of undesirable beam characteristics, and 3) suggest corrective measures wherever necessary.

We consider the effects on a particle with rigidity $(B\rho)$ arising from a single magnet of length ℓ with the following field errors:

Dipole error (for a quadrupole magnet this arises from misalignment) ΔB

Quadrupole error $\Delta B'$

Non-linear fields (2n-pole, n>2) $B^{(n-1)}$

FORMULAS

A. Effects of $\Delta B'$

1. Amplitude distortion $\Delta\beta$ is given by

$$\frac{\Delta\beta}{\beta} = -\frac{\beta_0}{2}\left(\frac{\Delta B'\ell}{B\rho}\right)\frac{\cos 2\nu(\pi-\theta)}{\sin 2\pi\nu} \tag{1}$$

where

$$\begin{cases} \ell = \text{length of magnet} \\ B\rho = \text{magnetic rigidity of beam} \\ \beta_0 = \text{amplitude function at magnet} \end{cases}$$

ν = betatron tune

$d\theta = \frac{1}{\nu\beta} d$(distance along orbit)

(magnet at $\theta = 0$)

2. Tune shift $\Delta\nu$ is given by

$$\Delta\nu = -\frac{\nu}{2\pi}\int_0^{2\pi} \frac{\Delta\beta}{\beta} d\theta = \frac{1}{4\pi}\beta_o\left(\frac{\Delta B'\ell}{B\rho}\right) \tag{2}$$

3. Half integer stop-band width (full) is given by

$$\delta\nu_{\frac{1}{2}} = \frac{1}{2\pi}\beta_o\left(\frac{\Delta B'\ell}{B\rho}\right) = 2\Delta\nu \tag{3}$$

4. Dispersion distortion —— the dispersion function η is given by

$$\frac{\eta}{\sqrt{\beta}} = \frac{\nu}{2\sin\pi\nu}\int_\theta^{\theta+2\pi} \frac{\beta^{3/2}}{\rho}\cos\nu(\pi+\theta-\phi)\,d\phi \tag{4}$$

The distortion of η arises simply through the amplitude distortion $\Delta\beta$ and the tune shift $\Delta\nu$ given by Eqs. (1) and (2).

5. Transition shift —— the transition-gamma γ_t is given by the integral of η, namely

$$\frac{1}{\gamma_t^2} = \frac{\nu}{2\pi R}\int_0^{2\pi} \frac{\eta\beta}{\rho} d\theta = \frac{\nu^2}{4\pi R\sin\pi\nu}\int_0^{2\pi} \frac{\beta^{3/2}}{\rho} d\theta \int_\theta^{\theta+2\pi} \frac{\beta^{3/2}}{\rho}\cos\nu(\pi+\theta-\phi)\,d\phi \tag{5}$$

where $2\pi R$ is the orbit circumference. The shift in γ_t arises again simply through $\Delta\beta$ and $\Delta\nu$.

B. Effects of ΔB

1. Orbit distortion Δx is given by

$$\frac{\Delta x}{\sqrt{\beta}} = -\frac{\sqrt{\beta_o}}{2}\left(\frac{\Delta B\ell}{B\rho}\right)\frac{\cos\nu(\pi-\theta)}{\sin\pi\nu} \tag{6}$$

If, in addition, there is $\Delta B'$, its effect on Δx enters simply through $\Delta\beta$ and $\Delta\nu$. Integrating Δx one gets the orbit length change which is generally negligibly small.

2. Dispersion distortion $\Delta\eta$ is given by

$$\frac{\Delta\eta}{\sqrt{\beta}} = \frac{\sqrt{\beta_0}}{2}\left(\frac{\Delta B\ell}{B\rho}\right)\frac{\cos\nu(\pi-\theta)}{\sin\pi\nu} = -\frac{\Delta x}{\sqrt{\beta}} \tag{7}$$

The effect of $\Delta B'$ similarly enters through $\Delta\beta$ and $\Delta\nu$.

3. Transition shift $\Delta\gamma_t$ is obtained by integrating $\Delta\eta$

$$\Delta\left(\frac{1}{\gamma_t^2}\right) = \frac{\nu}{2\pi R}\int_0^{2\pi}\frac{\beta\Delta\eta}{\rho}d\theta = \frac{\nu\sqrt{\beta_0}}{4\pi R\sin\pi\nu}\left(\frac{\Delta B\ell}{B\rho}\right)\int_0^{2\pi}\frac{\beta^{3/2}}{\rho}\cos\nu(\pi-\theta)d\theta \tag{8}$$

Again, the effect of $\Delta B'$ enters through $\Delta\beta$ and $\Delta\nu$.

C. Effects of $B^{(n-1)}$ for $n>2$

The local effect of ΔB is an orbit distortion Δx, a dipole moment change in the transverse distribution of the beam. That of $\Delta B'$ is an amplitude distortion $\Delta\beta$, a quadrupole moment change. This effect integrated over the orbit circumference gives the tune shift $\Delta\nu$ or the half integer resonance (stop-band) width $\delta\nu_{\frac{1}{2}}$. For B'' and higher order non-linear field errors the local effects, namely changes in the sextupole and higher multipole moments of the transverse distribution of the beam are not of much interest. The only interesting quantities are the integrated effects, namely the non-linear resonance widths. The full width $\delta\nu_{\frac{m}{n}}$ of the resonance $\nu = \frac{m}{n}$ is given approximately by

$$\delta\nu_{\frac{m}{n}} = 2\left|C_{\frac{m}{n}}\right|\left(\frac{\varepsilon}{\pi}\right)^{\frac{n-2}{2}} \tag{9}$$

where

ε = beam emittance

$$\left|C_{\frac{m}{n}}\right| = \frac{1}{2\pi}\left\{\sum_{\substack{\text{all}\\ \text{magnets}}}\left[\frac{1}{2^{n-1}(n-1)!}\left(\frac{B^{(n-1)}\ell}{B\rho}\right)\beta^{\frac{n}{2}}\right]^2\right\}^{1/2}$$

and where the non-linear field $B^{(n-1)}$ is assumed to be uncorrelated in different magnets.

NUMERICAL VALUES AND TOLERANCES

For order-of-magnitude estimates we take the strongest quadrupole in D.Johnson's insertion Type C ($\bar{p}$ Note No. 169). (There are actually two strongest quadrupoles working as a pair.) The parameters are roughly

$\ell = 7\text{m}, \quad \rho = \frac{3}{4} \times 10^3\text{m}, \quad B = \frac{4}{9} \times 10^2 \text{ kG}$

$\beta_0 = 800\text{m}, \quad \nu = 19.4, \quad B' = 10^3 \text{ kG/m}$

A. Effects of $\Delta B'$

1. Amplitude distortion: The last factor in Eq. (1) is of order unity. Hence we have

$$\left|\frac{\Delta\beta}{\beta}\right| = \frac{800\text{m}}{2}\,\frac{7\text{m}}{750\text{m}}\,\frac{\Delta B'}{B} = 4\text{m}\,\frac{\Delta B'}{B} \tag{10}$$

$\left|\frac{\Delta\beta}{\beta}\right|$ should be <10%, hence we have the tolerance

$$b_1 = \frac{\Delta B'}{B} < 6\times10^{-4} \text{ in}^{-1} \tag{11}$$

2. Tune shift: Eq. (2) gives

$$\Delta\nu = \frac{800\text{m}}{4\pi}\,\frac{7\text{m}}{750\text{m}}\,\frac{\Delta B'}{B} = 0.6\text{m}\,\frac{\Delta B'}{B} \tag{12}$$

$\Delta\nu$ should be much smaller than 0.01. This gives a tighter tolerance of $b_1 \ll 4\times10^{-4}$ in^{-1} which indicates that

$$b_1 \lesssim 1\times10^{-4} \text{ in}^{-1} \tag{13}$$

is a good choice.

3. Half integer stop-band: Eq. (3) gives

$$\delta\nu_{\frac{1}{2}} = 1.2\text{m}\ \frac{\Delta B'}{B} \tag{14}$$

For $\delta\nu_{\frac{1}{2}} < 0.01$ this requires $b_1 < 2\times10^{-4}$ in^{-1} showing again that the tolerance condition (13) is a good choice.

4. Dispersion distortion: Eq. (4) shows that in order-of-magnitude

$$\frac{\Delta\eta}{\eta} \sim 2\,\frac{\Delta\beta}{\beta} + \frac{\Delta\nu}{\nu} \tag{15}$$

With the tolerance condition (13) we get $\frac{\Delta\beta}{\beta} \lesssim 0.017$, $\frac{\Delta\nu}{\nu} \lesssim 0.0001$ and hence $\frac{\Delta\eta}{\eta} \lesssim 0.034$ which is quite acceptable.

5. Transition shift: Eq. (5) gives in order-of-magnitude

$$\frac{\Delta\gamma_t}{\gamma_t} \sim \frac{3}{2}\,\frac{\Delta\beta}{\beta} + \frac{\Delta\nu}{\nu} \lesssim 0.026 \tag{16}$$

for the tolerance (13). This is also very acceptable.

B. Effects of ΔB and alignment error

1. Orbit distortion: The last factor in Eq. (6) is of order unity. Hence, at the quadrupole ($\beta=\beta_0$) we have

$$|\Delta x| = \frac{800\text{m}}{2}\,\frac{7\text{m}}{750\text{m}}\left|\frac{\Delta B}{B}\right| = 4\text{m}\left|\frac{\Delta B}{B}\right| \tag{17}$$

We would like $|\Delta x|$ to be less than, say, 4 mm or

$$\left|\frac{\Delta B}{B}\right| < 1\times10^{-3} \tag{18}$$

With $B = \frac{4}{9}\times10^2$ kG and $B' = 10^3$ kG/m this corresponds to an alignment tolerance of

$$|\delta x| < 1\times10^{-3}\ \frac{\frac{4}{9}\times10^2}{10^3}\text{m} = 0.044\text{mm} = 1.7 \text{ mil} \tag{19}$$

This is very tight indeed.

The effect of $\Delta B'$ on δx through $\Delta\beta$ and $\Delta\nu$ is clearly too small to be worth considering.

2. Dispersion distortion: Eq. (7) gives $\Delta\eta = -\Delta x$. Clearly any tolerable Δx value will give a tolerable $\Delta\eta$. Again the effect of $\Delta B'$ on $\Delta\eta$ is totally negligible.

3. Transition shift: Eq. (8) gives in order-of-magnitude

$$\left|\frac{\Delta\gamma_t}{\gamma_t}\right| \sim \frac{1}{2}\left|\frac{\Delta\eta}{\eta}\right| \tag{20}$$

With $\Delta\eta \sim 4$ mm and $\eta \sim 4$ m we have $\left|\frac{\Delta\gamma_t}{\gamma_t}\right| \sim \frac{1}{2} \times 10^{-3}$ which is totally acceptable. Furthermore the additional effect by $\Delta B'$ is also totally acceptable.

C. Effects of non-linear field $B^{(n-1)}$ $(n>2)$

The β value of 800 m at the strongest low-β quad is ~16 times greater than the average value of ~50 m at all other magnets. For the same error strength $\frac{B^{(n-1)}\ell}{B\rho}$ then, the contribution of one of this pair of low-β quads is equivalent to 16^n other magnets. Thus, for all non-linear resonances $(n>2)$ the contribution from this pair of quads dominates and we can write for each quadrupole

$$\left|C_{\frac{m}{n}}\right| \cong \frac{1}{2\pi}\frac{\ell}{\rho}\frac{b_{n-1}}{2^{n-1}}\beta_0^{\frac{n}{2}} \tag{21}$$

where

$$b_{n-1} \equiv \frac{1}{(n-1)!}\frac{B^{(n-1)}}{B} = \text{standard multipole coefficient.}$$

This gives

$$\delta\nu_{\frac{m}{n}} = \frac{1}{2\pi}\beta_0\frac{\ell}{\rho}b_{n-1}\left(\frac{a}{2}\right)^{n-2} \tag{22}$$

where

$$a \equiv \sqrt{\frac{\beta_0\varepsilon}{\pi}} = \text{half width of beam at quad.}$$

The assumed normalized beam emittance is 24 πmm mrad. Hence at

1000 GeV ($\beta\gamma$ = 1067) we have

$$\varepsilon = \frac{24}{1067}\ \pi\text{mm-mrad} = 0.0225\ \pi\text{mm-mrad}$$

$$a = \sqrt{800 \times 0.0225}\ \text{mm} = 4\ \text{mm} = 0.16\ \text{in}$$

and

$$\delta\nu_{\frac{m}{n}} = \frac{1}{2\pi}\ (800\times40\ \text{in})\ \frac{7\text{m}}{750\text{m}}\ b_{n-1}\left(\frac{0.16\ \text{in}}{2}\right)^{n-2} \tag{23}$$

$$= 48\ b_{n-1}\ (0.08)^{n-2}\ \text{in}^{n-1}$$

For the half integer stop-band n=2 and $\delta\nu_{\frac{1}{2}} = (48\ \text{in})\ \frac{\Delta B'}{B}$ agreeing with Eq. (14). It should be easy to make the non-linear fields in this quad less than those in the ring dipoles. We shall, therefore, impose the following relatively easy tolerance of

$$b_{n-1} < 1\times10^{-4}\ \text{in}^{-(n-1)} \qquad \text{for } n>2 \tag{24}$$

This gives

$$\delta\nu_{\frac{m}{n}} = 0.0048 \times (0.08)^{n-2}$$

which is acceptable. However it should be pointed out here that orbit distortions will add to a, thereby giving a larger apparent beam width. We allowed in Eqs. (18) and (19) an orbit distortion of $|\Delta x|$ = 4 mm. Added to the actual beam width this will double a and yield

$$\delta\nu_{\frac{m}{n}} = 0.0048 \times (0.16)^{n-2}$$

Although still tolerable this is close to the limit. Hence, the consideration of non-linear resonance widths leads also to the same tolerances on the orbit distortions and on the alignment errors.

DISCUSSIONS OF TOLERANCES AND CORRECTIONS

The tolerances for the low-β quadrupoles (actually only for the strongest pair) can be summarized simply as follows:

A Alignment tolerance

$$\delta x < 1.7 \text{ mil} \tag{25}$$

It is almost impossible to achieve this kind of accuracy by absolute measurement. The positioning of the beam in these two strong low-β quads must be adjusted by trim dipoles. One possible arrangement is to use a three-dipole orbit bump for each quadrupole with the middle dipole placed right next to the quad. The two outboard dipoles could be located at, say, A49 and B11 and the two inboard dipoles should be placed just inside (the side of BØ) the triplets of quadrupoles. The inboard dipoles also serve to position the beams at the collision point. These orbit bumps should, of course, be provided in both the horizontal and the vertical planes.

It is, however, doubtful that space can be made available next to the strong quadrupole for the middle dipole of the three-dipole bump. Therefore, a better arrangement is to install dipole coils (both horizontal and vertical) inside the low-β quadrupoles to move the center of the quad field. If the absolute alignment accuracy is, say, 20 mil or $\frac{1}{2}$ mm, the range of the dipole field needed is only

$$\pm 1000 \text{ kG/m} \times (0.5\times10^{-3} \text{ m}) = \pm 500 \text{ G} \tag{26}$$

These dipole coils can therefore be quite small and simple. They are adjusted to center the quad field on the orbit to within 1.7 mil (0.044 mm) or equivalently, to reduce the dipole field on the orbit to within 1000 kG/m $\times$ (0.044 10^{-3}m) = 44 G.

B. Field error tolerance

Conditions (13) and (24) can be combined into the single condition

$$b_{n-1} < 1\times10^{-4}\ \text{in}^{-(n-1)}, \qquad n>1 \tag{27}$$

For the quadrupole field error $b_1 (= \frac{\Delta B'}{B})$ since the pair of strong quadrupoles are connected in series and powered with one single adjustable supply, condition (27) gives simply the required identity of the gradients of the two quads and the required regulation of the power supply in time. All non-linear multipole coefficients (n>2) impose conditions on construction accuracy and power supply regulation. These conditions are, however, not difficult to meet.

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LOCATION AND STRENGTHS OF CORRECTION DIPOLES IN THE LOW-β INSERTION

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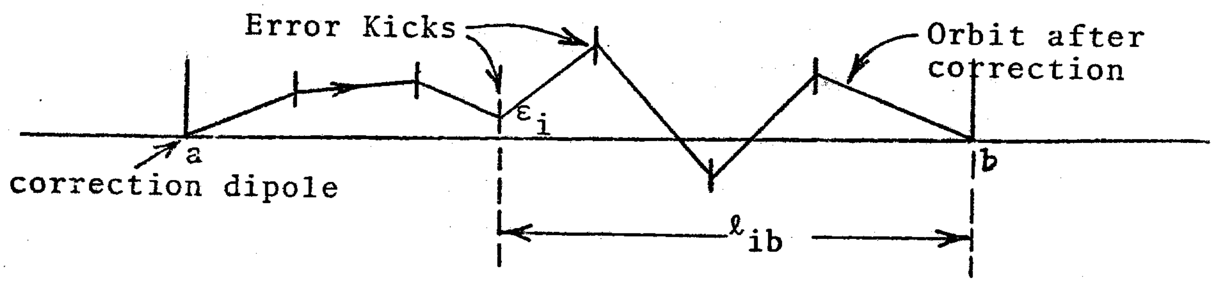
There seems to be a preference for separate correction dipoles instead of correction dipole coils installed in the aperture of the low-β quadrupoles. We give here the minimal requirements for the number, the locations and the strengths of these correction dipoles.

Normally, correction dipoles each capable of maximum strength $B\ell$ = 170kG in. = 4.3kGm are placed at locations A48, A49, B11 and B12. We shall show that in the low-β insertion at BO, in addition to these we should add two more dipoles either just inboard of QI4 and -QI4 (locations denoted as A50 and B10) or just outboard of QI4 and -QI4 (locations denoted as A50' and B10'). The correction dipoles are adjusted to make the orbit distortion zero at the dipoles as shown in the following diagram



All orbit deflections are approximated as local kicks. The error kick ε_i produced by a quadrupole with alignment error δx_i is

$$\varepsilon_i = (B'\ell)_i \, \delta x_i / B\rho \tag{1}$$

and the orbit distortion at b produced by ε_i is $\varepsilon_i \ell_{ib}$ where

$$\ell_{ib} = \sqrt{\beta_i \beta_b} \sin \phi_{ib} = \text{12 element of transfer matrix from i to b.} \tag{2}$$

If δx_i are random the expectation value of the orbit distortion at b produced by all ε_i is

$$\Delta x_b = \sqrt{\sum_i (\varepsilon_i \ell_{ib})^2} \qquad (3)$$

The correction dipole at a should be so adjusted as to cancell Δx_b, namely

$$\varepsilon_a \ell_{ab} = \varepsilon_a \sqrt{\beta_a \beta_b}\, \sin \phi_{ab} = -\Delta x_b \qquad (4)$$

ε_a then gives the strength of a required for the assumed values of δx_i. With this adjustment the expectation value of the orbit distortion at a location j in between a and b is then given by

$$\Delta x_j = \varepsilon_a \ell_{aj} - \sqrt{\sum_{i<j} (\varepsilon_i \ell_{ij})^2} \qquad (5)$$

The parameters for D. Johnson's Type E insertion at $\beta^* = 0.808$m and $B\rho = 33388$kGm (1 TeV) are given below. Since the insertion is antisymmetric, studying the orbit in one plane (x-plane) alone is adequate.

Location	$\phi_x/2\pi$	β_x(m)	$B'\ell$(kGm)
A49	5.95875	866.22	
Q2	5.95937	780.08	-1920
QI2	5.96036	700.48	4598
QI3	5.96221	311.65	-3552
QI3	5.96587	142.55	-3552
A50'	5.96868	116.02	
QI4	5.97221	95.31	3655
A50	5.97713	69.73	
BO	6.20837	0.808	
B10	6.43960	66.11	
-QI4	6.44414	140.85	-3655
B10'	6.44697	242.26	
-QI3	6.44803	365.98	3552
-QI3	6.44963	397.85	3552
-QI2	6.45278	211.60	-4598
-Q2	6.45763	135.14	1920
B11	6.46238	96.17	

For these parameters the performances of the various arrangements are the following.

A. Correction dipoles only at A49 and B11

Assumming alignment errors δx_i = 1mm and random for all the quadrupoles we get

Expectation value of orbit distortion at B11 due to alignment errors = 3.2mm

Required strength of dipole at A49 to give zero distortion at B11, $B\ell$ = 16.3kGm

Expection valve of orbit distortion at BO after correction = 8.7mm

With this arrangement we see that

1. Correction dipole strength required is to large.
2. Even after correction the orbit distortion at BO is too large.

B. Correction dipoles at A49, A50, B10, and B11

Again for δx_i = 1mm and random we get

Required strength of A49 dipole to give zero distortion at A50, $B\ell$ = 4.6kGm

Required strength of B10 dipole to give zero distortion at B11, $B\ell$ = 9.1kGm

Orbit distortion at BO after correction = 0

With this arrangement and standard correction dipoles ($B\ell$ = 4.3kGm) we can correct for random misalignments of $\delta x_i \sim 1/2$ mm. The correction of orbit distortion at BO is perfect.

C. Correction dipoles at A49, A50', B10', B11

For δx_i = 1mm and random we get

Required strength of A49 dipole to give zero distortion at A50', $B\ell$ = 4.1kGm

Required strength of A50' dipole to give zero distortion at B10', $B\ell$ = 3.9kGm

Required strength of B10' dipole to give zero distortion at B11, $B\ell$ = 6.2kGm

Expectation value of orbit distortion at B0 after correction = 0.17mm

With this arrangement and standard correction dipoles ($B\ell$ = 4.3kGm) we can correct for random misalignment of $\delta x_i \sim 2/3$mm. The expectation value of orbit distortion at B0 after correction is acceptable.

The conclusions are the following.

1. In order to be able to control the orbit distortion at B0 one must use arrangements B or C.
2. If only standard correction dipoles with $B\ell$ = 170kG in. are used one must ensure that the quadrupoles are aligned to better than $\sim$1/2mm.