Fermilab

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HARMONIC ANALYSIS OF ENERGY DOUBLER DIPOLE

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This is the explanation of our harmonic analysis to measure the multipole component of the Energy Doubler dipole. The final formula which are used in the computer program are in summary.

If you would like a further explanation you may wish to read other pages.

1. Definition of the Field

The definition of the multipole coefficients C_N used in this measurement are written as follows:

$$H_y + iH_x = B_o \sum_{N=0}^{\infty} C_N Z^N$$

where $Z = X+iY$
B_o= field at center of magnet

right hand coordinate is taken looking from dwonstream. C_N is composed of normal and skew parts as:

$$C_N = bn + i\ an$$
$$n = 2N+2$$

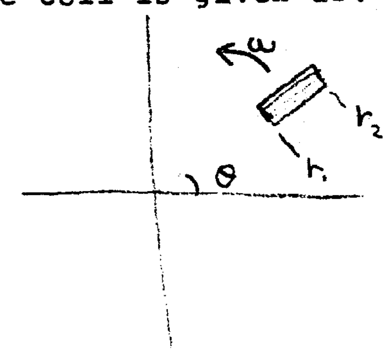
In the measurement results, bn and an are written as FFTB and FFTA respectively. For the convenience of calculation, complex field $\mathcal{H}$ is defined as:

$$\mathcal{H}(Z) = H_y + iH_x$$

so that multipole components are simply written as Tayler expansion coefficient of $\mathcal{H}(Z)$.

2. Voltage comes from Rotating Coils

Suppose we rotate a coil which has length L and width (r_2-r_1), then the component of the field perpendicular to the coil is given as:



$$\begin{aligned} H_0(z) &= (\vec{B}\cdot\hat{a}) \\ &= B_0\, \mathcal{R}e\, (i\mathcal{H}^*)^*(ie^{i\theta}) \\ &= B_0\, \mathcal{R}e \sum_{N=0}^{\infty} C_N z^N e^{i\theta} \\ &= B_0\, \mathcal{R}e \sum_{N=0}^{\infty} C_N r^N e^{i(1+N)\theta} \end{aligned}$$

Therefore, if the rotation speed is ω in the counter clock direction and the coil has T turns then the voltage on the coil is:

$$V'(\theta) = T \int \frac{dH_\theta}{d\theta} \frac{d\theta}{dt} da$$

$$= LT\omega \int_{r_1}^{r_2} \frac{dH_\theta}{d\theta} dr$$

$$= LT\omega B_0 R_e \left\{ \sum_{N=0}^{\infty} C_N \left[r^{N+1} \right]_{r_1}^{r_2} e^{i(N+1)\theta} \right\}$$

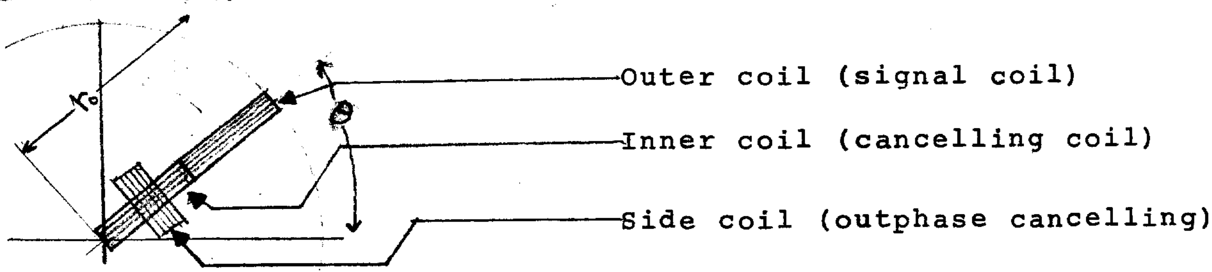
If we integrate this signal using a hardware integrator with a time constant RC

$$V(\theta) = \frac{1}{RC} \int V' dt$$

$$= \frac{1}{RC} \int \frac{1}{\omega} V'(\theta) d\theta$$

$$= \frac{LT}{RC} R_e \left\{ \sum_{N=0}^{\infty} C_N \frac{1}{N+1} e^{i(N+1)\theta} \left[r^{N+1} \right]_{r_1}^{r_2} + \text{const} \right.$$

3. <u>Actual Arrangement</u>



In the actual setup, two identical coils are placed in the same plane at different distances from the center of rotation. Another coil is used only to correct the imperfect alignment of the two coils. Essentially, measurement is made with two balanced coils.

In this case the Integrated voltage in each coil are

$$V_{out}(\theta) = \frac{LT}{RC} B_0 R_e \left[\sum_{N=0}^{\infty} C_N \frac{r_0^{N+1}}{N+1} e^{i(N+1)\theta} \left\{ 1 - \left(\tfrac{1}{2}\right)^{N+1} \right\} \right]$$

$$V_{in}(\theta) = \frac{LT}{RC} B_0 R_e \left\{ \sum_{N=0}^{\infty} C_N \frac{r_0^{N+1}}{N+1} e^{i(N+1)\theta} \left(\tfrac{1}{2}\right)^{N+1} \right\}$$

If we take the difference between these voltages,

$$V(\theta) = \frac{LT}{RC} B_0 R_e \left[\sum_{N=0}^{\infty} C_N \frac{r_0^{N+1}}{N+1} e^{i(N+1)\theta} \left\{ 1 - \left(\tfrac{1}{2}\right)^{N} \right\} \right]$$

Then all the dipole component is cancelled and only the higher multipole components are observed.

4. Evaluation of Coefficients

If the observed signal V (θ) is analyzed by Fourie transformation;

$$V(\theta) = \sum_{N=0}^{\infty} A_n \cos(n\theta + \alpha_n)$$

then comparison of this formula to another formula gives

$$A_n \cos(n\theta + \alpha_n) = \frac{LT}{RC} R_e \left[C_N \frac{r_0^{N+1}}{N+1} e^{i(N+1)\theta} \left\{1 - \left(\frac{1}{2}\right)^N\right\}\right]$$

$$= \frac{LT}{RC} B_0 \frac{r_0^{N+1}}{N+1} \left\{1 - \left(\frac{1}{2}\right)^N\right\} \left(b_n \cos\frac{n}{2}\theta - a_n \sin\frac{n}{2}\theta\right)$$

If one uses the result of analysis of V_{out} (θ), then since $C_0=1$, the dipole amplitude can be taken as a standard.

$$A_s = \frac{1}{2} \frac{LT}{RC} B_0 r_0$$

$$b_n = \frac{(N+1) A_n \cos\alpha_n}{2A_s r_0^N \left\{1 - \left(\frac{1}{2}\right)^N\right\}}$$

$$a_n = \frac{(N+1) A_n \sin\alpha_n}{2A_s r_0^N \left\{1 - \left(\frac{1}{2}\right)^N\right\}}$$

5. Adjustment of Phase

i) If dipole signal has phase of φ, then we need to rotate the coordinate by

$$V(\theta - \varphi) = V(\theta')$$

$$= \sum_{N=0}^{\infty} A_n \cos\left(\frac{n}{2}\theta' + \alpha_n\right)$$

$$= \sum_{N=0}^{\infty} A_n \cos\left\{\frac{n}{2}(\theta' - \varphi) + \left(\alpha_n - \frac{n}{2}\varphi\right)\right\}$$

Therefore, one needs to subtract $\frac{m}{2}\varphi$ from each α_n

ii) If rotated in a clockwise direction

$$v(-\theta) = V(\theta')$$

$$= \sum_{N=0}^{\infty} A_m \cos\left(\frac{m}{2}\theta' + \alpha_m\right)$$

$$= \sum_{N=0}^{\infty} A_m \cos\left\{-\frac{m}{2}\theta' + (-\alpha_m)\right\}$$

Therefore, one needs to flip the sign of the skew component

iii) If measured from upstream

$$v(180^\circ-\theta) = -V(\theta')$$

$$= \sum_{N=0}^{\infty} A_m \cos\left(\frac{m}{2}\theta' + \alpha_m + 180^\circ\right)$$

$$= \sum_{N=0}^{\infty} A_m \cos\left\{\frac{m}{2}(180^\circ-\theta') + \left(\frac{m+1}{2}\times 180^\circ - \alpha_m\right)\right\}$$

Therefore, one needs to flip the sign of even pole skew component and odd pole normal component.

Summary

$$bn = \frac{(N+1)\ An\ \cos(\alpha n - \frac{n}{2}\varphi)}{2A_s r_o^N\ (1-(\frac{1}{2})^n)} (UD)^N$$

$$an = \frac{(N+1)\ An\ \sin(\alpha n - \frac{n}{2}\varphi)}{2A_s\ r_o^N\ (1-(\frac{1}{2})^N)} (UD)^{N+1} * (RD)$$

where $n = 2N+2$

$$UD = \begin{cases} +1 & \text{:downstream} \\ -1 & \text{:Upstream} \end{cases}$$

$$RD = \begin{cases} +1 & \text{:C.C.W} \\ -1 & \text{:C.W.} \end{cases}$$

φ : dipole phase

A_n : npole amplitude

r_o : Outer coil radius

A_s : Standard amplitude of dipole signal

	dipole	quadrupole	sextupole
n	2	4	6
N	0	1	2

Note 1: assuming liniality, A_s can be written as

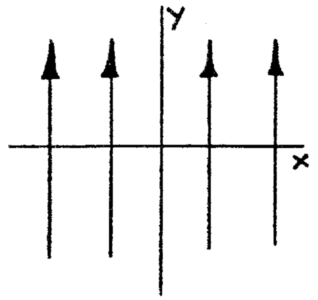
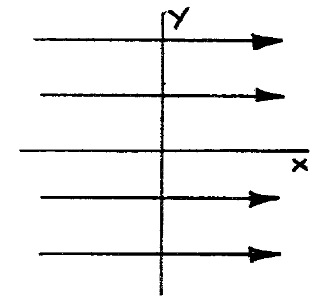
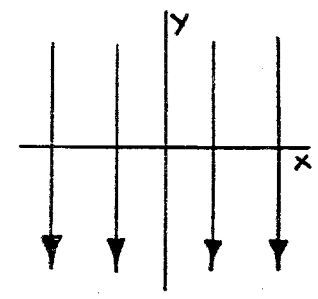
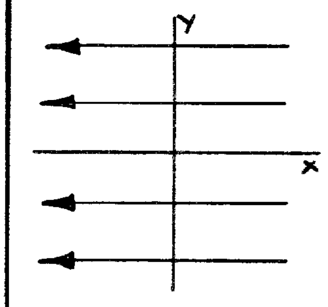
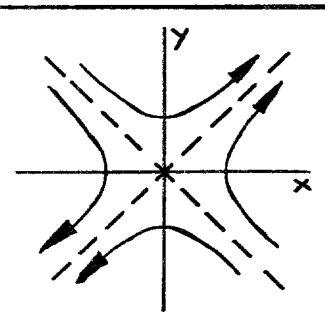
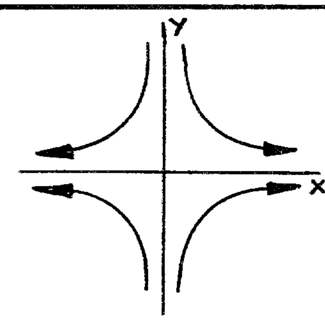
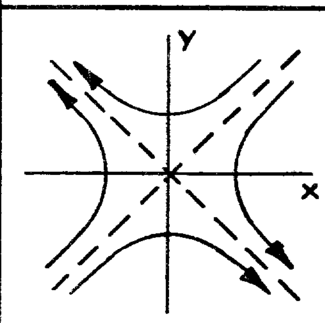
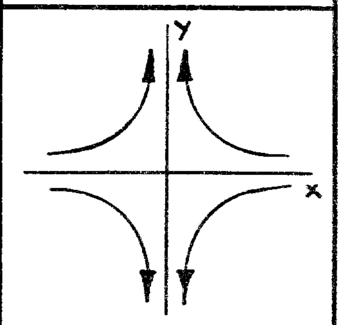
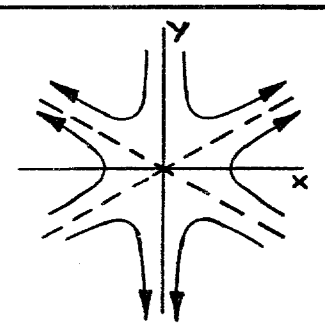
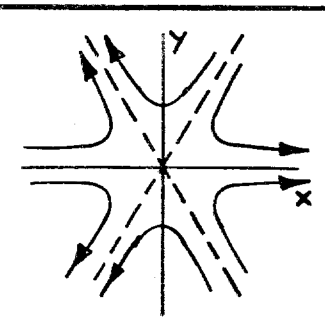
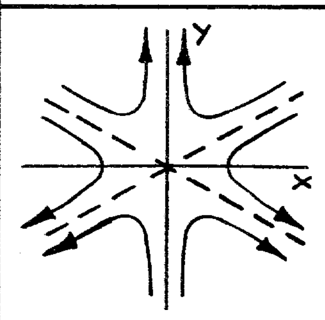
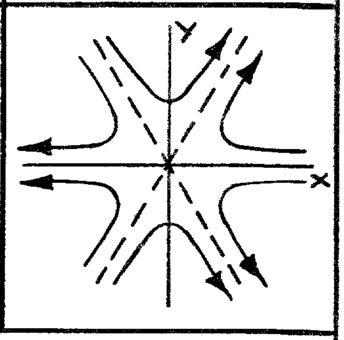
$$A_s = A_{\text{dipole at }1000A} * \frac{I}{1000}$$

Note 2: in 8 foot coil $r_o = 2.30$cm

Note 3: usually coil is rotated in C.W. direction

DEFINITION OF HARMONIC FIELD COMPONENT

(LOOKS FROM DOWNSTREAM, X=OUTSIDE, Y=UPWARD)

	DIPOLE MAGNET FOCUSING MAGNET		DEFOCUSING MAGNET	
	NORMAL	SKEW	NORMAL	SKEW
N=0 DIPOLE				
N=1 QUADRUPOLE				
N=2 SEXTUPOLE				
GENERAL EXPRESSION	$iH_x + H_y = \sum_{N=0}^{\infty} c_N Z^N$ where $c_N = b_n + i a_n$ $Z = x + iy$ $n = 2N + 2$		$iH_x + H_y = \sum_{N=0}^{\infty} - c_N Z^N$ where $c_N = b_n + i a_n$ $Z = x + iy$ $n = 2N + 2$	