Fermilab

Parametric Fits for the Maximum Luminosity and Tune Shift in $e^{\pm}$ Colliding Beams-Relevance to $\bar{p}p$ Colliding Beams

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We are interested in $\bar{p}p$ colliding beams which is a system of head-on collision of bunched beams. The only way of getting some empirical information about such a system is perhaps through the e^+e^- colliding beams which is also a regime of head-on collision of bunched beams. The difference is that for electron beams synchrotron radiation damping is a dominant effect.

The measured maximum luminosity L_{max} and tune-shift ξ_{max} are plotted as functions of the beam energy E for the storage rings ADONE and SPEAR by H. Wiedemann (SLAC-PUB-2320, PEP Note 299, May 1979) and are reproduced in Figures 1 and 2. In these plots the vertical arrows (↑) indicate the energies at which the tune-shift ξ_{max} saturates - remains constant at that value for higher energies.

1. It is instructive to know how these values were measured.

a. At each energy the beam current and size (β-function) were varied until the maximum luminosity is obtained. The luminosity is then measured directly from nuclear event rate.

b. To measure the tune-shift one needs the beam size (σ_x, σ_y) which is difficult to measure. So one gets the beam size through the luminosity. For two identical beams

$$L = nf\frac{N^2}{4\pi\sigma_x\sigma_y} \tag{1}$$

$$\xi_y = \frac{e^2}{2\pi}\frac{N}{E}\frac{\beta_y^*}{\sigma_y(\sigma_x+\sigma_y)} \qquad \left(\begin{array}{l}\text{for } \xi_x \\ \text{interchange x and y}\end{array}\right) \tag{2}$$

where

e = electronic charge

N = no. of particles per bunch

n = no. of bunches in each beam

f = revolution frequency

σ_x, σ_y = horizontal and vertical beam sizes (rms widths of Gaussian distributions)

β_y^* = vertical amplitude function.

Partially eliminating σ_x and σ_y we get

$$\xi_y = 2\, e^3 \frac{\beta_y^*}{1+\frac{\sigma_y}{\sigma_x}} \frac{L}{enfNE} = \frac{2\, e^3 \beta_y^*}{1+\frac{\sigma_y}{\sigma_x}} \frac{L}{IE} \tag{3}$$

where $I \equiv enfN$ = beam current. It is true that we still need σ_y/σ_x, but this varies only within rather tight limits:

round beam $\quad \sigma_y/\sigma_x = 1$

flat ribbon $\quad \sigma_y/\sigma_x \to 0$.

Thus, ξ_y is calculated from measured L, I, E, β_y^* and estimated σ_y/σ_x.

c. Strictly speaking ξ is not the tune-shift. It is related to the deflection of the orbit by a beam bunch considered as a thin lens

$$\xi \equiv -\frac{\beta}{4\pi} \frac{\Delta y'}{y} = \frac{\beta}{4\pi} \frac{B'\ell}{B\rho} .$$

Going through the beam bunch is, then, equivalent to the transfer matrix

$$\begin{pmatrix} y \\ y' \end{pmatrix}_{\text{after}} = \begin{pmatrix} 1 & 0 \\ -\frac{4\pi\xi}{\beta} & 1 \end{pmatrix} \begin{pmatrix} y \\ y' \end{pmatrix}_{\text{before}} .$$

In one revolution the transfer matrix becomes

$$\begin{pmatrix} 1 & 0 \\ -\frac{4\pi\xi}{\beta} & 1 \end{pmatrix} \begin{pmatrix} \cos 2\pi\nu + \alpha\sin 2\pi\nu & \beta\sin 2\pi\nu \\ -\gamma\sin 2\pi\nu & \cos 2\pi\nu - \alpha\sin 2\pi\nu \end{pmatrix}$$

$$= \begin{pmatrix} \cos 2\pi\nu + \alpha\sin 2\pi\nu & \underline{\qquad} \\ \underline{\qquad} & \cos 2\pi\nu - \alpha\sin 2\pi\nu - 4\pi\xi\sin 2\pi\nu \end{pmatrix} .$$

Thus the tune-shift $\Delta\nu$ is given by

$$\cos 2\pi(\nu+\Delta\nu) = \frac{1}{2}\text{Trace} = \cos 2\pi\nu - 2\pi\xi\sin 2\pi\nu \qquad (4)$$

$$\cos 2\pi(\nu+\Delta\nu) = \cos 2\pi\nu \underbrace{\cos 2\pi\Delta\nu}_{\sim 1} - \sin 2\pi\nu \underbrace{\sin 2\pi\Delta\nu}_{\sim 2\pi\Delta\nu} .$$

Therefore $\xi \cong \Delta\nu$ when small. We shall however continue to refer to ξ as the tune-shift.

2. Interpretation of the L_{max} and ξ_{max} curves

At lower values of ξ_{max} the beam-beam forces cause a diffusion (Arnol'd diffusion?) or an antidamping of the beam which is counteracted by the synchrotron radiation damping. At some ξ value, further increase in ξ will increase the beam size so much that increasing beam current will no longer increase the luminosity. These are the values plotted for ξ_{max} and L_{max}. The synchrotron radiation damping increases with energy, hence the maximum attainable ξ also increases with energy. This is true as long as $\xi_{max} \lesssim 0.06$.

At $\xi \cong 0.06$ presumably the stochasticity limit (overlapping of stochastic layers of neighboring resonances) is reached and the tune-shift can go no higher under whatever condition.

The synchrotron radiation damping rate is well-known and is given by

$$\frac{1}{\tau_{SR}} = \frac{C_\gamma}{4\pi} \frac{c}{R\rho} E^3 \qquad \text{for vertical beam size,} \tag{5}$$

where

$C_\gamma = 8.85\text{x}10^{-5}$ m/GeV^3

c = speed of light

$2\pi R$ = ring circumference

ρ = bending radius.

The beam-beam antidamping rate or the Arnol'd diffusion constant is not known, but we can make a good guess. Since the non-linear orbital beam-beam effect is measured by ξ, the "diffusion effect" from each encounters with one beam bunch should be proportional to ξ^2 and the antidamping rate should therefore be

$$\frac{1}{\tau_{BB}} = knf\xi^2 \tag{6}$$

where

n = number of bunches in each beam

f = revolution frequency

k = proportionality constant.

The maximum attainable ξ_{max} is given by

$$\frac{1}{\tau_{BB}} - \frac{1}{\tau_{SR}} = \text{some negative constant} = -A$$

$$knf\xi_{max}^2 - BE^3 = -A$$

or

$$\boxed{\begin{aligned} \xi_{max} &= \frac{1}{\sqrt{knf}} (BE^3-A)^{\frac{1}{2}} \\ B &\equiv \frac{C_\gamma}{4\pi} \frac{c}{R\rho} \end{aligned}} \tag{7}$$

with .

To get the energy dependence of L_{max} we eliminate N between the expressions (1) and (2) for L and ξ_y to get

$$\boxed{L_{max} = \frac{\pi n f}{e^4} \frac{\sigma_y^2}{\beta_y^{*2}} \frac{(\sigma_x+\sigma_y)^2}{\sigma_x\sigma_y} E^2 \xi_{max}^2} \tag{8}$$

where ξ_{max} is understood to be ξ_{ymax} and σ_x and σ_y are the beam sizes at maximum luminosity and should therefore be more-or-less constant independent of E.

Thus we have a 2-parameter (k and A) fit for ξ_{max} and a zero-parameter fit for L_{max}.

3. The fit

The listed parameters for ADONE and SPEAR are

	ADONE	SPEAR
$2\pi R$(m)	104	234
ρ (m)	5.0	12.7
β_y^*(m)	3.2	0.08
σ_x(m)	0.62	0.53
σ_y(mm)	0.35	0.014
f(MHz)	2.88	1.28
n	3	1

These parameters give

$$B = \begin{cases} 25.5 \ (\text{GeV})^{-3} \ \text{sec}^{-1} & \text{ADONE} \\ 4.46 \ (\text{GeV})^{-3} \ \text{sec}^{-1} & \text{SPEAR} \end{cases}$$

and the 2-parameter ξ_{max} fits to the measured data are

$$\begin{cases} 4776\xi_{max}^2 = 25.5E^3-2.1 & \text{ADONE} \\ 11590\xi_{max}^2 = 4.46E^3-4.35 & \text{SPEAR} \end{cases} \quad . \tag{9}$$

The zero-parameter L_{max} fits are

$$\begin{cases} L_{max} = 0.68\text{x}10^{32}\ E^2\ \xi^2_{max} & \text{ADONE} \\ L_{max} = 2.37\text{x}10^{32}\ E^2\ \xi^2_{max} & \text{SPEAR} \end{cases} \qquad (10)$$

where L is in $cm^{-2}\ sec^{-1}$ and E is in GeV. These fits are plotted as the curves in Figs. (1) and (2). Considering the very simple reasoning leading to the ξ_{max} fit and the zero-parameter fit for L_{max} these fits are remarkably good indeed. The only unsatisfactory feature is that the fitted parameters k and A for ξ_{max} are different for the two machines, namely

$$k = \begin{cases} 0.55\text{x}10^{-3} \\ 9.05\text{x}10^{-3} \end{cases} \qquad A = \begin{cases} 2.10 & \text{ADONE} \\ 4.35 & \text{SPEAR} \end{cases} .$$

The parameter A is related to the beam size when the luminosity is at maximum and is expected to be different for different storage rings. Indeed A is expected to be larger for SPEAR in which the beam size (vertical) is much smaller. On the other hand the parameter k is the proportionality factor between the antidamping rate due to Arnol'd diffusion and the "beam-beam effect per unit time", $nf\xi^2$, and should be the same for all machines. Any dependence of k on the beam parameters such as current, size, etc. is contrary to the thesis that the totality of non-linear beam-beam effects is dependent only on the tune-shift ξ. (However, one hastens to add that this thesis is only a matter of faith and not a proven fact.) The more than an order of magnitude difference between the k values for ADONE and SPEAR is annoying, especially since k is the crucial parameter needed for the evaluation of the $\bar{p}p$ system.

One may try to improve the situation by generalizing the beam-beam antidamping rate to

$$\frac{1}{\tau_{BB}} = knfF(\xi)$$

allowing that the "diffusion effect" from an encounter with a beam bunch could be any function F of ξ. We should, then, look for a function for which the fitting parameter k is the same for all machines. So far, we have not been able to discover such a function F. Experiments on the ISR by E. Keil et al (IEEE Trans. on Nucl. Sci., Vol. NS-22, No. 3, p. 1370, June 1975) using a non-linear lens and by B. Zotter (Proc. of X Int. Conf. on High Energy Accel., Protvino, USSR, Vol. 2, p. 23, 1977) using the high-β insertion seem to indicate an exponential form for F. But with $F(\xi) = e^{a\xi}$ the values of k fitted for the two machines are no closer together.

4. Application to the evaluation of $\bar{p}p$ colliding beams

For proton (or antiproton) beams the synchrotron radiation damping is negligible, the beam life-time is given simply by the beam-beam life-time or

$$\tau = \tau_{BB} = (knf\xi^2)^{-1} .$$

Take $\bar{p}p$ colliding beams in the Energy Doubler with

$n = 1$ (one bunch in each beam)

$f = 47.7 \times 10^3$ Hz

$\xi = 0.005$ (traditional upper limit)

and we get

$$\tau = \begin{cases} 25 \text{ min} & \text{if } k = 0.55 \times 10^{-3} \quad \text{(ADONE)} \\ 1.5 \text{ min} & \text{if } k = 9.05 \times 10^{-3} \quad \text{(SPEAR)} \end{cases} .$$

Both of these values for τ are too short for the colliding beams to be useful. Even for the favorable case of $k = 0.55 \times 10^{-3}$ we must reduce ξ by a factor 2 to 0.0025. This then increases the life-time to ~100 min which is just barely long enough. The addition of some damping through either stochastic or electron cooling will be very useful.

One may be surprised and incredulous of these short life-times. Believers will say that this is what is always expected of head-on collision of bunched beams. Non-believers will say that the discrepancy between the values of k for ADONE and SPEAR makes the entire program untrustworthy and the conclusions invalid. All one can be sure of is that a great deal of more work must be done.

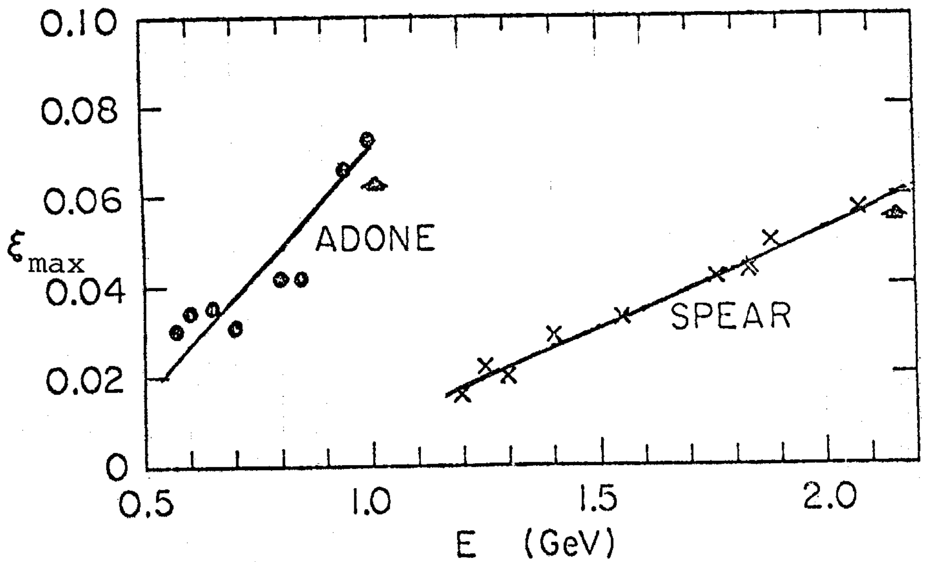

Figure 1 ξ_{max} versus E

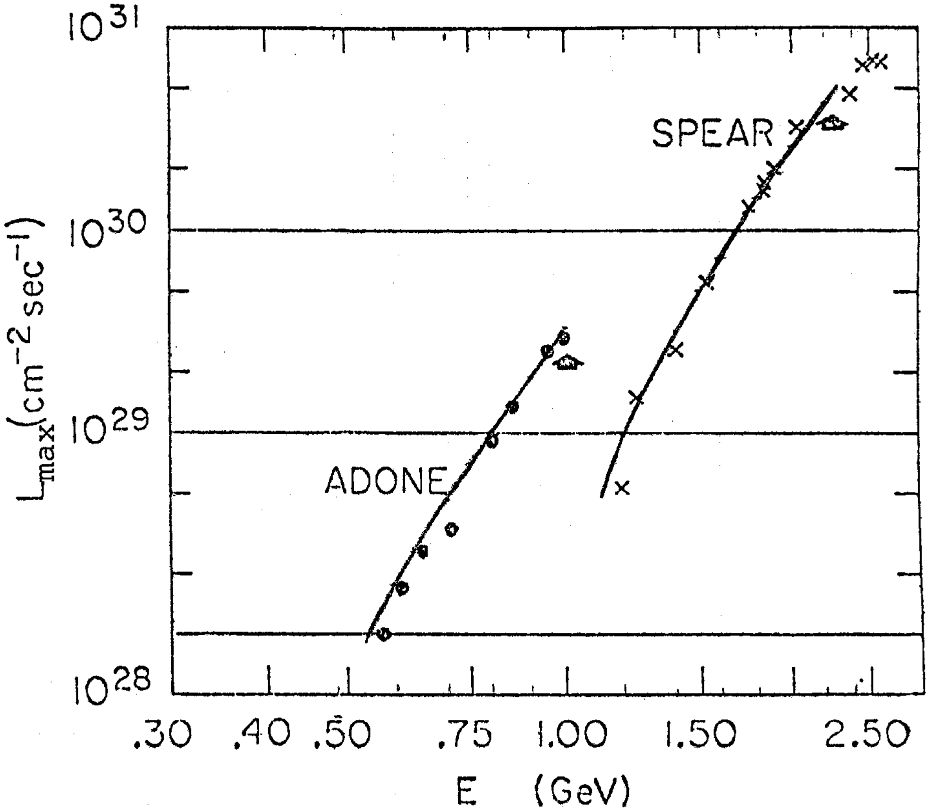

Figure 2 L_{max} versus E