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SIMPLE PHYSICS LIMITATIONS ON PULSED BENDING MAGNETS

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Pulsed magnets are used at an accelerator for injection, extraction, switching and abort functions on the beam. Simple physics determines the basic relations governing these devices. I have collected them for my own use and in hope that someone else will find them useful.[1]

Using an ideal magnet model, relations can be found among the deflection, the rise time, and the magnet parameters. Assume the magnet is of magnetic material of infinite permeability and is of rectangular cross section gap height G, width w and length ℓ. It is excited by a coil of N turns which carry current I.

Magnetic Field

The simple old story gives the magnetic field by Ampere's law.

$$\int B \cdot dl = \mu_o I \Rightarrow B = \frac{\mu_o IN}{G} \qquad \text{(MKS)} \tag{1}$$

Inductance

Using energy relations we will find the inductance

$$\text{Energy} = \frac{1}{2} LI^2 = \frac{1}{2} \frac{B^2 V}{\mu_o}$$

where V = volume = $wG\ell$. (Note that bend and inductance should be slightly modified by fringe field considerations.)

$$L = \frac{B^2}{I^2} \frac{wG\ell}{\mu_o} = \frac{\mu_o N^2 w\ell}{G} \tag{2}$$

or get the same result from Faraday's Law

$$\oint E \cdot dl = \frac{d}{dt} \int_s da\, \vec{B}\cdot\vec{n} \Rightarrow \text{ for 1 loop. } \quad V = \frac{d}{dt}(Bw\ell)$$

for N loops

$$V = Nw\ell \ \frac{dB}{dt} = Nw\ell \ \frac{\mu_o N}{G} \ \frac{dI}{dt} \Rightarrow L = \frac{\mu_o N^2 w\ell}{G}$$

Deflection

For a particle of momentum p one gets a deflection θ given by

$$p\theta \ (\text{GeV/c}) = .3 \ (B\cdot\ell) \ (\text{Tesla-m}) \tag{3}$$

where B is the magnetic field and ℓ is the path length in the field.

Time Response

We can find the response limitations directly from Faraday's Law above

$$\begin{aligned} V &= N \frac{d}{dt} (Bw\ell) \\ &= Nw \frac{d}{dt} (B\cdot\ell) \end{aligned} \tag{4}$$

Since the deflection $p\theta$ is proportional to $B\cdot\ell$ we see that the fundamental items in the response are the driving voltage V, the number of turns N, the width of the field w and the deflection required $p\cdot\theta$.

Most applications of pulsed magnets involve one of two types of pulse shape. If the magnet requires a flat pulse [because the beam would see the rise and/or fall time] then a delay line discharge (lumped parameter or cable delay) is frequently used for the pulsing circuit. If the beam is not present during the rising pulse or if the application tolerates beam motion, then a capacitive discharge system is often used, producing a half sine wave response. I will calculate properties and relations for each of these in turn.

Square Response Systems

If one builds a magnet such that it has a transmission line characteristic, then one has a characteristic velocity

$$v = (LC)^{-1/2}$$

where L, C are inductance and capacitance per unit length. The transmission time is then

$$T = \frac{\ell}{v} = \ell \sqrt{LC} \tag{5}$$

but $L_{TOT} = L\ell \quad C_{TOT} = C\ell$

$$T = \sqrt{L_{TOT}\, C_{TOT}} \tag{6}$$

Since the characteristic impedance of the line is $Z = \sqrt{\frac{L}{C}}$ we also have

$$T = \frac{L_{TOT}}{Z} = \frac{L_{TOT}\, I}{V}$$

We note that this time for the voltage and current wave form to propagate along ideal delay line magnet will be the rise time of the $\int B \cdot dl$ or $p \cdot \theta$. (Assuming negligible voltage pulse rise time)

$$\tau = \frac{L_{TOT}\, I}{V} = \frac{\mu_o N^2\, w\ell\, I}{GV} = \frac{\mu_o NI}{G} \cdot \frac{\ell\; Nw}{V}$$

$$\tau = \frac{B \cdot \ell\; Nw}{V} = \frac{p\theta\; Nw}{.3V} \tag{7}$$

where the substitutions for L, B involve the assumption that the rectangular aperture magnet is connected as a transmission line. This relation provides a fundamental limit on the available rise time for a magnet given by the aperture of the beam (machine) and the required bend.

The required current and the length available along the beam line are the other principal parameters of the problem. Using previous results we can obtain some other useful relations.

$$I = \frac{BG}{\mu_o N} = \frac{B \cdot \ell G}{\mu_o N \ell} = \frac{p\theta\ G}{.3\mu_o N\ \ell} \tag{8}$$

alternatively

$$I = \frac{G}{w}\ \frac{\tau V}{\mu_o N^2 \ell} \tag{9}$$

$$Z = \mu_o N^2\ \frac{\ell}{\tau}\ \frac{w}{G} \tag{10}$$

take N = 1 and then we have

$$Z = \left(\frac{w}{G}\right)\ \frac{\ell\,(m)}{\tau_{ns}} \times 1.26 \times 10^3 \text{ or } \tau\,(ns) = 25\left(\frac{50}{Z}\right)\left(\frac{w}{G}\right)\ell\,(m) \tag{11}$$

These relations all apply to a single kicker magnet. Often the requirements of a problem will demand several kickers. If there are N_K kickers driven by voltage V and current I_k, each of N_T turns and will length ℓ_k, gap G and width w then we find

$$\tau = \frac{p\theta\ N_T\ w}{.3\ N_k\ V} \tag{12}$$

$$I_k = \frac{p\theta\ G}{.3\mu_o N_T N_k \ell_k} \tag{13}$$

$$Z_k = \mu_o {N_T}^2\ \frac{\ell_k}{\tau}\ \frac{w}{G} \tag{14}$$

<u>Examples</u>

For concreteness let us consider approximate numbers for two kicker systems here at Fermilab. The first is the main ring extraction kicker[2] which deflects a 400 GeV beam across the extraction septum during a hole in the injected beam from booster. The kick is about 1/7 mr or $p\theta \sim 57$ MeV, which produces about 14 mm deflection at $\beta_{kick} \sim \beta_{septum} \sim 100$ m with phase advances $\sin\psi \sim 1$. A voltage of 60 kV on the charging line (it works at 80 kV if necessary for 500 GeV extraction) will produce 30 kV across a delay line magnet.

Thus we calculate for N_k kicker units (always use 1 Turn magnets)

$$N_k \tau = \frac{p\theta N_T W}{.3V} = 952 \text{ ns}$$

So to get the desired rise time takes several units. Two units is not quite as good as you would like but.... If we restrict ourselves to 6 m of magnet (not much more space is available) we then find

$N_k = 2$	$N_k = 4$	$N_k = 6$
$\ell_k = 3$	$\ell_k = 1.5$	$\ell_k = 1$
$\tau = 476$ ns	$\tau = 238$ ns	$\tau = 159$ ns
	$Z_k = 24\Omega$	

For each of these cases, $I_k = 1.25$ kA, but the total current from a delay line source will rise from 2.5 kA to 7.5 kA. The choice $N_k = 2$ is used. The pulse is supplied from a single 12Ω lumped parameter delay line thru a single thyratron pulse. The magnet, however, does not look like a delay line; more on that later.

In the booster, since the loss of one rf bucket per pulse results in 13 holes in the main ring beam, any more than this would be unfortunate.[3,4] Rise times of about 15 ns would be required to avoid hitting one bunch and it is difficult to obtain such a rise time with a thyratron pulser. Thus a magnet with a pulse delay time of about 25 ns is desired. This needs to provide about 20 mm of deflection to 8.9 GeV/c beam at $\beta \sim 20$ m. This gives $\theta \sim 1$ mr and $p\theta \sim 9 \times 10^{-3}$. We need $w \sim 7$ cm, and plan $V \sim 25$ kV (50 KV on charging line)

$$N_k = \frac{p\theta \; N_T W}{.3 \; V\tau} = 3.36$$

So the system was built conservatively with $N_k = 4$ while V is capable of operation at > 30 kV where 3 kickers will supply the required kick. We have about 6 m available which would allow ℓ_k = 1.5 m while G ~ w ~ 7 cm.

$$Z_k = \mu_o N_T^2 \frac{\ell_k}{\tau} \frac{w}{G} = 75\Omega$$

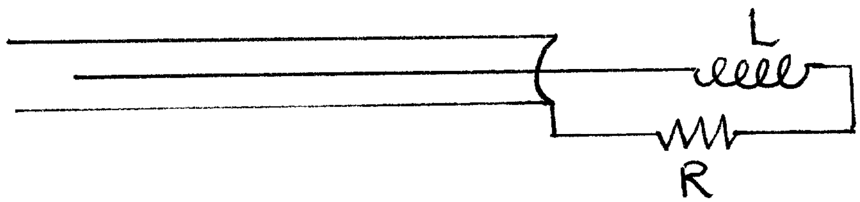
In fact when one fits 4 kickers into one long straight they really only get about 1 m and they were built with Z_k ~ 50Ω. Four separate 50Ω delay cables are used, each with its own thyratron switch which delivers 600A at 30 kV.

Building delay lines magnets is a demanding problem. Especially difficult are low impedance magnets (large capacitance per unit length). One useful technique is to short circuit the magnet at one end and drive it with a suitable pulser. This will produce twice the kick per ampere but the delay time will be twice as long per magnet (down and back). It does allow a 50Ω impedance magnet design to provide as much kick per unit length as a 25Ω magnet.[5]

The engineering parameters of fast kickers are mostly well-defined by the geometry of the machine and the time structure and momentum of the beam. However, the rectangular geometry discussed so far is artificial and for at least one notable case useful improvement is possible. The main ring extraction kickers need to provide a large open width for the low momentum beam. By adding space for ceramic beam pipes, one finds a net requirement of almost 15 cm width between the conductors. However, a generous allowance for good field region required is 2-3 cm. Allowing for inductance in the poor field region still allows a substantial reduction in the effective width. Such a design was produced by S. C. Snowdon in TM-420.[6]

Driving a Short (Lumped Inductance) Magnet from a Long Delay Line

The transmission line magnet is the ideal pulsed magnet which if driven with an ideal square pulse will give the ultimate response. If, however, the magnet is just a lumped inductance but the driving circuit puts a long pulse thru the coaxial driving system, then we find a circuit as follows



The coax has characteristic impedance Z = R and will be terminated by inductance L and resistance R. The voltage and current wave form in the cable will be related

$$V = IZ$$

and in addition to the incoming voltage V_I there will be a reflected wave V_o. The load accepts current I_L (t)

$$I_I - I_o = I_L \qquad \text{(current sum)}$$

$$V_I + V_o = L\frac{dI_L}{dt} + I_L R \qquad \text{(voltage sum)}$$

But

$$RI_L = I_I R - I_o R = V_I - V_o$$

$$V_I + (V_I - I_L R) = L\frac{dI_L}{dt} + I_L R$$

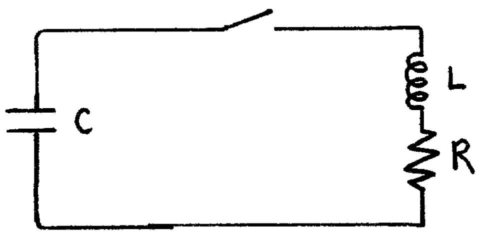
$$2V_I = L\frac{dI_L}{dt} + 2I_L R$$

$$\frac{V_I}{R} = \frac{L}{2R}\frac{dI_L}{dt} + I_L$$

Thus we see that the system exponential time is L/2R. The reflected voltage starts out at magnitude equal to the incoming pulse, driving the magnet twice as fast. This means that while a transmission line magnet will rise in time L/R, the lumped inductance will reflect energy but will be e^{-2} or ~ 13% from its final value in the same time L/R. However, it will also have effects at the falling edge (not so nice for injection). This is the driving technique employed for the main ring extraction kickers.

Half Sine Wave Response Systems

Typical magnet systems which allow a half sine wave response are described by a circuit shown here and with initial conditions



$I_L = 0 \quad V_C = V_o$

This will have sinsoidal response if R is small enough

with $\omega = (LC)^{-1/2}$ or a $\frac{1}{4}$ wave rise time

$$\tau = \frac{\pi}{2}\sqrt{LC}$$

But if all the energy transfers from capacitor to magnet then

$$\frac{1}{2}CV_o^2 = \frac{1}{2}LI_{max}^2$$

$$\sqrt{LC} = \frac{LI_{max}}{V_o}$$

But

$$I_{max} = \frac{B_{max}G}{\mu_o N} \qquad L = \frac{\mu_o N^2 w\ell}{G}$$

So

$$\tau = \frac{\pi}{2}\frac{B\cdot\ell\ wN}{V_o}$$

The capacitance must store the energy and/or supply the appropriate time scale for the pulse. Energy balance requires

$$CV^2 = LI^2 = \frac{B^2 wG\ell}{\mu_o}$$

$$C = \frac{(B\cdot\ell)^2}{V^2} \quad \frac{wG}{\mu_o \ell}$$

While the time scale says that

$$\tau = \frac{\pi}{2} \sqrt{L} \sqrt{C}$$

$$= \frac{\pi}{2} N \sqrt{\underline{\mu_o w\ell}} \sqrt{C}$$

or

$$C = \frac{\tau^2}{\pi^2 N^2} \frac{G}{\mu_o w\ell}$$

Another interesting design quantity is the RMS current. We find

$$\langle I^2 \rangle = \langle \frac{CV^2}{L} \rangle = \frac{C}{L} \frac{V_o^{\,2}}{2} N_p \tau$$

Where N_p is the number of pulses per second for the magnet. We find for a given τ

$$I_{RMS} = \frac{(B\cdot\ell)\ G}{\mu_o \ell\ N} \sqrt{\frac{N_p \tau}{2}}$$

Alternatively if we substitute for τ

$$I_{RMS} = \frac{G}{2\mu_o \ell} \sqrt{\frac{\pi\ (B\cdot\ell)^3\ w\ N_p}{NV_o}}$$

Discussions on this matter with S. C. Snowdon, F. E. Mills, K. Bourkland, J. D. McCarthy and C. M. Ankenbrandt have been very enlightening. I would like to also thank Dr. Ankenbrandt for encouragement to document this work and for reading the final manuscript.

1. I will use MKS units throughout, ($\mu_0 = 4\pi \times 10^{-7}$ h/m). This analysis is coherently presented in R. Bertolotto, et al., "The Fast Ejection System of the CERN 25 GeV Proton Synchrotron", P. 874, Int. Conf. on High Energy Accel., DUBNA, 1963.

2. R. A. Andrews, et al., "Extraction for the Main Ring at NAL", 1973 Particle Accel. Conf.

3. A. W. Maschke and L. W. Oleksiuk, "The NAL Booster-to-Main Ring Beam Transfer System", 1971 Particle Accel. Conf., p. 989.

4. E. L. Hubbard, Editor, "Booster Synchrotron", Fermilab TM-405, January, 1973.

5. E. M. Rowe and F. E. Mills, "TANTALUS I: A Dedicated Storage Ring Synchrotron Radiation Source", Particle Accelerators 1973, Vol. 4, P. 221-227.

   I thank Fred Mills for helpful discussions, for describing this technique and providing the reference.

6. S. C. Snowdon, "Kicker Magnet Design", Fermilab TM-420.