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NEUTRINO BEDPLATE DEFLECTIONS

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The Neutrino Target Area equipment is mounted on bedplates and transported into the target tube via a railway system. This equipment varies widely in weight and position on the bedplates for the various "target trains." When new "trains" are in the conceptual state of design, many iterations of the equipment are usually considered. Therefore, it is important to be able to determine these deflections rapidly and easily, since the deflections must fall within certain tolerable limits. Excessive deflections require the use of a strong back for the bedplate to remain within the necessary limits.

This paper presents the development of the equations and their adaption to a computer program used to calculate the deflections of the center and both ends of a simply supported beam with equal overhanging ends. The program is written to accept one (1) load on each end and two (2) intermediate loads. These conditions are typical for Neutrino Target Train Bedplates. The analysis is based on the method of superposition, and therefore, the deflection of any number of loads may be computed by running the program as often as required and adding the results.

For the nomenclature used in the derivation of the formulae, see Figure 1 and the accompanying legend.

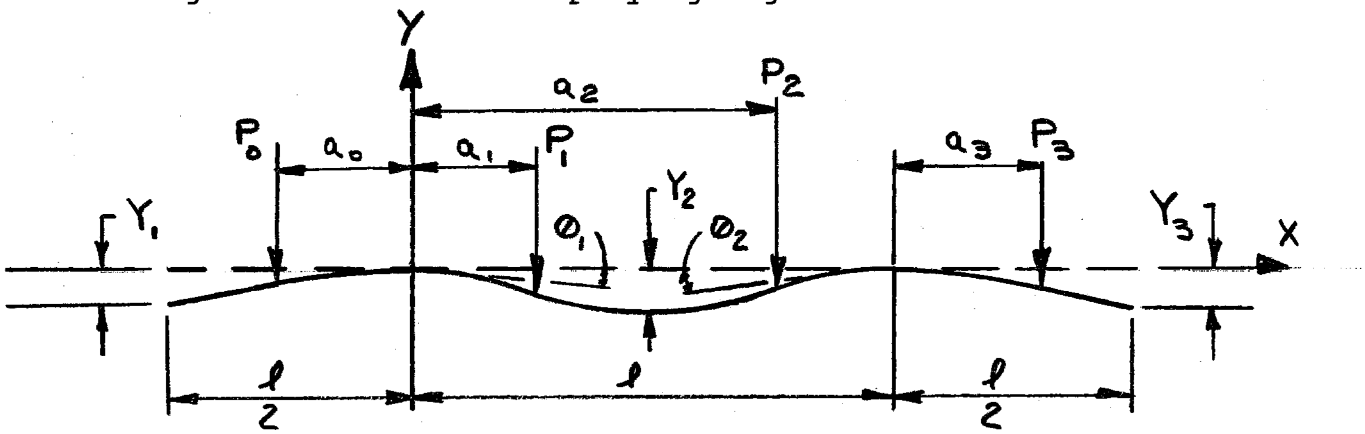

Figure I

P_0, P_1, P_2, P_3, - loads on the beam (lbs.)

a_0, a_1, a_2, a_3, - absolute distances to the above loads (inches).

ℓ -- Span between supports (inches)

ℓ_2 -- Length of overhang (inches)

Θ_1 -- Slope at left support - R_L (Radians)
Θ_2 -- Slope at right support - R_R (Radians) } CCW Rotation is positive

Y_1 -- Deflection at left end (inches)
Y_2 -- Deflection at center (inches) } Positive is down
Y_3 -- Deflection at right end (inches)

It should be noted that the deflection of the beam due to its own weight has been neglected. These equations assume the loads are centered laterally on the bedplate. Where the loads are eccentrically located along the width of the bedplate and the support stand is such that the load is not distributed for the width of the bedplate, a separate analysis must be made.

CENTER DEFLECTION

The equations developed for this deflection are from the fundamental equations found in Roark's 5th edition. For load P_1 where $a_1 < \ell/_2$:

$$y = \frac{P_1 a_1 (\ell - x)}{6EI\ell}\{2\ell b - b^2 - (\ell - x)^2\}$$ Roark's page 106 Case 12

Let $x = \ell/_2$ and $b = \ell - a_1$, we have

$$y = \frac{P_1 a_1 (\ell - \ell/_2)}{6EI\ell}\{2\ell(\ell - a_1) - (\ell - a_1)^2 - (\ell - \ell/_2)^2\}$$

$$y = \frac{P_1 a_1}{48EI}(3\ell^2 - 4a,^2)$$ Equation 1

For load P_2 where $a_2 > \ell/_2$

$$y = \frac{P_2 bx}{6EI\ell}\{2\ell(\ell - x) - (\ell - a_2)^2 - (\ell - x)^2\}$$ Roark's Page 106 Case 12

Let $x = \ell/_2$ and $b = (\ell - a\ )$, we have

$$y = \frac{P_2(\ell - a_2)(\ell/_2)}{6EI\ell}\{2\ell(\ell - \frac{\ell}{2}) - (\ell - a_2)^2 - (\ell - \frac{\ell}{2})^2\}$$

$$y = \frac{P_2(\ell - a_2)}{48EI}(8a_2\ell - 4a_2^2 - \ell^2)$$ Equation 2

The deflection at mid-span due to the overhanging loads P_0 and P_3 is derived as follows:

$$y = \frac{\ell}{6}\frac{M_0}{EI}\left(3x^2 - \frac{x^3}{\ell} - 2\ell x\right)$$ Roark's page 108 Case 19

Let $x = \ell/2$, we have

$$y = \frac{\ell}{6}\frac{M_0}{EI}\left\{3\left(\frac{\ell}{2}\right)^2 - \frac{\left\{\frac{\ell}{2}\right\}^3}{\ell} - 2\ell\left(\frac{\ell}{2}\right)\right\}$$

$$y = -\frac{M_0\ell^2}{16EI}$$ Equation 3, (minus sign indicates upward deflection.

For load P_0 and distance a_0, $M_0 = P_0a_0$

$$y = \frac{P_0a_0\ell^2}{16EI}$$ Equation 4

For load P_3 and distance a_3, $M_0 = P_3a_3$

$$y = \frac{P_3a_3\ell^2}{16EI}$$ Equation 5

The summation of equations 1,2,4 and 5 will yield the total mid-span deflection.

$$y_2 = \frac{P_1a_1}{48EI}(3\ell^2 - 4a_1^2) + \frac{P_2(\ell - a_2)}{48EI}(8a_2\ell - \ell^2 - 4a_2^2) - \frac{P_0a_0\ell^2}{16EI} - \frac{P_3a_3\ell^2}{16EI}$$

END DEFLECTIONS

The end deflections are determined from the slopes at the supports and the cantilever loading at the ends. The slopes are calculated based on CCW rotation as positive. Slope at supports for any central load is

$$\theta_{r_\ell} = -\frac{1}{6}\frac{W_1}{EI}\left(b\ell - \frac{b^3}{\ell}\right)$$ Roark's page 106 Case 12

$$\theta_{r_r} = +\frac{1}{6}\frac{W_1}{EI}\left(2b\ell + \frac{b^3}{\ell} - 3b^2\right)$$

Slope at supports for overhanging load

$$\theta_{r_\ell} = \frac{1}{3}\frac{M_0\ell}{EI}$$

$$\theta_{r_r} = \frac{1}{6}\frac{M_0\ell}{EI}$$

Roark's page 108 Case 19

Substituting for $M = P_n a_n$, $W = P_n$ and $x = \ell/2$ and making the appropriate summations, we have,

$$\Theta_1 = -\frac{P_1 a_1(\ell-a_1)(2\ell-a_1) - P_2 a_2(\ell-a_2)(2\ell-a_2)}{6EI\ell} + \frac{P_3 a_3 \ell}{6EI} + \frac{P_0 a_0 \ell}{3EI} . \quad \underline{\text{Equation 7}}$$

$$\Theta_2 = \frac{P_1 a_1(\ell^2-a_1^2) + P_2 a_2(\ell^2-a_2^2)}{6EI\ell} - \frac{P_3 a_3 \ell}{3EI} - \frac{P_0 a_0 \ell}{6EI} . \quad \underline{\text{Equation 8}}$$

The deflection of the cantilever loads at their point of application is

$$y = \frac{1}{6}\frac{W}{EI}(-a^3 + 3a^2\ell - 3a^2 x) \qquad \text{Roark's page 104 Case 2}$$

Let $x = o$ and $\ell = \frac{\ell}{2}$ (length of overhang), the deflection is then,

$$y = \frac{1}{12}\frac{W}{EI}(3a^2\ell - 2a^3)$$

The total deflection at the end must include the slope at the support multiplied by the length of the overhang. Deflection at the left end,

$$Y_1 = \frac{1}{12}\frac{P_0}{EI}(3a_0^2\ell - 2a_0^3) + \Theta_1 \frac{\ell}{2} \qquad \underline{\text{Equation 9}}$$

Deflection at the right end,

$$Y_3 = \frac{1}{12}\frac{P_3}{EI}(3a_3^2\ell - 2a_3^3) - \Theta_2 \frac{\ell}{2} . \qquad \underline{\text{Equation 10}}$$

COMPUTER PROGRAM

The equations outlined in the preceding discussion have been programmed on the Tektronix 4051 computer, using basic language. This unit is located on the 12th floor of the Central Lab Bldg., in the Neutrino Section. The program is stored on magnetic tape and is available to anyone. Attached to this paper is a printout of the program and an example problem.

```
LIST
100 INIT
110 PAGE
120 PRINT "THIS PROGRAM CALCULATES THE END AND CENTER DEFLECTIONS FOR A"
130 PRINT "SIMPLY SUPPORTED OVERHANGING BEAM. THE WEIGHT OF THE BEAM    "
140 PRINT "HAS BEEN NEGLECTED."
150 PRINT "WHAT IS E,";
160 INPUT E
170 PRINT "WHAT IS I,";
180 INPUT I
190 PRINT "WHAT IS L,";
200 INPUT L
210 PRINT "WHAT IS LOAD P0,";
220 INPUT P0
230 PRINT "WHAT IS DISTANCE A0,";
240 INPUT A0
250 PRINT "WHAT IS LOAD P1,";
260 INPUT P1
270 PRINT "WHAT IS DISTANCE A1,";
280 INPUT A1
290 PRINT "WHAT IS LOAD P2,";
300 INPUT P2
310 PRINT "WHAT IS DISTANCE A2,";
320 INPUT A2
330 PRINT "WHAT IS LOAD P3,";
340 INPUT P3
350 PRINT "WHAT IS DISTANCE A3,";
360 INPUT A3
370 K0=P0*A0*L/(3*E*I)
380 K1=-P1*A1*(L-A1)*(2*L-A1)/(6*E*I*L)
390 K2=-P2*A2*(L-A2)*(2*L-A2)/(6*E*I*L)
400 K3=P3*A3*L/(6*E*I)
410 K=K1+K2+K3+K0
420 R0=-P0*A0*L/(6*E*I)
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430 R1=P1*A1*(L↑2-A1↑2)/(6*E*I*L)
440 R2=P2*A2*(L↑2-A2↑2)/(6*E*I*L)
450 R3=-P3*A3*L/(3*E*I)
460 R=R0+R1+R2+R3
470 D0=P0*(3*A0↑2-2*A0↑3)/(12*E*I)+K*L/2
480 D1=P1*A1*(3*L↑2-4*A1↑2)/(48*E*I)
490 D2=P2*(L-A2)*(8*A2*L-L↑2-4*A2↑2)/(48*E*I)
500 D3=-P3*A3*L↑2/(16*E*I)
510 D4=-P0*A0*L↑2/(16*E*I)
520 D=D1+D2+D3+D4
530 W=P3*(3*A3↑2-2*A3↑3)/(12*E*I)-R*L/2
540 PRINT "SLOPE AT LEFT SUPPORT=";K;"RADIANS"
550 PRINT "SLOPE AT RIGHT SUPPORT=";R;"RADIANS"
560 PRINT "LEFT END DEFLECTION=";D0;"IN."
570 PRINT "CENTRAL DEFLECTION=";D;"IN."
580 PRINT "RIGHT END DEFLECTION=";W;"IN."
590 END
```

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THIS PROGRAM CALCULATES THE END AND CENTER DEFLECTIONS FOR A
SIMPLY SUPPORTED OVERHANGING BEAM. THE WEIGHT OF THE BEAM
HAS BEEN NEGLECTED.
WHAT IS E,30E6
WHAT IS I,112
WHAT IS L,120
WHAT IS LOAD P0,4500
WHAT IS DISTANCE A0,31
WHAT IS LOAD P1,4500
WHAT IS DISTANCE A1,15.5
WHAT IS LOAD P2,6500
WHAT IS DISTANCE A2,67.7
WHAT IS LOAD P3,6500
WHAT IS DISTANCE A3,14.2
SLOPE AT LEFT SUPPORT=-1.054276629E-4RADIANS
SLOPE AT RIGHT SUPPORT=2.647345395E-4RADIANS
LEFT END DEFLECTION=-0.0126536731647IN.
CENTRAL DEFLECTION=0.0241741251736IN.
RIGHT END DEFLECTION=-0.0167097361607IN.
```