national accelerator laboratory

TM-703
1600.00
November 16, 1976

PRESSURE DROP OF SINGLE PHASE AND TWO PHASE HELIUM FLOW

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ABSTRACT

Formulas for the pressure drop of both helium streams of the Energy Doubler are presented as a function of the cross section area, wetted perimeter, length and mass flow rate. These formulas provide reasonable values and are intended as a tool for guiding the design of the cryostats and junction boxes. An instructive graph applicable to a hypothetical 400 ft long cryostat is also presented.

INTRODUCTION

The cooling of the Energy Doubler superconducting coils is done by the elegant scheme[1] of having the wires in contact with pressurized (~2 atm) liquid (single phase) which is continuously exchanging heat with boiling helium (two phase). The higher pressure being more effective in providing uniform cooling.

In the implementation of this scheme the single phase flows through a series of ~20 cryostats and junction boxes, at the end of which it is expanded into a liquid and gas mixture (two phase) at ~1.2 atm and returned, flowing through the same cryostats and junction boxes in the reverse order. In the present design (E22-15) a 5-5/9" i.d., .036" wall stainless steel tube separates the two flows. The pressurized liquid inside this tube flows along the cryostat by forced convection, with a transversal component in free convection. This transversal flow pattern carries the heat generated in the superconducting wire to the returning two phase flow.

In steady operation, by conservation of mass, both streams have the same mass flow rate $\dot{m}$. The temperature is determined by the local pressure in the two phase stream. We impose this temperature to be kept between 4.5 and 4.2°K, therefore, the maximum pressure drop over the two phase stream should be less than 4.52 psi. The change in enthalpy from liquid at 4.5K to gas at 4.2K is 20.1 J/g. The required mass flow rate is determined by the heat dissipated in the system. For instance: with a total heat dissipation of 800 W (i.e., 40 W/magnet) to be carried out, a minimum $\dot{m} = \frac{800}{20.1} \simeq 40.$ g/sec is required.

SINGLE PHASE PRESSURE DROP

In the calculations that follow we use the following notations:

A = cross section area

ω = wetted perimeter

$D = 4\frac{A}{\omega}$ equivalent diameter

v = velocity of flow

ρ = density

μ = viscosity

$\dot{m} = \rho v A$ mass flow rate

$Re = \rho v D/\mu = 4\ \dot{m}/\mu\omega$ Reynolds number

f = friction coefficient

ΔP = pressure drop

L = length

For a turbulent flow with $10{,}000 < Re < 120{,}000$ we have

$$\Delta P = 4f\frac{L}{D}\ \rho\frac{v^2}{2}$$

and $f = .046\ Re^{-.2}$

after some algebra this yields:

$$\frac{\Delta P_{1\phi}}{L} = .017\ \mu^2\ \omega^{1.2}\ \dot{m}^{1.8}/\rho A^3$$

(CGS units)

or in more convenient form:

$$\boxed{\frac{\Delta P_{1\phi}}{L}\ (\text{psi/foot}) = C(P,T)\ \frac{\omega^{1.2}}{A^3}\ \dot{m}^{1.8}} \qquad (1)$$

where the units are: ω(cm), A(cm^2) and $\dot{m}$(g/sec) and C(P,T) is given[2] by Table I.

TABLE I

Coefficient C(P,T) for Expression (1)

P \ T	4.5K	4.424K	4.0K
2.0 atm	7.63×10^{-6}	7.20×10^{-6}	7.20×10^{-6}
1.6 atm	7.75×10^{-6}	7.26×10^{-6}	7.26×10^{-6}
1.2 atm		7.99×10^{-6}	

The restrictions on the Reynolds number can be expressed as:

$$.088 < \frac{\dot{m}}{\omega} < .936\ \frac{g}{\text{cm sec}}$$

In sudden contraction or expansion of cross sections, changes in the momentum of the fluid cause a pressure drop, $\Delta P_{1\phi_M}$. In terms of the smaller cross section area involved, $A_1 = \frac{\pi}{4}D_1^2$, for the case of circular pipes its expression is:

$$\Delta P_{1\phi_M} = \frac{1}{2\rho}K\dot{m}^2 A_1^{\ 2} \qquad \text{(CGS units)} \qquad (2)$$

where the resistance coefficient K is presented in Figure 1 and the factor $\frac{1}{2\rho}$ in Table II.

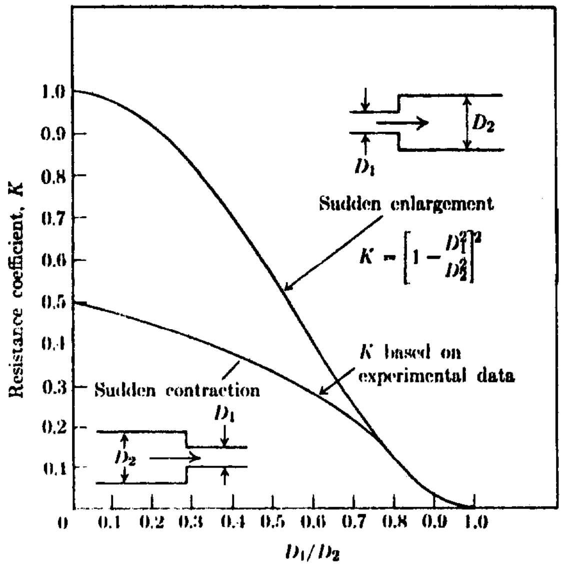


Figure 1. Resistance due to sudden expansions and contractions.

TABLE II

Values of $\frac{1}{2\rho}$ (cm^3/g)

P \ T	4.5K	4.0K
2.0 atm	4.013	3.715
1.6 atm	4.107	3.762

TWO PHASE PRESSURE DROP

The composition of the two phase stream is characterized along its path by its quality x defined as

$$x = \frac{m_G}{m_G + m_L}$$

where m_G and m_L are the mass of the gas and liquid fractions respectively. The pattern of the flow can be evaluated with the help of Baker's diagram[3] shown in Figure 2. We will restrict the calculations to the patterns in which the gas and the liquid have the same velocity (bubble flow and plug flow. In this case x can be expressed in terms of mass flow rates:

$$x = \frac{\dot{m}_G}{\dot{m}_L + \dot{m}_G}$$

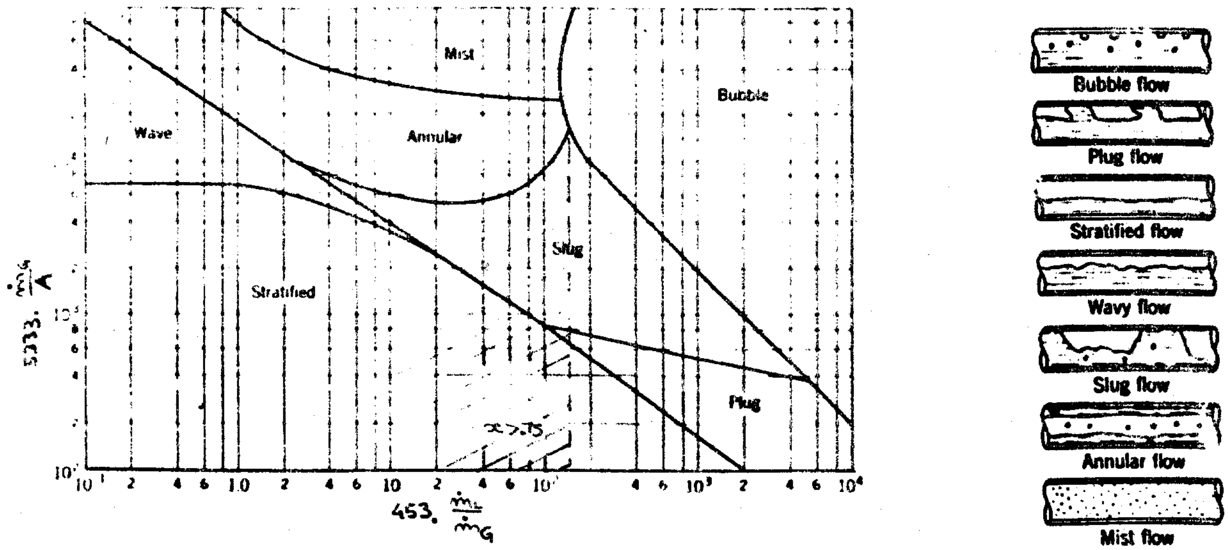


Figure 2. Baker's two-phase flow pattern regions for helium.

From Baker's diagram we should have $453\ \frac{\dot{m}_L}{\dot{m}_G} > 150$ for this condition to be valid. Therefore this calculation applies only to x<.75 (i.e., at least 1/4 of the mixture in liquid state).

Since heat is continuously being added to the flow it is a diabatic flow. Its pressure drop due to friction $\left.\frac{dP_{2\phi}}{dL}\right)_f$ can be given in terms of the pressure drop, $\left(\frac{dP_{1\phi}}{dL}\right)_o$, that one would obtain if the quality x were zero by the expression

$$\left.\frac{dP_{2\phi}}{dL}\right)_f = (1-x)^{1.8}\,\phi_L^{2}\left(\frac{dP_{1\phi}}{dL}\right)_o$$

where we assume a Reynolds number larger than 2000 for both gas and liquid components. Here $\emptyset$ is a parameter given by the Martinelli-Nelson correlation.[4] Over a finite length L we have

$$\left(\frac{\Delta P_{2\phi}}{L}\right)_f = \left(\frac{\Delta P_{1\phi}}{L}\right)_o \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} (1-x)^{1.8} \; \emptyset_L^{\,2} dx \tag{2A}$$

where $\left(\frac{\Delta P_{1\phi}}{L}\right)_o$ is given by (1). For liquid helium boiling at 1.2 atm ($\therefore$4.424K) the integral was evaluated and can be given by: $13.13\ (x_2^{1.62} - x_1^{1.62})$. The requirement above on the Reynolds number (>2000) can be expressed as $\left\{\begin{array}{l}\dot{m}_G > .0067\omega \text{ g/sec} \\ \dot{m}_L > .0153\omega \text{ g/sec}\end{array}\right.$

where the wetted perimeter ω is given in cm.

The pressure drop due to momentum change is:

$$\Delta P_M = \frac{\emptyset_m \; \dot{m}^2}{A^2 \rho_L} \tag{3}$$

$$\text{with } \emptyset_m = \left[\frac{(1-x_2)^2}{R_{L_2}} - \frac{(1-x_1)^2}{R_{L_1}}\right] + \left(\frac{x_2^{\,2}}{R_{G_2}} - \frac{x_1^{\,2}}{R_{G_1}}\right) \frac{\rho_L}{\rho_G} \quad \text{(CGS units)}$$

where R_L and R_G are the volume fractions of the liquid and gas phase respectively. For boiling helium at 1.2 atm, when we neglect the pressure drop due to momentum change, equation (2A) can be written as

$$\boxed{\Delta P_{2\phi}(\text{psi}) = 105.\times 10^{-6} \; \frac{\omega^{1.2}}{A^3} \; \dot{m}^{1.8} \; \frac{x_2^{\,1.62} - x_1^{\,1.62}}{x_2 - x_1} \; L} \tag{4}$$

where L is given in feet but $\dot{m}$, A and ω in CGS units.

Imposing a total pressure drop of $\Delta P_{2\phi}$ = 3 psi over 400 feet of magnets with 1/4 of the mass flow in liquid form at the end (x_2 = .75) and initial quality resulting from the isoenthalpic expansion of 1.6 atm helium at 4.5K into 1.2 atm at 4.424K ($\therefore x_1$ = .0209), we get:

$$3 = 36.0\times10^{-3}\,\frac{\omega^{1.2}}{A^3}\,\dot{m}^{1.8} + .19\times10^{-3}\,\frac{\dot{m}^2}{A^2} \quad ; \qquad (5)$$

an expression which is presented in a graphical form in Figure 2, and whose last term is a small pressure drop correction due to change in momentum. In this figure we also inserted two lines representing the geometric relationship between A and ω for a cylindrical pipe ($A = \omega^2/4\pi$) and for a cylindrical anulus ($A = \omega\delta/2$) with gap δ. The values of δ relevant to the Energy Doubler cryostat are explicitly indicated.

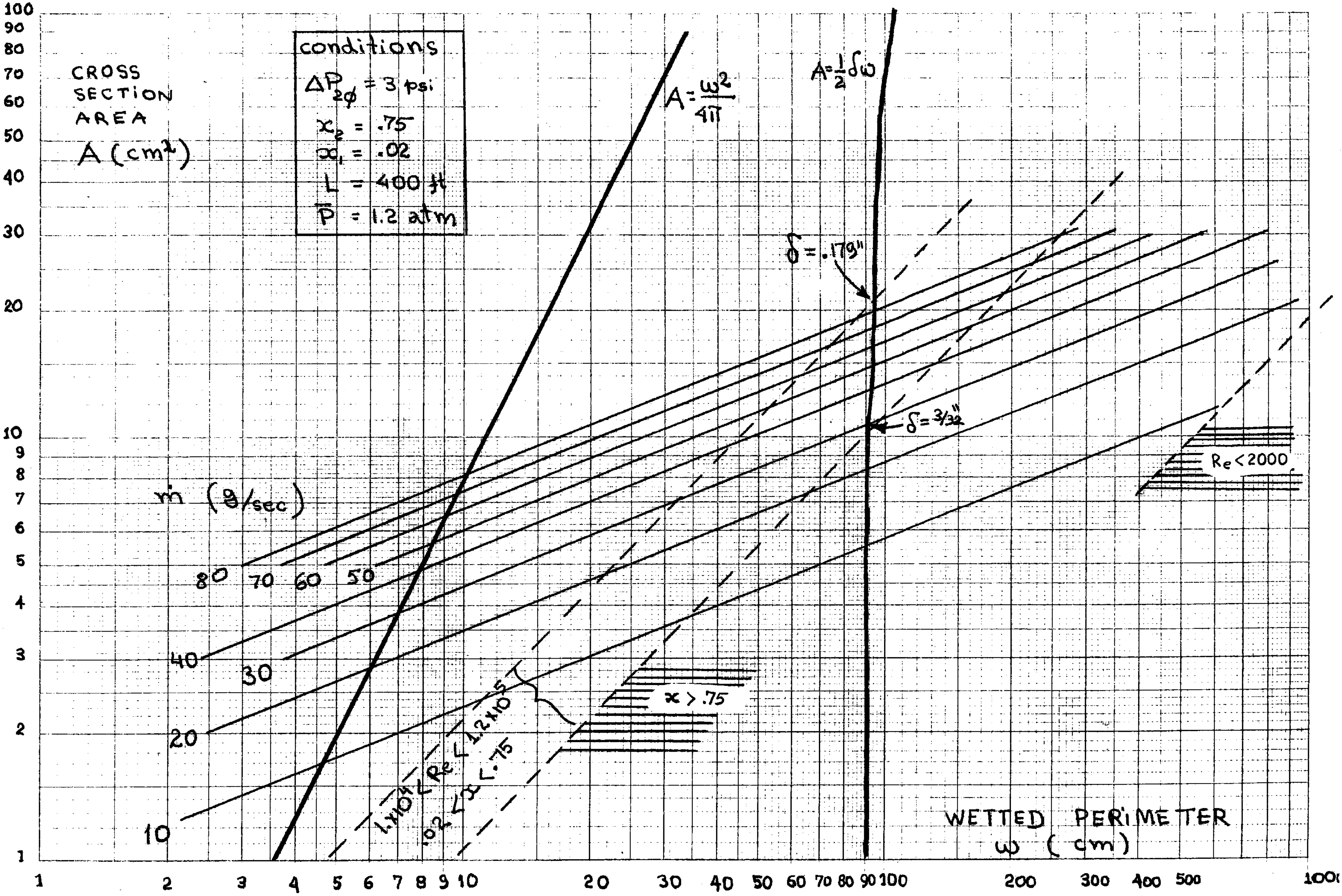
Dashed lines delineate the regions over which $10{,}000<Re_L<120{,}000$ and $.02<x<.75$ are simultaneously satisfied. These conditions reduce to

$$\frac{10{,}000\ \mu_L\omega}{4(1-.75)} < \dot{m} < \frac{120{,}000\ \mu_L\omega}{4(1-.02)} \qquad \text{where} \quad \mu_L = 30.6\times10^{-6}\,\frac{g}{cm\ sec}$$

and only over this region is the calculation valid.

COMMENT

This calculation is based on the widely used Martinelli-Nelson correlation determined from studies of different mixtures. However, in the one published case[5] in which the pressure drop of boiling liquid helium in a 3 mm i.d., 11 ft long tube was measured[5] the authors found this correlation to give values too high by a factor of 3. Applying expression (5) to their very small diameter case might not be warranted. Unpublished measurements made on the Doubler loop of the Protomain were consistent with the Martinelli correlation.[6] In the actual case of the Energy Doubler we have junction boxes with complicated flow paths which have not been taken into account in the example given, but can be estimated with the help of expressions (1), (2), (3) and (4).



CROSS SECTION AREA
A (cm²)
conditions
ΔP2φ = 3 psi
x2 = .75
x1 = .02
L = 400 ft
P̄ = 1.2 atm
A = ω²/4π
A = ½δω
δ = .179"
δ = 3/32"
ṁ (g/sec)
80 70 60 50
40
30
20
10
x > .75
10^4 < Re < 1.2x10^5
.02 < x < .75
Re < 2000
WETTED PERIMETER
ω (cm)

REFERENCES

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2. Calculated with data tabulated in NBS Technical Note 631.

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4. Ibid, p. 522.

5. A. de LaHarpe, S. Lehangre, J.Mollard and C.Johannes, Adv. Cryo. Engr. 14, 170 (1969).

6. S.Stoy, private communication.