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ESTIMATES CONCERNING BEAM NEUTRALIZATION FOR POPAE

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The allowed neutralization due to tune shift and electron-proton instability are estimated. Preliminary estimates show that this neutralization is achievable without a field gradient in the bending magnets.

When the beam is partially neutralized by electrons trapped in the beam's electric field, the tune shift due to the beam itself can be written[1]

$$\Delta Q_{sp.ch.} = \frac{N_p r_p R}{\pi b(a+b) Q \beta^2 \gamma} \left[\frac{1}{\gamma^2} - \eta + \varepsilon_1 \frac{b(a+b)}{h^2} \left(\beta^2 + \frac{1}{\gamma^2} - \eta \right) + \varepsilon_2 \frac{b(a+b)}{g^2} \beta^2 \right]$$

where η = fractional neutralization and all other symbols are as in the CERN Synchrotron Design Handbook. Thus, for a change in neutralization

$$\frac{\partial \Delta Q_{sp.ch.}}{\partial \eta} \approx - \frac{N_p r_p R}{\pi b(a+b) Q \beta^2 \gamma} \quad ; \quad \underline{\text{NOTE}} \quad \frac{\partial \Delta Q}{\partial \eta} \propto N_p$$

For preliminary POPAE parameters

$$\frac{\partial \Delta Q}{\partial \eta} \approx - \frac{2\text{x}10^{15}\text{x}1.5\text{x}10^{-18}\text{x}1.5\text{x}10^{3}}{\pi\text{x}1.5\text{x}10^{-3}\text{x}7.5\text{x}10^{-3}\text{x}4\text{x}10^{1}\text{x}4.3\text{x}10^{2}} \qquad \begin{array}{l} I = 10A \\ E = 400 \text{ GeV} \end{array}$$

≈ -7.4. If $\Delta Q'$ is the allowable change in tune shift due to neutralization of the space charge then $\eta_{allowed} \approx \frac{\Delta Q'}{7.4}$

Based on ISR experience where $\Delta Q_{sp.ch.} \sim$ few x 10^{-2}, let's take $\Delta Q' \simeq 10^{-2}$ as being reasonable so $\eta \gtrsim 10^{-3}$ seems ok.

Note that for fixed energy $\eta_{allowed} \propto \frac{1}{N_p}$

so that if we are ok at full current, we are still ok at lower current provided that the sweep out time for the electrons varies no more rapidly than the inverse of the beam current.

One must also avoid electron proton instability thus setting an upper limit on the number of electrons that may accumulate in the beam. To check on this, we use the criterion given by Keil[1]

$$Q_p < \frac{8Q}{3\pi}\sqrt{\frac{\Delta Q}{Q}\frac{\Delta Q_e}{Q_e}}$$

Q = betatron tune ~ 40

Q_p = proton bounce freq.

Q_e = electron bounce freq.

ΔQ's = tune spreads which give Landau damping

$$Q_e^2 \simeq \frac{2N_p r_e R}{\pi b(a+b)\beta^2} \quad ; \quad Q_p^2 \simeq \frac{2\eta N_p r_p R}{\pi\beta^2\gamma b(a+b)}$$

$$Q_e^2 \simeq \frac{2x2x10^{15}x2.81x10^{-15}x1.5x10^3}{\pi x1.5x10^{-3}x7.5x10^{-3}} = 0.48x10^9 \simeq 5x10^8$$

$$Q_p^2 \simeq \frac{2\eta x2x10^{15}x1.5x10^{-18}x1.5x10^3}{\pi x1.5x10^{-3}x7.5x10^{-3}x4.27x10^2} \simeq 0.59\eta x10^4 \simeq 6x10^2\eta$$

The effective betatron tune spread across the beam (ΔQ) may be ~ 0.1 so $\Delta Q/Q \sim 2.5 \times 10^{-3}$. The electron bounce frequency will vary both across and along the beam owing to variations in the beam charge density. The variations along the beam will be caused by the changing betatron amplitude and dispersion functions. Changes across the beam will be due to the stacking process and subsequent diffusion pr ses. We ignore the latter and estimate from the $\beta(s)$'s and X_p's of the preliminary parameters

$$\frac{\Delta Q_e}{Q_e} \simeq -\frac{1}{2}\frac{\Delta(ab)}{ab} \simeq -.04$$

so $$Q_p \lesssim 40\sqrt{2.5\times10^{-3}\times.4\times10^{-1}} \simeq 0.4 \text{ or } Q_p^2 < 0.16$$

Thus the requirement on η is $6\times10^{2}\eta \lesssim 0.16$

or $\eta \lesssim 2.7\times10^{-4}$

Within the accuracy of this limit, then the electron proton instability gives the tightest requirement on η. For the present purposes then we take as our criterion on that

$$\boxed{\eta_{max} \lesssim 2\times10^{-4}}$$

Again, returning to the stability criterion, note that

$$Q_p^2 \leqslant Q^2 \cdot \frac{\Delta Q}{Q}\;\frac{\Delta Q_e}{Q_e} \quad \text{or}$$

$$\eta_{allowed} N_p \propto \frac{1}{\sqrt{N_p}}\;,\ \gamma^2 \text{ constant}$$

so $\eta_{allowed} \propto \dfrac{1}{N_p^{3/2}}$ so that if we are ok at $N_{p\,max}$ we should be ok at injection again provided that the electron drift velocity towards the clearing electrodes doesn't fall faster than $N_p^{3/2}$.

Electron Formation

If all electrons struck out of atoms by the protons are captured in the beam potential well, then the rate of accumulation of ions in the beam is given directly by the minimum dE/dX. As our model, we choose N_2 as the residual gas.

$$\frac{dE}{dX} = 1.82 \text{ MeV/gm/cm}^2 \quad ; \rho_{N_2} = 1.25\text{x}10^{-3} \text{ gm/cm}^3$$

so $\frac{dE}{dX} = 2.28$ KeV/cm at STP. The energy loss per ion pair for N_2 is 33.3 eV

so $\frac{dN_e}{dX} = \frac{2.28\text{x}10^3}{33.3} = 68$ ion pairs/cm path at STP.

Thus the creation rate per proton per cm of path is

$n_c = .68\text{x}10^2 \text{ x } \frac{P}{760} \text{ x } f_o$ where P is the N_2 pressure in Torr and f_o is the revolution frequency. The <u>total</u> ionization rate = n_c × circumference

$$N_c = \frac{.68}{7.6} \text{ x}3\text{x}10^{10}P \cong 2.7P\times10^9\text{/proton/sec}$$

This is to be compared with the experimental number from the ISR[2] of $N_c = 1.4P\times10^9$ which takes account of the gas composition and the fact that a few of the δ-rays escape the beam directly.

We use the ISR number in what follows and have thus

$$n_c \simeq \frac{.34}{7.60} \; P \, f_o = 4.5\text{x}10^{-2}P \, f_o \text{ electrons/cm}$$

Assume that the clearing electrodes are separated by a distance L and take one of them as the origin. At equilibrium the number of electrons per cm of path per beam proton is $n(s) \simeq \frac{n_c}{v_D} \cdot s$ where s is the distance from the origin along the beam orbit and v_D is the drift velocity of the electrons - assumed to be along the orbit also, though this is not strictly true.

By definition $\eta = \frac{N_-}{N_p}$, $N_- = \overline{n(s)} \cdot C$

so $\eta = \overline{n(s)} \cdot C$ where C = circumference

$$\overline{n(s)} = \frac{1}{L}\int_o^L \frac{n_e}{v_D} \cdot sds = \frac{n_e}{2v_D} \cdot L$$

and $\eta = \frac{n_c LC}{2v_D}$. Inverting to get the v_D required for a given η we have

$v_D \geqslant \frac{n_c LC}{2\eta}$. Substituting for n_c

$$v_D \geqslant \frac{4.5x10^{-2} Pf_o \cdot L \cdot C}{2\eta} \quad ; \; f_o C = 3x10^{10} \text{ cm/sec}$$

For $P \simeq 10^{-10}$ Torr, $L = 5.9x10^2$ cm $\eta = 2x10^{-4}$

$$v_D \geqslant \frac{4.5x10^{-2} x10^{-10} x5.9x10^2 x3x10^{10}}{2x10^{-4}} \cong 4x10^5 \text{ cm/sec}$$

Thus for sufficient clearing

$$\boxed{v_D \geqslant 4x10^3 \text{ m/sec}}$$

The trajectories of the electrons captured in the beam potential well and the bending magnetic field are very complicated and depend upon many parameters. For purposes of estimate here only very rough parameterizations are made. Besides their cyclotron motion about the bending magnetic field lines* the electrons will generally be oscillating along these field lines under the influence of the beam field. The electrons will drift along the beam orbit if there is a radial gradient in the bending field. The drift velocity due to this effect is

*Where there is no bending field, the "thermal velocity" of the electrons directed parallel to the beam will serve to carry them along towards the clearing electrodes. Typical velocities will correspond to a few volts of $v \simeq 10^{-6}$ m/sec.

$$V_D = \frac{1}{\omega_c} \frac{\nabla_\perp B}{B_\perp} \left(\frac{1}{2} \overline{v_\perp^2} + \overline{v_{||}^2} \right)$$

ω_c = cyclotron freq. = $\frac{eB}{m}$

$v_\perp$ = orbital velocity of electron perpendicular to the bending field and will be characterized by some kinetic energy eV $v_\perp \simeq \sqrt{\frac{2eV}{m}}$

$v_{||}$ = velocity of electron along bending field lines.

$\overline{v_{||}^2}$ will depend upon the electron bounce frequency and amplitude of oscillation.

Within the beam, the electron field will be radial only

$E = \frac{e\rho r}{2\varepsilon_o}$ where ρ = charge density

$$\rho \sim \frac{2x10^{15}}{2x10^{-3}x8x10^{-3}x10^{4}} \sim \frac{1}{8} \text{ x } 10^{17}/m^3$$

so $$E_r \sim \frac{1.6x10^{-19}}{2} \times \frac{1}{8} \text{ x } 10^{17} \text{ x } \frac{r}{10^{-9}} 36\pi \simeq 36\pi\times10^6 r \sim 10^8 r \text{ V/m}$$

For a radially directed electron in this field

$$m\ddot{r} = eEr = 10^8 er \qquad \ddot{r} - \frac{e}{m} x10^8 r = 0$$

so $$\omega_e \simeq \sqrt{\frac{e}{m} x10^8} = \sqrt{\frac{1.6x10^{-19}}{9.1x10^{-31}} x10^8} = \sqrt{.18x10^{20}} = .42x10^{10}$$

$f_e \sim .07x10^{10} = 7x10^8$Hz. For an amplitude of oscillation of 10^{-3}m

then $v_{||} \simeq 4x10^{-3}x7x10^8 = 2.8x10^6$m/sec

For eV ≃ 30 volts $v_\perp \simeq \sqrt{\frac{2x30}{.5x10^6}} \text{ x } 3x10^8$ m/sec ≃ $3.3x10^6$ m/sec

Thus $v_{\perp} \sim v_{\parallel} \sim 3\text{x}10^6$ m/sec

For $B = 1.8\text{T}$ $\omega_c = \dfrac{1.6\text{x}10^{-19}\text{x}1.8}{9.11\text{x}10^{-31}} = .32\text{x}10^{12}$

For a gradient similar to the ISR, i.e. $\nabla_{\perp} B \simeq 3\text{T/m}$

$$v_D \sim \frac{10^{-12}}{.32} \times \frac{3}{1.8} \left(\frac{1}{2}\text{x}9+9\right)\text{x}10^{12} \sim 70.3 \text{ m/sec}$$

which is hopelessly small.

In addition to any gradient drift, there will be a drift along the orbit due to any radial electric field, $v_D = \dfrac{E_{\perp}}{B}$. For an electron at a position > 1 mm radially out from the beam core then

$v_D \gtrsim \dfrac{10^5}{1.8} \sim .5\text{x}10^5$ m/sec which is more than adequate to clear the beam to the estimated level. Since this velocity is essentially 10 times the required velocity, it may be necessary to have clearing electrodes at every quadrupole, for example, the quads being spaced at intervals of 4 magnets, and still leave a factor of two safety.

Clearly there is a problem at the center of the beam where the radial component of the field and therefore the drift velocity is small. Electrons will tend to diffuse out of this region due to heating by impacts of the proton beam, by scattering on the residual gas and by $\vec{E} \times \vec{B}$ drift due to the longitudinal component of the beam's field. The proton heating and gas scattering diffusion need looking at, but the last effect - drift perpendicular to both B and the orbit direction - is very effective since a very small amount of drift carries the electron quickly into a high radial field region. To estimate the magnitude of the longitudinal component of the beam electric field, which owes its existence to changing beam shape along the orbit, note that in 30 m the beam width changes by ~ 4 mm as x_p changes. This gives an angle ~ $\frac{2}{30}$ mr to be applied to the transverse component of the beam field so $E_L \sim \frac{10^{-3}}{15}\ E_r \sim \frac{10^{-3}}{15} \times 10^8 \times 2 \times 10^{-3} \cong 13$ V/m

so $v \simeq \frac{13}{1.8} \sim 10$ m/sec. Which should be adequate.

To sum it all up:

The ISR clearing works and relies very little upon gradient drift so POPAE probably doesn't have to have gradient magnets for ion clearing.

[1] E. Keil, Intersecting Storage Rings, CERN 72-14.

[2] R. Calder et al, Vacuum Conditions for Proton Storage Rings, 9th International Conference on High Energy Accelerators, SLAC, May 1974.