national accelerator laboratory

TM-502
0482

ENERGY DOUBLER DIPOLE MARK II: NONLINEARITY AND TOLERANCE REQUIREMENT

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June 14, 1974

I. Introduction

Mark II is one of several superconducting dipoles which have been designed for the energy doubler.[1] The shape of the vacuum pipe is of race-track type made of two half circles (radius 0.875") and a rectangle (0.75"x1.75"). With an iron shield, the design value of the maximum field is 44.4 kG for the conductor current of 2,815 A. Although Mark II may not be the final choice for the energy doubler, it is worthwhile to investigate, from the beam dynamics viewpoint, its expected performance as well as tolerance requirements on the conductor alignment. The purpose of this note is to provide a starting point for the future discussion on energy doubler dipoles. There is as yet no design of superconducting quadrupoles for the doubler.

There are a series of reports by G. Parzen and others[2,3] on tolerances for superconducting dipoles and quadrupoles to be used for ISABELLE. Since ISABELLE is a storage accelerator, tolerances they hope to achieve are more stringent than those required for the energy doubler. Based on the experience from the main-ring operation, E.J.N. Wilson has reported on tolerances one would have to meet for superconducting magnets.[4] However, his discussion is of general nature and does not refer to any specific design.

II. Field Nonlinearity in Mark II

A perfectly constructed dipole has the median plane field of the form

$$B_y(x) = B_o \left(1 + b_2x^2 + b_4x^4 + \cdots\cdots\right) \tag{1}$$

where x is the radial distance from the axis. In general, the most important is the sextupole field, b_2x^2 term, that will produce a linear dependence of tunes on particle momentum.* Both the field within the magnet body and the integrated (over the length, 20 feet, of the magnet) field of Mark II have been analyzed in this form:

	field within the magnet body	integrated field
b_2	$-\ 4.98\ m^{-2}$	$-\ 6.65\ m^{-2}$
b_4	$+\ 3.13\ 10^4\ m^{-4}$	$+\ 2.98\ 10^4\ m^{-4}$
b_6	$-\ 6.55\ 10^7\ m^{-6}$	$-\ 6.35\ 10^7\ m^{-6}$
b_8	$+\ 3.03\ 10^{10}\ m^{-8}$	$+\ 2.86\ 10^{10}\ m^{-8}$

The amount of sextupole field in the integrated field is ~ 30% more than that in the magnet body, indicating the contribution from the edge. These values should be compared with the relative strength of sextupole component in the main ring at 8 GeV,

$$b_2 = -\ 1.49\ m^{-2} \quad \text{in B1 dipoles,}$$

$$= -\ 0.98\ m^{-2} \quad \text{in B2 dipoles.}$$

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*Intrinsic third-integer resonances driven by systematic sextupole fields, $3\nu_x$ and $\nu_x + 2\nu_y$ = (number of superperiods) x (integer), are usually avoided by a proper choice of tunes.

The momentum dependence of tunes (chromaticity) can be calculated from these values and machine parameters. From SYNCH run at $\nu_x = \nu_y = 19.40$,

	$\Sigma\beta_x(X_p)^n$	$\Sigma\beta_y(X_p)^n$
$n = 1$	$1.331\ 10^5\ m^2$	$1.218\ 10^5\ m^2$
3	$1.787\ 10^6\ m^4$	$1.330\ 10^6\ m^4$
5	$3.368\ 10^7\ m^6$	$2.076\ 10^7\ m^6$

where the summation is over all 774 dipoles and the doubler is assumed to have the same lattice structure as the main ring. By combining with the chromatic aberration of quadrupoles, $\Delta\nu_{x,y} = -22\ (\Delta p/p)$, one finds

$$\Delta\nu_x = -1.165\ \delta_p + 0.138\ \delta_p{}^3 - 0.00829\ \delta_p{}^5 \tag{2}$$

$$\Delta\nu_y = 1.024\ \delta_p - 0.102\ \delta_p{}^3 + 0.00511\ \delta_p{}^5 \tag{3}$$

where $\delta_p \equiv (\Delta p/p)$ in 10^{-3}. Effects of b_8 are negligible for $|\delta_p| \lesssim 1$. Since the lowest injection energy of the doubler is ~ 100 GeV where the momentum spread of the beam is $|\delta_p| \lesssim 0.3$, some sort of chromaticity correction is of course an absolute necessity. If the main ring correction sextupole is simply scaled with one-half aperture, one needs 3' long sextupoles at all mini-straights and the current is ~ 30 A at 1,000 GeV. Dipoles for ISABELLE are designed to have $|b_2|$ less than 0.5 m^{-2} at 40 kG.

Higher multipole fields b_4, b_6, etc. are not important up to $|(\Delta p/p)| \simeq 0.7 \times 10^{-3}$, which is much higher than the expected momentum spread of the beam at 100 GeV. However, in Eqs. (2)

and (3), δ_p can also be interpreted as a pulse-to-pulse fluctuation of the injection field or of the injection energy and the higher multipoles may become important. Of many nonlinear resonances driven by the systematic nonlinear fields, $5\nu_x = 6 \times 16 \equiv 96$ ($\nu_x = 19.2$) and its related coupling resonances, $3\nu_x + 2\nu_y = 96$ and $\nu_x + 4\nu_y = 96$, are the only ones near the expected operating point ($\nu_x = \nu_y = 19.38 \sim 19.40$). In the lowest-order approximation and ignoring effects from other multipole fields, one finds, from $b_4 = 2.98\text{x}10^4\ m^{-4}$,

$$\text{total width of the resonance}\ \ 5\nu_x = 96$$

$$= 7.7 \times 10^6\ m^{-1} \times (\text{emittance}/\pi)^{3/2}.$$

The expected horizontal emittance of the beam at 100 GeV is $0.3\pi \times 10^{-6}$ m so that, even with an emittance dilution factor of two due to a possible mismatching, the total resonance width is very small ($\simeq 0.004$).*

III. <u>Field Errors Caused by Random Coil Misalignments</u>

Unlike conventional magnets, superconducting magnets are likely to have all types of fields, regular as well as skew, since the field is very sensitive to the current distribution.[2,4] Alignment tolerances on the conductor position are therefore much more stringent. The analytical treatment on this subject by Month and

*For the main-ring lattice, 102nd (6x17) harmonics are an order of magnitude larger than others. The total resonance width for $5\nu_x = 102$ ($\nu_x = 20.4$) is 0.036 with $b_4 = 2.98\text{x}10^4\ m^{-4}$ when the emittance is $0.6\pi\text{x}10^{-6}$ m.

Parzen[2] is applied to Mark II with minor modifications. There are three blocks of conductors in each quadrant of Mark II. All twelve blocks are assumed to have the same rms error Δ_r in their radial positions and the same rms error Δ_{az} in their azimuthal positions. In order to simplify the analytical treatment, all blocks are assumed to be placed around a circle of radius R. Effects of the thickness of each block and of the rectangle between two half circles are thus ignored. It is further assumed that the effect due to the iron shield is small for relative field errors[2] and the alignment of conductors within each block is perfect. Choice of the value of R is somewhat arbitrary and here it is taken to be 1.2". If one calculates R from the central dipole field (36.0 kG for 2,815 A excluding the iron shield contribution), the value is somewhat larger, 1.306".

The current distribution in the k-th block is

$$j_k(r,\theta) = R^{-1} f_k(\theta)\ \delta\ (r-R) \tag{4}$$

and the total current in the block is

$$I_k = \iint j_k(r,\theta) r dr d\theta \ . \tag{5}$$

The azimuthal distribution $f_k(\theta)$ is constant inside and zero outside the block. The rms error in the median plane field for the n-th multipole is:

(a) regular field

$$(\Delta B_{y,n}/B_o)_{rms} = (|x|^n/R^{n+1})\ [\sum_k I_k(1)]^{-1}\ x$$

$$x\ \sqrt{n^2\ \Delta_r^{\ 2} \sum_k I_k^{\ 2}(n+1) + (n+1)^2\ \Delta_{az}^{\ 2} \sum_k J_k^{\ 2}(n+1)} \tag{6}$$

(b) skew field

$$(\Delta B_{x,\,n}/B_o)_{rms} = (|x|^n/R^{n+1})\ [\Sigma\ I_k(1)]^{-1}\ x$$

$$x\ \sqrt{n^2\ \Delta_r^{\,2}\ \sum_k J_k^{\,2}(n+1) + (n+1)^2\ \Delta_{az}^{\,2}\ \sum_k I_k^{\,2}(n+1)} \qquad (7)$$

where B_o is the central dipole field*

$$B_o = (\mu/2\pi)\ \sum_k I_k(1)/R \qquad (8)$$

and

$$I_k(n) = \int f_k(\theta)\ \cos(n\theta)d\theta, \qquad (9)$$

$$J_k(n) = \int f_k(\theta)\ \sin(n\theta)d\theta. \qquad (10)$$

<u>dipole field</u> (n = 0)

regular $(\Delta B_y/B_o) = 6.52\ m^{-1}\ \Delta_{az}$ (11)

skew $(\Delta B_x/B_o) = 9.67\ m^{-1}\ \Delta_{az}$ (12)

<u>quadrupole field</u> (n = 1)

regular $(\Delta B_y/B_o) = \sqrt{2.19x10^4 m^{-2}\ \Delta_r^{\,2} + 2.90x10^5 m^{-2}\ \Delta_{az}^{\,2}}\,|x|$ (13)

skew $(\Delta B_x/B_o) = \sqrt{7.26x10^4 m^{-2}\ \Delta_r^{\,2} + 8.76x10^4 m^{-2}\ \Delta_{az}^{\,2}}\,|x|$ (14)

<u>sextupole field</u> (n = 2)

regular $(\Delta B_y/B_o) = \sqrt{1.93x10^7 m^{-2}\ \Delta_r^{\,2} + 4.05x10^8 m^{-2}\ \Delta_{az}^{\,2}}\ x^2$ (15)

skew $(\Delta B_x/B_o) = \sqrt{1.80x10^8 m^{-2}\ \Delta_r^{\,2} + 4.34x10^7 m^{-2}\ \Delta_{az}^{\,2}}\ x^2$ (16)

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*$(\mu/2\pi) = 2 \times 10^{-6}$ kG-m/ampere.

IV. Tolerance Requirements

Using (11) - (16), one can estimate various effects of field errors on the beam. In the following, all necessary machine parameters are calculated for $\nu_x = 19.38$ and $\nu_y = 19.42$ with the quadrupole field gradient B'(kG/m) = 0.6007 x (cp in GeV).

(a) closed orbit distortion

$$(\Delta x)_{rms} \text{ in mm} = 0.152 \sqrt{\beta_x \text{ in m}}\ \Delta_{az}(\text{mil}) \tag{17}$$

$$(\Delta y)_{rms} \text{ in mm} = 0.219 \sqrt{\beta_y \text{ in m}}\ \Delta_{az}(\text{mil}). \tag{18}$$

For example, the maximum value of β(both x and y) in dipoles is 95 m so that, with Δ_{az} = 5 mil, $(\Delta x)_{rms}$ = 7.4 mm and $(\Delta y)_{rms}$ = 10.7 mm. Aside from the obvious reason of aperture reduction, closed-orbit distortions are harmful in that their dominant 19th and 20th harmonics can be coupled with systematic error harmonics and create resonances like $3\nu = 58$ (= 6x13 - 20) and $4\nu = 77$ (= 6x16 - 19) near the operating point. Hopefully, orbit distortions can be eliminated by correction dipoles (at low energies) and by quadrupole moves.

(b) half-integer stopbands

From (13), total stopband widths for $2\nu_{x,y} = N$ are

$$(\delta\nu_x)_{rms} = 0.0082\ \Delta_r(\text{mil}) \text{ and } 0.0299\ \Delta_{az}(\text{mil}) \tag{19}$$

and

$$(\delta\nu_y)_{rms} = 0.0083\ \Delta_r(\text{mil}) \text{ and } 0.0303\ \Delta_{az}(\text{mil}). \tag{20}$$

An azimuthal misalignment of $\Delta_{az} \gtrsim 4$ mil is dangerous, $\delta\nu_{x,y}$

$\gtrsim$ 0.12. For linear coupling resonances, $\nu_x + \nu_y = N$, total stopband widths are from (14),

$$(\delta\nu)_{rms} = 0.0134\ \Delta_r(\text{mil}) \text{ and } 0.0147\ \Delta_{az}(\text{mil}). \tag{21}$$

With Δ_{az} = 5 mil, correction quadrupoles needed for these resonances are expected to be $(B'\ell) \simeq 5.5$ kG each at 1,000 GeV placed at all mini-straights.

(c) <u>third-integer resonances</u>

Average sextupole field arising from random errors, (15), can contribute to the chromaticity of the machine but this is much smaller than the contribution from the systematic sextupole field b_2. For example, with Δ_{az} = 5 mil, $(\Delta\nu_x)_{rms} = 19|\Delta p/p|$.

If one defines the resonance width $\delta\nu$ of a third-integer resonance $3\nu_x = N$ or $3\nu_y = N$ such that, for $|\nu_{x,y} - N/3| \gtrsim (\delta\nu)/2$, the beam is entirely within the central stable region in phase space, one finds from (15) and (16), respectively,

$3\nu_x = N$

$$(\delta\nu_x)_{rms} = 0.00205\ \Delta_r(\text{mil})\ \sqrt{\text{emittance in } 10^{-6}\text{m}/\pi}$$
$$= 0.00938\ \Delta_{az}(\text{mil})\ \sqrt{\text{emittance in } 10^{-6}\text{m}/\pi}. \tag{22}$$

$3\nu_y = N$

$$(\delta\nu_y)_{rms} = 0.00638\ \Delta_r(\text{mil})\ \sqrt{\text{emittance in } 10^{-6}\text{m}/\pi},$$
$$= 0.0313\ \Delta_{az}(\text{mil})\ \sqrt{\text{emittance in } 10^{-6}\text{m}/\pi}.$$

With the expected horizontal emittance of $0.3\pi \times 10^{-6}$m at 100 GeV, Δ_{az} = 5 mil gives $(\delta\nu_x)_{rms} = 0.026$. Whether this would require harmonic collection sextupoles or not would depend on the tune spread of the beam after the chromaticity correction.

V. Concluding Remarks

Since there is no design of superconducting quadrupoles for the energy doubler, it is not possible to give a definite conclusion on dipole tolerances. Nevertheless, from results presented here, one may conclude that (1) it is desirable to redesign Mark II and reduce the amount of sextupole field by a factor of ~ 5, and that (2) the tolerance on the position of each conductor block should be at most 5 mil (rms) and preferably ~ 3 mil.

It is still too early to say anything definite on a possible scheme of slow extraction from the doubler.[5] One should simply note that, for a third-integer resonance extraction, average octupole field must be provided to eliminate the trapping of particles that are being spilled out of the shrinking central stable area.[2,6] Half-integer resonance extraction may or may not be possible depending on how far one can stretch the beam in the doubler field without seriously distorting the linear shape of the beam in phase space.

In preparing this report, discussions with S. Snowdon have been invaluable.

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