national accelerator laboratory

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STORED ENERGY IN DOUBLER TYPE SHEET CURRENT DIPOLES

S.C. Snowdon

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Summary

The energy stored in the magnetic field or alternatively the inductance is calculated for a sheet current density distribution that simulates the excitation currents in the energy doubler dipole. Results are given for the dual test dipole, the doubler dipole, and the muon beam dipole.

Complex Potential

In complex variable notation the magnetic field may be given by

$$H^* = i \frac{dW}{dz} \tag{1}$$

where the potential W is

$$W = - ii_o a \int_{-\pi/2}^{\pi/2} \left[\ln(z-\Delta-ae^{i\theta}) + \ln\left(z-\frac{b^2}{\Delta+ae^{-i\theta}}\right)\right] \cos\theta d\theta$$

$$- ii_o a \int_{\pi/2}^{-\pi/2} \left[\ln(z+\Delta-ae^{i\theta}) + \ln\left(z+\frac{b^2}{\Delta-ae^{-i\theta}}\right)\right] \cos\theta d\theta \quad . \tag{2}$$

It is assumed in Eq. (2) that the sheet current density is $i_o \cos\theta$. Figure 1 gives the appropriate geometrical quantities. If one lets $s = e^{i\theta}$ then

$$W = ii_o\, a \int_{C_1} \left[\ln(z-\Delta-as) + \ln\left(\frac{az-(b^2-\Delta z)s}{a+\Delta s} \right) \right] (1+\frac{1}{s^2})\, ds$$

$$+ ii_o\, a \int_{C_2} \left[\ln(z+\Delta-as) + \ln\left(\frac{az-(b^2+\Delta z)s}{a-\Delta s} \right) \right] (1+\frac{1}{s^2})\, ds \,. \qquad (3)$$

Integration gives

$$W = ii_o\, a \Bigg\{ - 2\pi i \frac{z}{a} + 2i \ln\left(\frac{ia-z+\Delta}{ia-z-\Delta} \cdot \frac{ia+z-\Delta}{ia+z+\Delta} \right)$$

$$+ \left(\frac{z-\Delta}{a} - \frac{a}{z-\Delta} \right) \cdot \ln\left(\frac{ia+z-\Delta}{ia-z+\Delta} \right)$$

$$+ \left(\frac{z+\Delta}{a} - \frac{a}{z+\Delta} \right) \cdot \ln\left(\frac{ia-z-\Delta}{ia+z+\Delta} \right)$$

$$- i\pi \left(\frac{az}{b^2-\Delta z} + \frac{az}{b^2+\Delta z} \right)$$

$$+ 2i \ln\left[\frac{i(b^2-\Delta z)-az}{i(b^2+\Delta z)+az} \cdot \frac{i(b^2-\Delta z)+az}{i(b^2+\Delta z)-az} \right]$$

$$+ \left(\frac{az}{b^2-\Delta z} - \frac{b^2-\Delta z}{az} \right) \cdot \ln\left[\frac{i(b^2-\Delta z)+az}{i(b^2-\Delta z)-az} \right]$$

$$+ \left(\frac{az}{b^2+\Delta z} - \frac{b^2+\Delta z}{az} \right) \cdot \ln\left[\frac{i(b^2+\Delta z)-az}{i(b^2+\Delta z)+az} \right] \Bigg\}. \qquad (4)$$

The arguments of the logarithmic terms have been arranged so that the solution is appropriate to the inside region.

Stored Energy

Note first that the complex potential is

$$W = A_z + iV,$$

where A_z is the vector potential and V is the scalar potential. Our interest here is with the vector potential which is continuous through the current sheet. The stored energy is given by

$$E = \text{Real}\ \frac{1}{2} \int_V A_z J_z^* \, dv, \qquad (6)$$

which for our case becomes

$$E = \frac{1}{2} \ell a i_o \oint_c A_z \cos\theta d\theta \quad \text{(ergs)}. \qquad (7)$$

Inductance

Assuming a one turn current sheet one has

$$E = \frac{1}{2} L\, I_o{}^2 = \frac{1}{2} L_o (2ai_o)^2. \qquad (8)$$

For N turns one then has

$$L = \frac{N^2 \ell}{4 i_o a} \oint_c A_z \cos\theta d\theta \quad \text{(emu)} . \qquad (9)$$

Results

Application of the previous formulas is made using input data appropriate for the dual test dipole, the doubler dipole and the muon beam line dipole. See Table 1.

Figure 1. Legend

0 - Center

F - Origin of Right Hand Circular Sheet Current

F' - Origin of Left-Hand Circular Sheet Current

z_1 = Image Position of z_o

a = Radius of Current Sheet $(F-z_o)$

b = Inner Radius of Iron (0-P)

Δ = Offset (F'-0 or 0-F)

$$H^* = -2iI\left(\frac{1}{z-z_o} + \frac{1}{z-z_1}\right)$$

$$z_o = \begin{Bmatrix} \Delta + ae^{i\theta} & \text{Right Hand Sheet} \\ -\Delta + ae^{i\theta} & \text{Left Hand Sheet} \end{Bmatrix}$$

$$z_1 = \frac{b^2}{z_o^*}$$

$$I = i_o a \cos\theta d\theta$$

Note that the origin at θ is F for $-\pi/2<\theta<\pi/2$ and F' for $\pi/2<\theta<3\pi/2$.

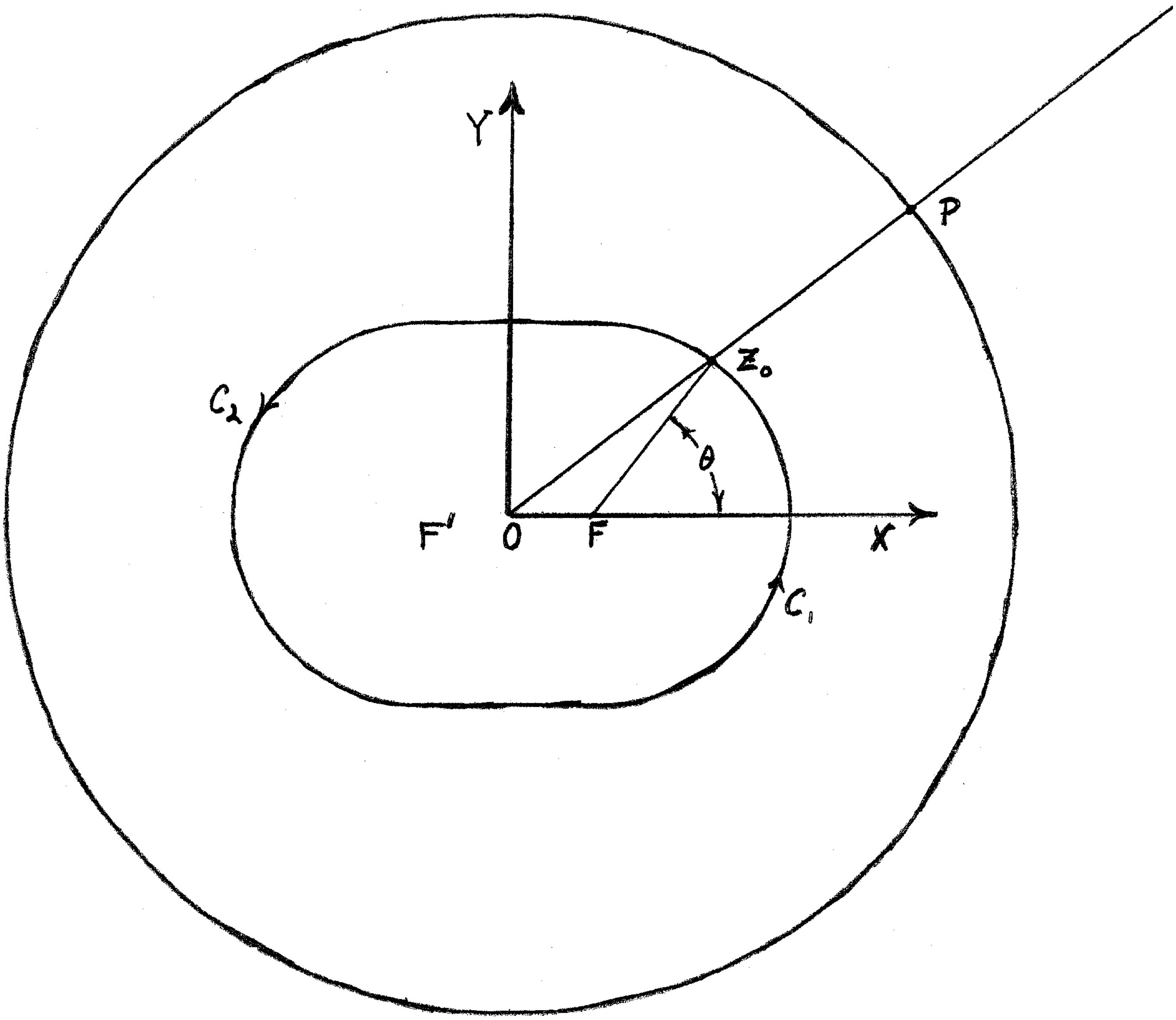

Figure 1. Current Sheet Dipole

Table 1. Current Sheet Dipoles

	Test	Doubler	Muon
Conductor Current (A)	2625	2815	2380
Number of Conductors	130	140	306
Current Sheet Radius (in)	1.340	1.1325	2.489
Current Sheet Offset (in)	.000	.375	1.000
Inner Radius of Iron (in)	3.050	3.0625	5.750
Central field (kG)	37.6	45.5	40.5
Inductance (mHy)	5.8	90.	246
Stored Energy (kJ)	19.9	356.	698.