national accelerator laboratory

TM-464
0428

EDDY CURRENTS IN SUPERCONDUCTING DIPOLE BANDING

S.C. Snowdon

December 28, 1973

PURPOSE

The banding used to contain the coils in a superconducting dipole might be simplified if stainless steel were used rather than the fabricated plastic bands now envisaged. The use of metal bands depends on the magnitude of the fields and the associated power loss arising from eddy currents. An estimate of these effects is made using excitation currents and eddy currents confined to cylindrical sheets. Also, for simplicity no outer magnetic shield is included. For reasonable bands and a frequency of 1 Hz the field distortion is less than $1 \div 10^6$ and the power dissipated is less than 1 W/magnet.

VECTOR POTENTIAL OF EXCITING CURRENTS

All geometrical quantities are depicted in Fig. 1. The exciting current is assumed to be

$$i_z = i_0 \cos\phi \text{ (abA/cm)} \tag{1}$$

and confined to the cylindrical sheet at $\rho = a$. Then for the guage in which $\nabla \cdot \vec{A} = 0$,

$$A_\rho = 0 \qquad A_\phi = 0 \qquad A_z = \left\{ \begin{array}{l} A_1 \rho \\ \dfrac{B_1}{\rho} \end{array} \right\} \cos\phi \quad \begin{array}{l} 0<\rho<a \\ a<\rho<\infty \ . \end{array} \tag{2}$$

The corresponding fields are

$$H_\rho = \frac{1}{\rho}\frac{\partial A_z}{\partial \phi} = \begin{Bmatrix} -A_1 \\ -B_1 \dfrac{1}{\rho^2} \end{Bmatrix} \sin\phi, \quad H_\phi = -\frac{\partial A_z}{\partial \rho} = \begin{Bmatrix} -A_1 \\ \dfrac{B_1}{\rho^2} \end{Bmatrix} \cos\phi \ . \tag{3}$$

At $\rho = a$: $H_\phi^+ - H_\phi^- = 4\pi i_z$, $H_\rho^+ = H_\rho^-$ or

$$A_1 = 2\pi i_0 \qquad B_1 = 2\pi i_0 \ a^2. \tag{4}$$

FIELDS AND POTENTIALS FROM EDDY CURRENT SHEET AT $\rho = b$

In order to satisfy $\nabla \cdot \vec{i} = 0$ for the surface eddy currents at $\rho = b$ choose the following harmonic expansion.

$$i_z = \cos\phi \ \Sigma_n i_n \sin \frac{n\pi z}{d} \ , \ i_\phi = -\sin\phi \ \Sigma \frac{n\pi b}{d} i_n \cos \frac{n\pi z}{d} \ . \tag{5}$$

To be consistent with the gauge previously chosen, one again seeks a vector potential that satisfies $\nabla \cdot \vec{A} = 0$. However, the most general vector satisfying this condition may be written as

$$\vec{A} = \vec{L} \ W_1 + \nabla \times \vec{L} \ W_2 \tag{6}$$

where, for the problem at hand the $\vec{L}$ - operator is most conveniently expressed as

$$\vec{L} = \vec{k} \times \nabla \quad (\vec{k} = z \text{ - unit vector}) \ . \tag{7}$$

The properties of the $\vec{L}$ - operator[1] suggest that

$$\nabla^2 \ W_1 = 0 \ , \qquad \nabla^2 \ W_2 = 0 \tag{8}$$

will insure that $\vec{A}$ satisfies $\nabla \times \nabla \times \vec{A} = 0$. In this case, however, Eq. (6) may be replaced by

$$\vec{A} = \vec{L} \ W + \nabla \ \Phi \tag{9}$$

where

$$W = W_1 \qquad \nabla^2 \Phi = 0 \ . \tag{10}$$

Thus, to permit a connection with the dipole excitation potentials, choose

$$W = \begin{Bmatrix} \Sigma_n \frac{d}{n\pi} C_n \ I_1(\frac{n\pi\rho}{d}) \ \cos \frac{n\pi z}{d} \\ \Sigma_n \frac{d}{n\pi} D_n \ K_1(\frac{n\pi\rho}{d}) \ \cos \frac{n\pi z}{d} \end{Bmatrix} \sin\phi \qquad \begin{matrix} 0<\rho<b \\ b<\rho<\infty \end{matrix} \tag{11}$$

$$\Phi = \begin{Bmatrix} \Sigma_n \frac{d}{n\pi} E_n \ I_1(\frac{n\pi\rho}{d}) \ \cos \frac{n\pi z}{d} \\ \Sigma_n \frac{d}{n\pi} F_n \ K_1(\frac{n\pi\rho}{d}) \ \cos \frac{n\pi z}{d} \end{Bmatrix} \cos\phi \qquad \begin{matrix} 0<\rho<b \\ b<\rho<\infty \end{matrix} \ . \tag{12}$$

The vector potential due to eddy currents becomes

$$A_\rho = -\frac{1}{\rho}\frac{\partial W}{\partial \phi} + \frac{\partial \Phi}{\partial \rho} = \begin{Bmatrix} \Sigma_n [-\frac{d}{n\pi\rho} C_n \ I_1(\frac{n\pi\rho}{d}) + E_n \ I_1'(\frac{n\pi\rho}{d})] \ \cos \frac{n\pi z}{d} \\ \Sigma_n [-\frac{d}{n\pi\rho} D_n \ K_1(\frac{n\pi\rho}{d}) + F_n \ K_1'(\frac{n\pi\rho}{d})] \ \cos \frac{n\pi z}{d} \end{Bmatrix} \cos\phi \tag{13}$$

$$A_\phi = \frac{\partial W}{\partial \rho} + \frac{1}{\rho}\frac{\partial \Phi}{\partial \phi} = \begin{Bmatrix} \Sigma_n [C_n \ I_1'(\frac{n\pi\rho}{d}) - \frac{d}{n\pi\rho} E_n \ I_1(\frac{n\pi\rho}{d})] \ \cos \frac{n\pi z}{d} \\ \Sigma_n [D_n \ K_1'(\frac{n\pi\rho}{d}) - \frac{d}{n\pi\rho} F_n \ K_1(\frac{n\pi\rho}{d})] \ \cos \frac{n\pi z}{d} \end{Bmatrix} \sin\phi \tag{14}$$

$$A_z = \frac{\partial \Phi}{\partial z} = \begin{Bmatrix} -\Sigma_n E_n \ I_1(\frac{n\pi\rho}{d}) \ \sin \frac{n\pi z}{d} \\ -\Sigma_n F_n \ K_1(\frac{n\pi\rho}{d}) \ \sin \frac{n\pi z}{d} \end{Bmatrix} \cos \phi \tag{15}$$

The corresponding fields become

$$H_\rho = -\frac{\partial^2 W}{\partial \rho \partial z} = \begin{Bmatrix} \Sigma_n \frac{n\pi}{d} C_n \ I_1'(\frac{n\pi\rho}{d}) \ \sin \frac{n\pi z}{d} \\ \Sigma_n \frac{n\pi}{d} D_n \ K_1'(\frac{n\pi\rho}{d}) \ \sin \frac{n\pi z}{d} \end{Bmatrix} \sin\phi \tag{16}$$

$$H_\phi = -\frac{1}{\rho}\frac{\partial^2 W}{\partial\phi\partial z} = \left\{\begin{matrix} \Sigma_n \frac{1}{\rho} C_n \; I_1(\frac{n\pi\rho}{d}) \; \sin\frac{n\pi z}{d} \\ \Sigma_n \frac{1}{\rho} D_n \; K_1(\frac{n\pi\rho}{d}) \; \sin\frac{n\pi z}{d} \end{matrix}\right\} \cos\phi \tag{17}$$

$$H_z = -\frac{\partial^2 W}{\partial z^2} = \left\{\begin{matrix} \Sigma_n \frac{n\pi}{d} C_n \; I_1(\frac{n\pi\rho}{d}) \; \cos\frac{n\pi z}{d} \\ \Sigma \frac{n\pi}{d} D_n \; K_1(\frac{n\pi\rho}{d}) \; \cos\frac{n\pi z}{d} \end{matrix}\right\} \sin\phi \tag{18}$$

$$E_\rho = -j\omega A_\rho, \quad E_\phi = -j\omega A_\phi, \quad E_z = -j\omega(A_z + A_z(\text{excit.})) \; . \tag{19}$$

BOUNDARY CONDITIONS

At $\rho = b$ the following conditions produce the corresponding conditions among the coefficients

$$H_\rho^+ = H_\rho^- \quad : \quad C_n \; I_1'(\frac{n\pi b}{d}) = D_n \; K_1'(\frac{n\pi b}{d}) \tag{20}$$

$$H_\phi^+ - H_\phi^- = 4\pi i_z \quad : \quad D_n \; K_1(\frac{n\pi b}{d}) - C_n \; I_1(\frac{n\pi b}{d}) = 4\pi b i_n \tag{21}$$

$$H_z^- - H_z^+ = 4\pi i_\phi \quad : \quad D_n \; K_1(\frac{n\pi b}{d}) - C_n \; I_1(\frac{n\pi b}{d}) = 4\pi b i_n . \tag{22}$$

$$E_\rho^+ - E_\rho^- = 4\pi\sigma \quad : \quad \text{Will not need the surface charge.} \tag{23}$$

$$E_\phi^+ = E_\phi^- \quad : \quad E_n \; I_1(\frac{n\pi b}{d}) = F_n \; K_1(\frac{n\pi b}{d}) \; . \tag{24}$$

The eddy currents $\vec{i}$ in the surface, $\rho = b$, are related to the electric field $\vec{E}$ by Ohm's law: $s\,\vec{E} = \vec{i}$, where s is the surface conductivity. Hence

$$s\,E_\phi = i_\phi \quad : \quad C_n \; I_1'(\frac{n\pi b}{d}) - \frac{d}{n\pi b} E_n \; I_1(\frac{n\pi b}{d}) = \frac{\frac{n\pi b}{d}}{j\omega s b} b i_n . \tag{25}$$

Finally, the condition, $s\,E_z = i_z$, gives for $b > a$

$$-j\omega s \; 2\pi \frac{a^2}{b} i_0 + j\omega s \; \Sigma_n E_n \; I_1(\frac{n\pi b}{d}) \sin\frac{n\pi z}{d} = \Sigma_n i_n \sin\frac{n\pi z}{d} \; . \tag{26}$$

BANDING CONDITION

The surface currents are to exist in a width c with a repeat distance of d. Hence the first term in Eq. (26) may be multiplied by

$$\left\{\begin{array}{ll} 0 & 0<z<\frac{1}{2}(d-c) \\ 1 & \frac{1}{2}(d-c)<z<\frac{1}{2}(d+c) \\ 0 & \frac{1}{2}(d+c)<z<\infty \end{array}\right\} = \frac{4}{\pi} \sum_{n=1,3,5} \frac{1}{n} \sin\frac{n\pi}{2} \sin\frac{n\pi c}{2d} \sin\frac{n\pi z}{d} \,. \tag{27}$$

Thus, combining Eq. (27) with Eq. (26) gives

$$-\, 8j\omega sb \,\frac{a^2}{b^2}\, i_0 \,\frac{\sin\frac{n\pi}{2}}{n} \sin\frac{n\pi c}{2d} + j\omega s\, E_n\, I_1\!\left(\frac{n\pi b}{d}\right) = i_n. \tag{28}$$

COEFFICIENTS

From Eqs. (20-28) one has

$$C_n = 4\pi\left(\frac{n\pi b}{d}\right) K_1'\left(\frac{n\pi b}{d}\right) bi_n \tag{29}$$

$$D_n = 4\pi\left(\frac{n\pi b}{d}\right) I_1'\left(\frac{n\pi b}{d}\right) bi_n \tag{30}$$

$$E_n = \frac{\left(\frac{n\pi b}{d}\right)^2}{I_1\left(\frac{n\pi b}{d}\right)} \left[4\pi\, I_1'\left(\frac{n\pi b}{d}\right) K_1'\left(\frac{n\pi b}{d}\right) - \frac{1}{j\omega sb}\right] bi_n \tag{31}$$

$$F_n = \frac{\left(\frac{n\pi b}{d}\right)^2}{K_1\left(\frac{n\pi b}{d}\right)} \left[4\pi\, I_1'\left(\frac{n\pi b}{d}\right) K_1'\left(\frac{n\pi b}{d}\right) - \frac{1}{j\omega sb}\right] bi_n \tag{32}$$

$$i_n = \frac{8j\omega sb\, \frac{a^2}{b^2} \sin\frac{n\pi}{2} \sin\frac{n\pi c}{2d}}{n\left\{\left(\frac{n\pi b}{d}\right)^2 \cdot \left[4\pi\, j\omega sb\, I_1'\left(\frac{n\pi b}{d}\right) K_1'\left(\frac{n\pi b}{d}\right) - 1\right] - 1\right\}}\, i_o \tag{33}$$

MEDIAN PLANE FIELD

$$H_y(x,z) = -\ 2\pi i_0 + \frac{1}{x}\ \Sigma_n\ C_n\ I_1(\frac{n\pi x}{d})\ \sin\frac{n\pi z}{d}\quad . \tag{34}$$

Averaging this over z gives

$$\left\langle H_y(x)\right\rangle_{Av} = -\ 2\pi i_0 + \frac{2}{\pi x}\ \Sigma_n\ \frac{1}{n}\ C_n\ I_1(\frac{n\pi x}{d}) \tag{35}$$

which, upon expanding in x gives

$$\left\langle H_y(x)\right\rangle_{Av} = -\ 2\pi i_0 + \frac{1}{d}\ \Sigma_n\ C_n[1 + \frac{1}{2}(\frac{n\pi x}{2d})^2 + ---]. \tag{36}$$

At x = 0 one has

$$\frac{\Delta H}{H} = \frac{\left\langle H_y(0)\right\rangle_{Av} - (-\ 2\pi i_0)}{-\ 2\pi i_0} = -\ \frac{\Sigma_n\ C_n}{2\pi i_0\ d}\quad . \tag{37}$$

In addition

$$\frac{H''}{H} = \frac{\left\langle H_y''(0)\right\rangle_{Av}}{-\ 2\pi i_0} = -\ \frac{\pi\ \Sigma_n\ n^2\ C_n}{8\ i_0\ d^3} \tag{38}$$

POWER DISSIPATED IN BANDS

$$P = \text{Real}\left\{\frac{1}{2}\iint\vec{E}\cdot\vec{i}^{*}\,bd\phi dz\right\} \tag{39}$$

$$P = \frac{b}{2s}\ \Sigma_n\ \Sigma_m\ i_n\ i_m^*\iint\left\{\cos^2\phi\ \sin\frac{n\pi z}{d}\ \sin\frac{m\pi z}{d}\right.$$

$$\left. + \frac{n\pi b}{d}\cdot\frac{m\pi b}{d}\ \sin^2\phi\ \cos\frac{n\pi z}{d}\ \cos\frac{m\pi z}{d}\right\}d\phi dz$$

$$P = \frac{bd}{8s} \Sigma_n [1 + (\frac{n\pi b}{d})^2] i_n i_n^* \text{ (ergs/sec - one band)} \tag{40}$$

NUMERICAL RESULTS

In order to apply to the doubler dipole put

$a = 1.1325'' = 2.8766$ cm $\quad b = 2.0'' - .375'' = 4.1275$ cm

$c = .375'' = .9525$ cm $\quad d = 1.0'' = 2.54$ cm

$f = \omega/2\pi = 1$ Hz $\quad s = \frac{.100\text{x}2.54}{50\text{x}10^{-6}} \text{ x } 10^{-9} = 5.08\text{x}10^{-6}$ (emu)

$H_o = 45$ kG .

This gives

$\omega sb = .0001317 \quad a/b = .6969 \quad \pi b/d = 5.1051$

$\sin \frac{\pi c}{2d} = .5556 \quad I_1'(5.1051) = 24.554 \quad K_1'(5.1051) = - .004018.$

Thus

$$i_1 = \frac{8j\text{x}.0001317\text{x}.5556\text{x}.6969\text{x}.6969}{-4\pi j\text{x}.001317\text{x}24.554\text{x}.004018\text{x}(5.1051)^2-1-5.1051\text{x}5.1051} i_0$$

$$= \frac{.0002843j}{-.00426j-27.0620} = - 10.51\text{x}10^{-6} \, ji_0$$

i_n = negligible for n = 3, 5, 7 etc. because of

$K_1'(5.1051 \, n)$

$C_1 = 4\pi\text{x}5.1051\text{x}.004018\text{x}4.1275\text{x}10.51\text{x}10^{-6} \, ji_0 = 11.18 \, ji_0$

C_n = negligible for n = 3, 5, 7, etc.

$$i_0 = H_0/2\pi = 45000/2\pi = 7162 \text{ abA/cm}$$

$$\Delta H/H = - C_1/(2\pi\, i_0\, d) = - .70\text{x}10^{-6}\ j$$

$$H''/H = - \pi C_1/(8 i_0\, d^3) = - 1.73\text{x}10^{-6}\ j/\text{in/in.}$$

$$P/d = b[1 + (\pi b/d)^2]\ i_1\ i_1^* /8s = .0040 \text{ W/in.}$$

$$P(240'') = .95 \text{ W/magnet}$$

REFERENCE

1. S.C. Snowdon, Properties of the L-Operator, MURA Technical Note TN-506, Oct. 19, 1964.