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A Rotating Coil Device for Harmonic Analysis of Magnetic Fields

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A coil rotating at a uniform speed can be used to measure not only the strength of the field but also it can provide data which can be used in determining the coefficients of a harmonic series, thus describing the field completely within the measured volume. The use of a rotating coil to measure field strengths at a point has been developed quite extensively by the Rawson-Lush Co.[1] The technique for using a rotating coil in harmonically analyzing a magnetic field has been developed by J. Cobb et. al. at SLAC.[2,3,4] A device similar to that now used at SLAC has been constructed here at NAL with some additional design features which enable it to be used in high fields and cold bore conditions. This report describes the construction and operation of this device together with a brief review of the basic principles involved.

I. Theory of Operation

A voltage is generated by a wire filament moving in a magnetic field which is related to the time rate of change of flux by Faraday's Law of Induction

$$E(t) = \frac{-d\phi}{dt} \quad . \tag{1}$$

The flux linking a circuit of area S is defined by

$$\Phi = \int_S \vec{B} \cdot \vec{A} \ . \tag{2}$$

The time derivative of Φ follows from the identity

$$\frac{d}{dt} \int \vec{B} \cdot d\vec{A} = \int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A} + \int_S \left[\vec{V} \ (\vec{\nabla} \cdot \vec{B}) - \vec{\nabla} \times (\vec{V} \times \vec{B}) \right] \cdot d\vec{A} \quad .$$

In this case $\frac{\partial \vec{B}}{\partial t}$ and $\vec{\nabla} \cdot \vec{B} = 0$ leaving

$$E(t) = - \int_S \vec{\nabla} \times (\vec{B} \times \vec{V}) \cdot d\vec{A}$$

which from Stokes theorem becomes

$$E(t) = - \int_C (\vec{B} \times \vec{V}) \cdot d\vec{\ell} \quad . \tag{3}$$

the vector $\vec{V}$ is the velocity of the circuit element $d\vec{\ell}$, and $\vec{B}$ is the external applied field. The integral is taken around the perimeter of the coil. It is convenient to write these quantities in cylindrical coordinates as follows:

$$\vec{V} = \frac{d\vec{r}}{dt} = \dot{r}\,\hat{r} + r\dot{\theta}\,\hat{\theta} + \dot{z}\,\hat{z}$$

$$\vec{B} = B_r\,\hat{r} + B_\theta\,\hat{\theta} + B_z\,\hat{z}$$

$$d\vec{\ell} = \hat{r}\,dr + \hat{\theta}\,r d\theta + \hat{z}dz \quad .$$

In this application only the angular coordinate is a function of time, thus

$$(\vec{B} \times \vec{V}) \cdot d\vec{\ell} = r\dot{\theta}\,B_r\,d_z - r\dot{\theta}\,B_z dr \quad .$$

The coil is constructed so as to be rectangular in shape with one side of the rectangle forming the axis of rotation (see Figure 1).

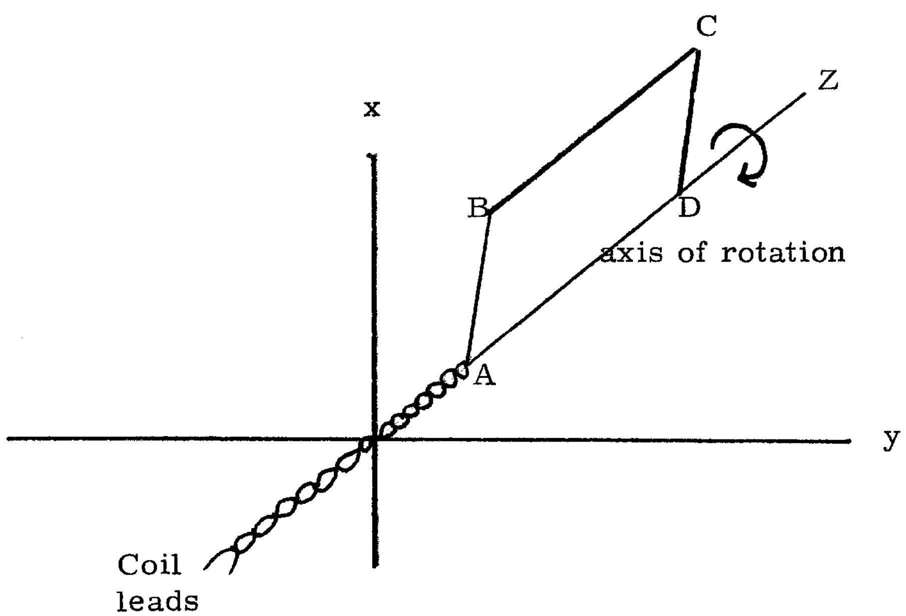

Figure 1. Rotating coil with N turns.

To do the integral around the coil circuit ABCD two cases are considered; either the coil is entirely inside the magnet so that only a 2-dimensional field is present with $B_z = 0$, or the ends of the coil AB and CD are well outside the magnet where $\vec{B} = 0$. In either case the only contribution to the integral comes from the radial component of field B_r (since along AD, $\dot{\theta} = 0$).

$$E(r,\theta) = -r\dot{\theta} N \int_B^C B_r(r,\theta,z)\, dz \quad . \tag{4}$$

Since there are no free currents in the volume to be measured ($\vec{\nabla} \times \vec{B} = 0$), $\vec{B}$ can be determined from a scalar potential $\vec{B} = -\mu_o \vec{\nabla} V$ which from $\vec{\nabla} \cdot \vec{B} = 0$ leads to Laplaces Eq. $\nabla^2 V = 0$. For the 2-dimensional case with the coil well inside the magnet, Laplaces Equation becomes

$$r \frac{\partial}{\partial r}\left(r \frac{\partial V}{\partial r}\right) + \frac{\partial^2 V}{\partial \theta^2} = 0 \quad .$$

This is solved by separation of variables, $V = R(r)\ \Theta(\theta)$. The equations in R and Θ are

$$\frac{\partial^2 \Theta}{\partial \theta^2} + \lambda\, \Theta = 0 \quad \text{and} \quad r \frac{\partial}{\partial r}\left(r \frac{\partial R}{\partial r}\right) - \lambda R = 0$$

where λ is the constant of separation. The Θ equation has the solution $\Theta_n = a_n \sin n\theta + b_n \cos n\theta$ where n is integer in order that Θ be continuous and single valued in the interval $0 \le \theta \le 2\pi$ ($\lambda = n^2$). The radial equation has the solution $R = Ar^n + \frac{B}{r^n}$. In this case B must be zero in order that $R\ (r=0)$ remain finite.

The 2-dimensional solution for the scalar potential is an infinite sum of terms

$$V(r,\theta) = \sum_{n=0}^{\infty} r^n (a_n \sin n\theta + b_n \cos n\theta) \quad .$$

This solution must transform smoothly to the case where $B_z \neq 0$. Therefore, only the coefficients a_n and b_n are functions of the z coordinate.

$$V(r,\theta,z) = \Sigma_{n=0}^{\infty} \; r^n \; (a_n(z) \sin n\theta + b_n(z) \cos n\theta).$$

From Eq. (4) only the radial field component is determined directly from the measurement

$$B_r(r,\theta,z) = -\Sigma_{n=0}^{\infty} \; \mu_o \, nr^{n-1} \; (a_n(z) \sin n\theta + b_n(z) \cos n\theta).$$

The component B_θ follows from $\vec{\nabla} \times \vec{B} = 0$:

$$B_\theta = -\Sigma_{n=0}^{\infty} \; \mu_o \; nr^{n-1} \; (a_n(z) \cos n\theta - b_n(z) \sin n\theta).$$

Eq. (4) then becomes

$$E(r,\theta) = r\,\dot{\theta}\,N\,\Sigma_{n=0}^{\infty}\,\mu_o \; nr^{n-1} \int_B^C [a_n(z) \sin n\theta + b_n(z) \cos n\theta]\, dz$$

Let $\ell\,\bar{a}_n = \int_B^C a_n(z)dz$ and $\ell\,\bar{b}_n = \int_B^C b_n(z)dz$

(ℓ is the length of the coil along BC) and combine the sine and cosine terms with

$$\bar{c}_n = \sqrt{\bar{a}_n^2 + \bar{b}_n^2} \quad , \quad \alpha_n = \tan^{-1} \frac{\bar{a}_n}{\bar{b}_n} \quad , \text{ and } \quad \omega = \dot{\theta}$$

to get

$$E(r,\theta) = r\,\omega\,N\ell \; \Sigma_{n=0}^{\infty} \; \mu_o \; nr^{n-1} \; \bar{c}_n \sin(n\theta + \alpha_n). \tag{5}$$

The measured quantities are E_n the peak voltage of the n^{th} harmonic and the phase angle of α_n of the n^{th} harmonic taken with respect to the fundamental and averaged over the z coordinate. The peak amplitude of the n^{th} harmonic is

$$\bar{B}_n = \mu_o \, nr^{n-1} \, \bar{c}_n$$

(since mks units are employed the components $\bar{B}_n$ of magnetic induction are in tesla), and

$$E_n = r\,\omega\,N\ell \; \bar{B}_n \; . \tag{6}$$

For a dipole magnet the fundamental is given by n=1, for quadrupoles n=2.

When measuring, it is usually convenient to orientate the coordinate system (Fig. 1) so that the phase angle of the fundamental is zero.

The radius r in Eq. (6) is the distance of the coil segment BC (Fig. 1) from the axis of rotation. Obviously it should be chosen so that the volume swept by the rotating coil encompasses the volume in which knowledge of the field is desired. The quantity N is the number of turns in the coil. The angular velocity ω is given, of course, in radians/sec, ℓ and r in meters. The length ℓ of the coil is chosen depending on the type of measurement desired. If a discrete knowledge of the field at many points along the magnet aperture is required, a short coil must be used. In general, it has been found that short coils (less than 2 inches) are quite easy to wind and balance. Where only the integrated field components are needed a long coil is the best choice since the time required for measurement can be reduced considerably.

II. Construction

The essential features of a rotating coil device built and tested at NAL are shown in Figure 2. The coil is rotated at a uniform speed by a 2-speed, reversible motor which is synchronized with the line frequency. The motor couples to the coil shaft through an O-ring belt and flywheel. This type of drive mechanism prevents transmission of harmonics found in the driving motor. Besides the motor speeds of 900 and 1800 RPM, the flywheel-pulley combination has been constructed with 2 speed ratios, thus providing 4 speeds in all. The designed speeds are 3, 6, 13.43, and 26.86 Hz, the last two of which are not subharmonics of the line frequency. The slowest speed might be used for rotating long coils to alleviate the necessity for critical balancing of the coil. Attached to one end of the shaft is an optical encoder which provides 256 pulses per revolution and a once per revolution marker. Between the encoder and flywheel is a device to provide for fine adjustment of phase angle between the coil and the encoder. This device permits an angular displacement of 7° between them. This is accomplished by holding the flywheel stationary and rotating the outer shell of the clutch assembly. A rotation of 1° of this cylinder causes a displacement of approximately 1/2 min of arc between the encoder and coil.

The coil voltage (which is the quantity $E(r, \theta)$ given by Eq. (5)) is transmitted through slip rings to the analyzing device. The slip rings are connected to the coil with a twisted pair of leads running down the center of the hollow rotating shaft. The analyzing device may be a wave analyzer or an analog to digital voltage converter on line to a digital computer. In the latter case the waveform is then subjected to a Fourier Analysis to yield amplitudes and phases in Eq. (5). If a minicomputer system is not available, a commercial wave analyzer can be used. A Quantech Model 2449 has been used at NAL with generally satisfactory results. However, there is no provision on this unit to measure the phase angles. This can be accomplished by displaying the input waveform and the restored harmonic from the wave analyzer simultaneously on an oscilloscope. In the case of the Quantech wave analyzer, an additional phase difference, which is a function of frequency, is induced between the input signal and the restored output which must be taken into account when determining the correct phase.

The coil shown at the end of the shaft in Figure 2 is one designed for sensing the field in a superconducting prototype dipole where fields up to 40 kG are present. Even though the coil itself is balanced the shaft, which is 60 in. long, is not. Therefore, if left unsupported at the coil end, the vibration that is present will render the data useless. To prevent motion of the coil due to shaft vibration, a collection of expandable collets have been constructed which can be readily adapted to various magnet apertures. In using them, however, one has the disadvantage of not being able to center the coil on the magnetic center of, say, a quadrupole where the geometric and magnetic center may differ. A device that provides both support and translation in the x-y plane would be ideal. The long shaft, while suitable for short coils, obviously could not be used to support coils much longer than a couple of inches. For such coils one should provide supporting bearings at each end of the coil. For the coil shown in Figure 2 the wire bundle consists of 60 turns of # 40 awg wire which forms a square cross-section 1/32 in. on a side. It is desirable to keep this coil cross-section as small as possible. In general, it should be small compared to the wave length of the highest harmonic that one wishes to detect. Typically, the harmonics are a small fraction of the fundamental, 0.1% or less. Therefore, the coils should be constructed with enough turns so that the fundamental produces a peak to peak voltage that is compatible with the analyzing device and still give the desired resolution. Thus, if one wishes to resolve harmonics that comprise only 0.01% of the signal, a 14 bit ADC with a range of ±10 volts would be adequate. In this case, the coil should have enough turns to produce a 20 vpp signal.

The rotating device described here has been subjected to field testing for approximately two months. Using this instrument, measurements were made on a prototype superconducting energy doubler dipole magnet by B. Strauss and D. Sutter at NAL.[5] The most serious difficulty encountered thus far is a tendency for the angular velocity to drift over ±0.1% instead of the desired ±0.01%. This drift is not uniform in nature but rather more or less sporadic. The cause of this fault is most probably due to excessive shaft friction in the nylon bearings. One solution to this problem would be in using stainless steel ball bearings and a shaft which has tighter tolerances on the uniformity of wall thickness, and straightness. The shaft now being used and all rotating parts are of nonconducting material to prevent shaft loading at high fields and subsequent field distortion. In addition, the bearings are designed to run without lubrication so that cold bore superconducting magnets could be measured. At any rate it appears as though the clearance of 0.001 to 0.0015 in clearance is insufficient for smooth operation of the nylon-teflon bearings used in this device. Perhaps a better choice of material for these would be graphite.

The variation in angular velocity described above is observed as an oscillation of the peak amplitude of the harmonic when measured with the Quantech wave analyzer, and causes a jitter in the waveforms on the oscilloscope, making it difficult to measure the phase angles by this technique (even without this complication, the phase angle measurement via the oscilloscope is very imprecise as is also pointed out by J. Cobb[4]). When used with a computer oriented system one can expect a substantial improvement in both accuracy and time required for analysis. Such a system should be capable of resolving harmonics to the order of ±0.01%.

References

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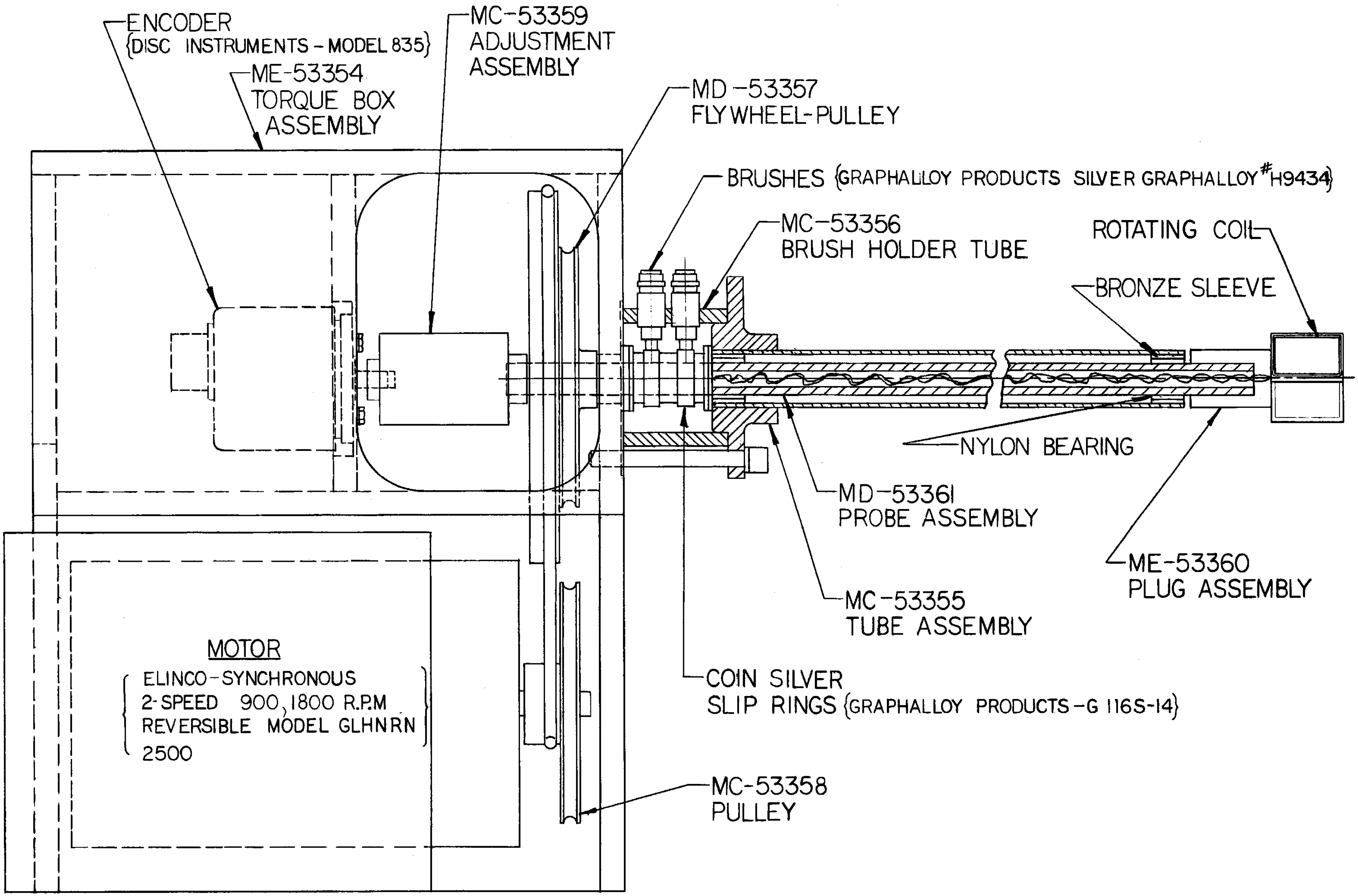
ENCODER
{DISC INSTRUMENTS - MODEL 835}
ME-53354
TORQUE BOX
ASSEMBLY
MC-53359
ADJUSTMENT
ASSEMBLY
MD-53357
FLYWHEEL-PULLEY
BRUSHES {GRAPHALLOY PRODUCTS SILVER GRAPHALLOY #H9434}
MC-53356
BRUSH HOLDER TUBE
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PLUG ASSEMBLY
MC-53355
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COIN SILVER
SLIP RINGS {GRAPHALLOY PRODUCTS-G 116S-14}
MOTOR
ELINCO-SYNCHRONOUS
2-SPEED 900, 1800 R.P.M
REVERSIBLE MODEL GLHNRN
2500
MC-53358
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FIGURE 2.