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SCALING OF LUMINOSITY IN COLLIDING BEAMS

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We investigate here the dependence of the maximum attainable luminosity on the parameters of the colliding-beam storage rings under ideal conditions. Means of establishing such ideal beams in such ideal storage rings are ignored. The configuration of the storage is assumed to be that of the energy doubler.

A. Beam-Intensity Limitation

We assume the beam to be surrounded first by two circular arcs of infinitely conducting ($\sigma=\infty$) walls and then further out by a circular infinite permeability ($\mu=\infty$) wall as shown in Fig. 1.

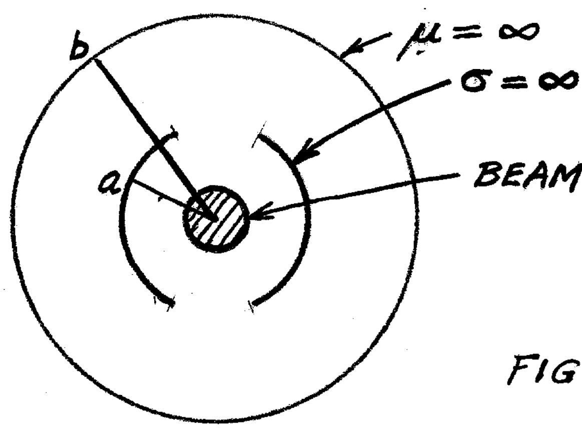

FIGURE 1

For a high-energy beam ($\gtrsim$200 GeV) the shift of the incoherent horizontal tune $\Delta\nu$ is given approximately by

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$$\Delta\nu \cong -\frac{r_p}{10}\frac{R^2}{\nu}\frac{\varepsilon}{a^2}\frac{\lambda}{\gamma}$$

where

r_p = classical proton radius = 1.535×10^{-18}m,

R = radius of ring = 10^3m,

ν = unshifted tune = 20.25,

a = radius of the $\sigma=\infty$ walls (Fig. 1),

γ = beam total energy in mc^2 unit,

λ = linear density of beam,

and ε is a parameter of the order of unity whose exact value depends on the geometry of the $\sigma=\infty$ walls. Note that in this approximation (tune-shift is dominated by image charges) neither the cross-section of the beam nor the radius b of the circular $\mu=\infty$ wall enters in the formula. This gives

$$\lambda = \frac{10}{r_p}\left(\frac{-\nu\Delta\nu}{R^2\varepsilon}\right)\gamma a^2. \tag{1}$$

The stochasticity limit of $\Delta\nu$ was determined empirically to be ~ -0.025. At 200 GeV (γ=214) this gives for the maximum value of λ

$$\hat{\lambda} = \left(\frac{7.06 \times 10^{14}}{\varepsilon}\ m^{-3}\right) a^2. \tag{2}$$

The equivalent maximum current $\hat{I}$ is given by

$$\hat{I} = ec\hat{\lambda} = \left(\frac{3.39 \times 10^4}{\varepsilon}\ A/m^2\right) a^2. \tag{3}$$

With a = 2.5 cm = 0.025m we get

$$\hat{\lambda} = \frac{4.4 \times 10^{11}}{\varepsilon} \text{ m}^{-1}, \qquad \hat{I} = \frac{21.2}{\varepsilon} \text{ A}.$$

B. Luminosity

The luminosity per unit length of the head-on collision of two beams of linear densities λ_1 and λ_2, and identical cross-section is

$$\frac{L}{\ell} = 2c\frac{\lambda_1\lambda_2}{A}$$

where A is the cross-sectional area of the beams. If $\lambda_1 = \lambda_2 = \hat{\lambda}$ and the beams have the same energy and uniform density over a circular cross-section with radius r we get

$$\frac{\hat{L}}{\ell} = \frac{200c}{r_p^2}\left(\frac{-\nu\Delta\nu}{R^2\varepsilon}\right)^2 \gamma^2 \frac{a^4}{\pi r^2} \qquad (4)$$

and for 200 GeV

$$\frac{\hat{L}}{\ell} = \left(\frac{0.95 \times 10^{38}}{\varepsilon^2} \text{ m}^{-5} \text{ sec}^{-1}\right) \frac{a^4}{r^2}\ .$$

We see that the maximum luminosity is proportional to the 4th power of the radius of the superconducting bore and inversely to the square of the beam radius. To get high luminosity we should have large a (large magnet bore) and small r (low-β insertion). Taking

$$a = 2.5 \text{ cm} = 0.025\text{m}$$

$$r = 1 \text{ mm} = 0.001\text{m}$$

we get

$$\frac{\hat{L}}{\ell} = \frac{3.7 \times 10^{33}}{\varepsilon^2} \text{ cm}^{-2} \text{ sec}^{-1}/\text{m}.$$