national accelerator laboratory

TM-163
0510

THE EFFECT OF A HIGH-BETA INSERTION ON RESONANT EXTRACTION

E. D. Courant

April 11, 1969

It has recently been suggested that the lattice of the extraction straight section of the main ring be modified so as to produce a high local value of the amplitude function $\beta(s)$ at the azimuth of the septum. Lattices accomplishing this have been devised by L. C. Teng and G. J. Bellendir.

The advantage claimed for this modification is that, if β is substantially larger in the extraction region than in the main part of the lattice, the corresponding aperture requirement is also larger there, and if the present aperture is then still sufficient in the extraction section, the aperture in the main lattice can be reduced with a corresponding cost saving.

Implicit in this argument is the assumption that the present design aperture is larger than needed for orbit containment, and is as large as it is only to make room for the blowup of the beam which is part of the resonant extraction process. Be this as it may (and a reduction in main-lattice aperture would certainly tighten the tolerances on magnet errors- whether by too much is another question), there is a simple argument why the high-beta section will tend to reduce extraction efficiency.

We assume that we wish to extract at the same radial excursion x_e from the equilibrium orbit as in the standard design (say 4 cm). Resonant extraction at $\nu = m/3$ (m = 61 in our case) is accomplished by means of a set of sextupole magnets having an m-th azimuthal harmonic; these produce a triangular separatrix in phase space, and the beam parameters are slowly varied so that the beam gradually gets squeezed outside this separatrix. The mechanism is analyzed in detail in four reports by K. Symon.[1, 2, 3, 4] When a particle blows up, its radial position on every third turn jumps by a distance Δx which increases "faster than exponentially" from turn to turn because of the nonlinearity of the driving force, i.e. the ratio

$$\frac{\Delta x_{n+1}}{\Delta x_n} ,$$

Where Δx_n is the difference between the position at the 3n-th turn to that at the (3n-3)-th turn, increases with n. The particle losses in extraction are proportional to the ratio of septum thickness to the final Δx; therefore a large Δx is desirable.

For a given strength of the sextupoles exciting the resonance, x_n at a given azimuth is proportional to the value of $\beta^{1/2}$ at that azimuth. Therefore, at a given value of the extraction radius x_e, the turn number n at which a particle reaches that radius decreases as β is increased. But then, because (as we have just seen) the turn separation ratio

---

1, 2, 3, 4 K. R. Symon, FN-130, FN-134, FN-140, FN-144 (1968)

$\Delta x_{n+1}/\Delta x_n$ increases with n, this ratio will be smaller at the given extraction radius, and it can easily be shown that the actual turn separation Δx_n will decrease as β is increased.

Quantitatively, consider eq. (30) of FN-130. Note (by referring to eq. (14) as is done in FN-130) that there is a typographical error; eq. (30) should read

$$\Delta x = \beta^{1/2}\Delta X = 18\pi A\beta^{1/2}X_e^2 = 18\pi A\beta^{-1/2}x_e{}^2 \qquad (30)$$

The quantity A is a measure of the sextupole strength (see FN-134) and depends on the β-values at the sextupole locations and not at the extraction point. Thus if A is kept constant, the jump Δx decreases as $\beta^{-1/2}$ as β is increased.

One could, of course, increase A (the sextupole strength). But, as has also been pointed out by A. Maschke[5], this would necessitate a larger change of ν during the extraction process, leading to a more pronounced rotation of the phase space diagram during the extraction process and consequently to a larger angular spread and effective emittance of the extracted beam.

The conclusion is that a high-beta insertion is not desirable for the resonant extraction process.

---

5 Private communication