TM-103
0106

IMPROVED LATTICE PARAMETERS FOR THE 100-BeV STORAGE RINGS

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September 8, 1969

A. INTRODUCTION

The purpose of this report is to discuss the lattice of the Storage Ring proposed for the NAL synchrotron during the 1968 Design Study in greater detail then was then reported[(1)], and to present certain revisions and refinements of that lattice. The parameters presented in Table I of this report constitute our current design recommendations for the lattice of this facility.

The principal design improvements made since presentation of the former lattice[(1)] are the following:

i) The horizontal tune ν_x has been set to a value (14.35) that Courant has calculated will maximize injection efficiency.[(2)]

ii) The vertical β_y, and consequently the beam height, has been reduced from 4m to 2m at the beam crossings, which enhances the luminosity.

iii) The momentum-dispersion matching of the crossing insertions to the normal cells has been improved, so that the fluctuation in the cells is reduced from 20% to 3.5%.

Table I. Lattice Structure and Orbit Parameters

(*Numbers in parentheses are bending magnet fields and quadrupole field gradients at 100 BeV.)

1. Normal Cell (C)

a. Length of bending magnet (B1 and B2)	4.2m	(19.99 kG)*
b. Length of cell quadrupole magnet (QF and QD)	1.4m	(+234.1,-236.5 kG/m)
c. Length of minimum separation between magnets (0)	0.3m	
d. Length of short straight section (00)	1.6m	
e. Cell structure	(QF/2)00(B1)0(B2)0-(QD)00(B2)0(B1)0(QF/2)	
f. Total length of cell	24m	
g. Number of cells in each ring	48	

2. Injection Insertion (I)

a. Length of end quadrupole (QDI = QD/2)	0.7m	(-236.5 kG/m)
b. Length of bending magnet (BI)	4.2m	(19.99 kG)
c. Lengths of quadrupoles in triplets		
QFI1	1.56m	(200 kG/m)
QDI2	2.20m	(-205 kG/m)
QFI3	.74m	(215 kG/m)
d. Drift lengths between		
QDI and BI (LI1)	7.56m	
BI and BI (0)	0.3m	
BI and QFI1 (LI2)	2.20m	
QFI1 and QDI2 (LI3)	4.67m	
QDI2 and QFI3 (LI4)	2.68m	
QFI3 and crossing point (inside) (LI5)	37.15m	
Crossing point and QFI3 (outside) (LI6)	27.58m	
e. Insertion point in normal cell	Midpoint of QD	
f. Structure of insertion I (in beam direction from inside to outside)	(QDI)LI1(B2)0(B1)0(BI)0(BI) LI2(QFI1)LI3(QDI2)LI4(QFI3)LI5 LI6(QFI3)LI4(QDI2)LI3(QFI1)LI2 (BI)0(BI)0(B1)0(B2)LI1(QDI)	
g. Total length of I-insertion	145.75m	
h. Number of I-insertions	3	

3. Experimental insertion (E)

a.	Length of end quadrupole		(234.1 kG/m)
	(outer end QFEO)	.90m	
	(inner end QFEI)	.87m	
b.	Length of bending magnet (BE)	4.2m	(19.99 kG)
c.	Lengths of quadrupoles in outer triplet		
	QDE1	3.64m	(-200 kG/m)
	QFE2	3.56m	(200 kG/m)
	QDE1	3.13m	(-200 kG/m)
	Lengths of quadrupoles in inner triplet		
	QDE4	2.84m	(-200 kG/m)
	QFE5	3.24m	(200 kG/m)
	QDE6	3.60m	(-200 kG/m)
d.	Drift lengths between		
	QFE0 and B1 (LE1)	2.61m	
	BE and BE(0)	0.3m	
	BE and QFE1 (LE2)	.30m	
	QFE1 and QDE2 (LE3)	4.12m	
	QDE2 and QFE3 (LE4)	2.75m	
	QDE3 and crossing point (LE5)	31.21m	
	Crossing point and QDE4 (LE6)	39.99m	
	QDE4 and QFE5 (LE7)	2.68m	
	QFE5 and QDE6 (LE8)	5.87m	
	QDE6 and FE (LE9)	1.08m	
	B1 and QFE1	2.61m	
e.	Insertion point in normal cell	Midpoint of QF	
f.	Structure of insertion E (in beam direction from outside to inside)	(QFE0)LE1(B1)0(B2)0(BE)0-(BE)0(BE)0(BE)LE2(QDE1)-LE3(QFE2)LE4(QDE3)LE5-LE6(QDE4)LE7(QFE5)LE8-(QDE6)LE9(BE)0(BE)0(BE)-0(BE)0(B2)0(B1)LE1(QFEI)	
g.	Total length of E insertion	168.38m	
h.	Number of E insertions	3	

4. Superperiod

a.	Half cells	
	from mid-QF to mid-QD	$\frac{C}{2}$
	from mid QD to mid-QF	$\frac{\bar{C}}{2}$
b.	Structure of superperiod	$ECCCCCCC\frac{C}{2}I\frac{\bar{C}}{2}CCCCCCCC$
c.	Total length of superperiod	698.13m
d.	Number of superperiods	3

5. Orbit properties

a. Betatron oscillation wave-numbers	
horizontal (ν_x)	14.35
vertical (ν_y)	17.35
Betatron oscillation wave-lengths	
horizontal	140.0m
vertical	120.7m
b. Betatron oscillation amplitude function (β)	
Normal cell	
β_x max	40.13m
β_y max	39.41m
β_x min	11.50m
β_y min	11.16m
Injection insertion	
β_x max	136m
β_y max	95m
Experimental insertion	
β_x max	634m
β_y max	801m
Experimental crossing point	
β_x	10m
β_y	2m
c. Momentum excursion function (x_p)	
Maximum in normal cell	2.54m
Minimum in normal cell	1.35m
Injection insertion	3.67m
Experimental insertion	0

iv) The injection insertions are more closely tailored to facilitate injection by maximizing momentum dispersion at the septum magnet and providing 180° of radial phase advance between the two kicker magnets.

v) The betatron tunes ν_x and ν_y are widely split (14.35, 17.35) compared to their former values (14.81, 14.85). Techniques used in obtaining these proposed parameters are described in the hope they will prove helpful for further refinements or new designs.

The proposed colliding beam storage rings(3) consist of two rings, each of 1/3 km average radius, that intersect each other at six points. The intersections occur in special insertions, three of which are designed for experiments and three for injection and beam dump. Each ring has three superperiods each containing one experimental type and one injection type insertion, and 16 normal cells. The superperiod has one outer 'sextant' with 8½ normal cells and one inner 'sextant' with 7½ normal cells, as proposed by Teng(3). The resulting intersection angle is 50 mrad, which is equal to the bending angle of one normal half-cell.

The normal cell, of FODO separated function design, is 24 meters and contains four 20 kG, 4.2m length bending magnets and two 1.4m long quadrupoles with gradients of +234 and -236 kG/m.

In the experimental insertions the beams are directed inward to maximize the downstream drift from the interaction region. They are designed to produce small β_y (2m) at the intersection, which enhances the luminosity, and to reduce the momentum excursion function x_p to zero in the interaction region in order to better localize the collision zone. The momentum matching to the normal cell at either end of the insertion is accomplished by a special end quadrupole, a drift space, and a set of six bending magnets. The betatron functions are matched by the two quadrupole triplets to produce vertical and horizontal waists at the crossing point[(5)].

In the injection insertions the beams travel outwards. The desired orbit properties are large β_x at both septum and kickers to reduce the needed kick[(6)], large dispersion x_p and zero dispersion slope x_p', and π phase advance between the kickers located just inside the end quadrupoles. In contrast to the experimental insertion, the two quadrupole triplets in the injection insertions are identical.

The new parameters are shown in Table I, which is a revision Design Study Report Table II-1[(7)].

B. STEPS IN THE CALCULATION

The starting point is a normal cell [F(OO)BOBODD(OO) BOBOF, 24m long] with gradients $\pm$248.5 kG/m, which minimizes the β function. Its betatron functions from computations with SYNCH[(8)] are used to calculate the values for the momentum matching elements at the ends of the insertions. The length of these elements are then accounted for in the calculation of the overall ring geometry which fixes the lengths available for betatron matching. The betatron matching of the insertions to the cells is performed by TRANSPORT[9] for each type of insertion and iterated so as to maximize the drift lengths through the intersections. The ν values for the ring are then found from SYNCH, and then adjusted to make $\nu_x \approx n + 1/3$ for three turn injection by adjusting the normal cell quadrupoles. As this adjustment disturbs the matching, the whole procedure is repeated an adequate number of times to achieve acceptable matching at the desired tune. As one might hope, the process is rapidly convergent; more details on each step are given in order below.

1. Momentum Match in Experimental Insertion

The momentum excursion functions x_p and x_p' at the beginning of the insertion (middle of QF) are represented by the vector

$$X_{pi} = \begin{pmatrix} x_p \\ x'_p \\ 1 \end{pmatrix}$$

The desired vector at the end of the momentum matching section consisting of QFE, drift E and bending section B6 (BOBOBOBOBOB) is

$$X_{pf} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} .$$

The transfer matrix for the matching section is

$$M = (B6)(E)(QFE) \approx \begin{pmatrix} 1 & \ell_6 & \ell_6^2/2\rho_6 \\ 0 & 1 & \ell_6/\rho_6 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & E & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos K\ell_F, & \frac{\sin K\ell_F}{K}, & 0 \\ -k\sin K\ell_F, & \cos K\ell_F, & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where ℓ_6 is the length of the six magnet section B_6 and ρ_6 is the radius of curvature.

The slight approximation in the B6 matrix given above is removed in practice by calculating it with SYNCH. The solution of the required matrix equation,

$$MX_{pi} = X_{pf},$$

is obtained by suitable choice of E and ℓ_F. This is easy since E does not appear in the second row of M, so that ℓ_F obtained from the second equation leads to E from the first by direct substitution. The two ends of the insertion only differ in that the sign of x_p' is opposite. Both solutions occur as roots of the second equation when it is solved as a quadratic in $\sin(K\ell_F)$.

2. Momentum Matching in The Injection Insertion

There are no special conditions at the crossing point in the injection insertions; hence a symmetric design can be used. The symmetric design allows separation of the momentum and betatron matching problems. Solution of the former problem is obtained by placin- he four bending magnets following the cell QD quadrupole so that the vector leaves them as if it had proceeded in a straight line from the center of the kickers; and by setting the T_{22} element between the kickers and the insertion center to zero. The $T_{22} = 0$ condition simultaneously quarantees that the kickers will fit the injected beam to the circulating beam, and that the off-momentum orbit will be flat ($x_p' = 0$) in the middle of the insertion. Hence there will be a real focus for the betatron matching triplets in the kickers. A consequence of this procedure is that the phase advance of the insertion is close to π.

Explicit calculation of the length $\underline{a}$ of the drift space between the quad QD and the bending section B4 required for the momentum match proceeds as follows. The transfer matrix from the end of the quad QD to the end of the four bending magnets is

$$M_{12} = (B4)(A) = \begin{pmatrix} 1 & 17.69826 & .88259736 \\ 0 & 1. & .09973832 \\ & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where the drift length is $\underline{a}$, and the matrix B4 for the four bending magnets was calculated with SYNCH. Let X_{p1} be the dispersion vector at the end of QD. At the end of the four bending magnets it will be $X_{p2} = M_{12}X_{p1}$. The off-momentum orbit enters the triplet as if it had started from the axis a distance $d=\frac{x_2}{x_2'}$ upstream from the end of B4, with slope x_2' with no bending magnets present. We adjust the distance $\underline{d}$ to carry one back from the end of B4 to the center of the kicker. Thus,

$$d = a + \ell_4 - \ell_k,$$

where ℓk is the distance from the end of QD to the center of the kicker, and the length a is obtained from the equation

$$a + \ell_4 - \ell_k = \left| \frac{(x_p)}{(x'_p)} \right|_2 = \frac{(x_p)_1 + (a+17.69826)(x_p')_1 + .88259736}{(x_p')_1 + .09973832}$$

with ℓ_4 = length of B4, ℓ_k = length of the kicker we obtain a = 7.5566m. The reason we used four bending magnets was that it leads to a reasonable length for a.

3. Geometrical Conditions

With the lengths of the momentum matching sections determined as above one can find out how much insertion space is left for the betatron matching and free straight section. Denote by s_1, s_2, s_3 and s_4 the straight sections on either side of two consecutive joining points as in the following diagram. Two relations among these are obtained from the condition that the sum of the vector displacements along inner and outer arcs are equal. These equations are (where $\Delta_4 = R_c - R_4$ and $\Delta_6 = R_c - R_c$)

$$s_1 - s_2\cos\alpha + s_3\cos(\beta+\theta+\gamma) - s_4\cos(\alpha+\beta+\theta+\gamma) =$$

$$- R_6\sin\alpha + t_2\cos(\alpha+\beta) - t_1\cos\beta - \Delta_6[\sin(\alpha+\beta) - \sin\beta]$$

$$+ \Delta_4[\sin(\alpha+\beta+\varphi) - \sin(\beta+\theta)] + t_4\cos(\alpha+\beta+\varphi) - \tau_3\cos(\beta+\theta)$$

$$+ R_4[\sin(\alpha+\beta+\varphi+\gamma) - \sin(\beta+\theta+\gamma)]$$

$$s_2 - s_3\sin(\beta+\theta+\gamma) + s_4\sin(\alpha+\beta+\varphi+\gamma) =$$

$$- R_6(\cos\alpha-1) - t_2\sin(\alpha+\beta) + t_1\sin\beta - \Delta_6\left[\cos(\alpha+\beta)-\cos\beta\right]$$

$$+ \Delta_4\left[\cos(\alpha+\beta+\varphi) - \cos(\beta+\theta)\right] - t_4\sin(\alpha+\beta+\varphi) + t_3\sin(\beta+\theta)$$

$$+ R_4\left[\cos(\alpha+\beta+\varphi+\gamma) - \cos(\beta+\theta+\gamma)\right]$$

A third relation is obtained from desired ring circumference:

$$s_1 + s_2 + s_3 + s_4 = \frac{2\pi R}{3} - 16L_c - (t_1+t_2+t_3+t_4)$$

where R is the average radius 1000/3 meters.

A fourth relation may be specified arbitrarily, and initially the symmetric choice

$$s_1 + s_2 = s_3 + s_4$$

was used. In later cycles of computation, however, where small changes were being made in just one insertion, the disruptive effects of these changes were reduced by holding the straights in the other insertion fixed at their value for the previous cycle. From these four relations the lengths s_1, s_2, s_3, and s_4 are determined.

4. Betatron Matching The Injection Insertion

The betatron matching problem is simply to carry the nearly upright input ellipses at the center of a cell QD quadrupole to upright ellipses at the center of the long drift length, subject to the constraint $T_{22} = 0$ on the matrix from the kicker position to the center of the insertion. One also wishes to maximize the drift length from the last quad to the center. This was solved readily with the program TRANSPORT[9].

5. Betatron Matching in The Experimental Insertion

Because one wishes to have β_y = min at the beam intersection, which is not midway between the ends of the insertion, the experimental insertion is asymmetric. However, a technique which led systematically to our best solution started by calculating a symmetric solution for a fictitious crossing point at the center with the desired min β_y. This solution was used to produce separate solutions for the upstream and downstream halves of the insertion by progressively shortening and lengthening respectively the central drift length in small steps while adjusting the triplets to hold the beam ellipses fixed at the starting point in the cell F quadrupole and at the crossing point. Such a systematic procedure is important because TRANSPORT cannot find a solution if the initial parameters are too far off, and the very small β_y at the crossing point implies sensitivity to small parameter changes.

The residual momentum mismatch in the lattice presented is manifested by $\pm 3.5\%$ fluctuation of x_p from cell to cell. It is induced by the fact that the bending magnets are not quite centered between the cell quadrupoles, and the consequent asymmetries in the x_p and β_y fractions were not taken account of in our matching calculations. Although such fluctuation does not seem a matter of practical concern it might be improved by refinement of the methods described above.

REFERENCES

(1) Proton-Proton Colliding-Beam Storage Rings for the National Accelerator Laboratory, Batavia, Ill. (1968) referred to throughout as the Design Study

(2) Ibid, p. 55

(3) Ibid, Ch. II

(4) A. A. Garren and E. Hubbard, Sixth International Conference on High Energy Accelerators, p. 325, CEA (1967)

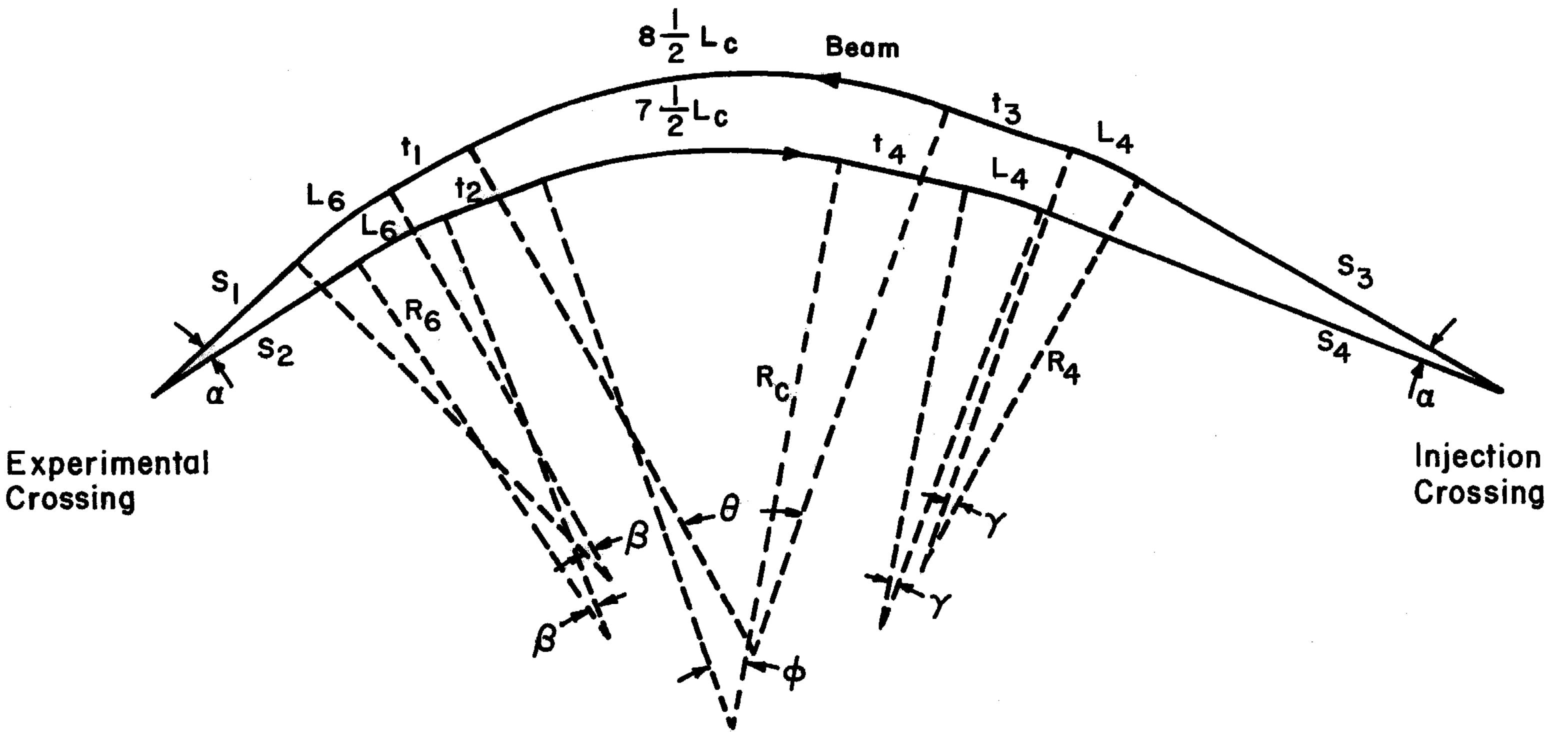
(5) P. L. Morton and J. R. Rees, SLAC-PUB 246 (1966)

(6) Design Study, p. 56

(7) Ibid, p. 20

(8) A. A. Garren and A. S. Kenny, "SYNCH, A Computer Code for Synchrotron Design and Orbit Analysis," LRL (1968)

(9) C. H. Moore, S. K. Howry, and H. S. Butler, "TRANSPORT, A Computer Program for Designing Beam Transport Systems," SLAC (1965)



S_1, S_2 betatron matching straights for experimental insertion

S_3, S_4 betatron matching straights for injection insertion

t_1, t_2 momentum matching drifts for experimental insertion

t_3, t_4 momentum matching drifts for injection insertion

α = intersection angle
β = bending angle of B6
θ = bending angle of 8 1/2 cells
ϕ = bending angle of 7 1/2 cells
γ = bending angle of B4

L_c normal cell length
L_4 four bending magnet arc length
L_6 six bending magnet arc length