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The Beam Dynamics of Slip Stacking

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Abstract

"Slip Stacking"[1] is the name used at Fermilab for a process of combining two bunched beams in a synchrotron into a single beam at the same harmonic number by bringing them close together in energy and capturing two bunches per bucket when the bunches are aligned in phase. Emittance is necessarily diluted because the two radio frequency systems act on both beams, leading to chaotic motion when the bunches are too close together. Hence, either the bunches are disrupted by close approach or the emittance is diluted by unoccupied phase space area at capture time. The dynamics of this process have been examined with the object of minimizing emittance dilution and relaxing demands on transient beam loading correction.

Introduction

Ankenbrandt proposed to increase intensity from the old Main Ring by (nearly) doubling the number of Booster injections per Main Ring cycle.[1] The Booster batches were to be injected on an off-momentum orbit and decelerated to join previously injected batches. The injection energy offset was chosen so that the decelerating batch would slip in phase sufficiently to open the same gap for another batch on the next Booster cycle. Even before the installation of the B0 overpass in 1982, the momentum aperture in the Main Ring might have been marginal, and reasonably successful high intensity operation for fixed target was attained by more conventional measures. A very similar scheme had been proposed earlier for combining beams from the CERN four-ring CPS Booster. Development efforts were terminated without complete success after difficulties were met with high intensity beam and a satisfactory alternative was identified.[2]

In AD 2001, with a new Main Injector (MI) synchrotron possessing momentum acceptance approaching two percent and an ambitious neutrino program scheduled to run concurrently with colliding beam operation, it is timely for Fermilab to re-examine the potential of slip stacking. The anticipated benefit is a possible doubling of intensity on antiproton and neutrino production targets with a small extension of the MI cycle time and some loss of effectiveness of the bunch rotation in the Debuncher for antiproton accumulation. In principle, no major hardware developments should be required, but the scenario is somewhat intricate and technical obstacles exist, so a significant manpower commitment in rf instrumentation, controls, and commissioning must be considered. This note treats four important concerns: minimizing emittance dilution by parameter choice in the context of single particle dynamics, reduction of beam loading excitation of the rf systems by parameter choice, dynamic effects of beam loading, and (to a lesser degree) beam loading compensation.

Detailed modeling of slip stacking for the MI was undertaken by Shukla.[3] His last published results were superceded by his later results which have not been widely circulated. Without trying to reconstruct his later work, one can say at the very least he demonstrated that, with single particle dynamics, two 0.1 eVs bunches from the Booster could be combined into a single 0.3 eVs bunch in the MI. This result has been confirmed and slightly improved using the ESME code[4] with parameters close to those of Shukla. It appears to remain valid in multiparticle modelling with suitable choices for the rf system parameter curves. Furthermore, the result appears to be close to a rather broad multi-parameter optimum, encouraging a hope that adjustments can be made to deal with collective effects while retaining a satisfactory final emittance.

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Basic Concepts

The pictorial concept that two bunch trains at different momenta exist in independent buckets is clearly a crude simplification. Both bunch trains experience the full effect of both rf systems. From the trigonometric identity

$$\sin a + \sin b = 2\, sin\frac{a+b}{2}\, cos\frac{a-b}{2} \tag{1}$$

one knows that both beams are seeing low frequency amplitude modulation, 100 % modulation for equal amplitudes. Suppose that bunches are kept at a constant energy separation corresponding to rf angular frequency $\pm\delta$ from the central orbit value and the rf amplitudes are $V_\circ$. The net rf waveform experienced by both is

$$V_{\mathrm{rf}} = 2V_\circ \sin\omega_\circ t \cos\delta t \quad . \tag{2}$$

This looks awful, yet at sufficient frequency separation the wrong-frequency excitation averages to zero in a very small fraction of a synchrotron oscillation period. Therefore, at some separation the phase motion of the two bunches must be practically independent. It is useful to express δ in units of the small-amplitude synchrotron frequency ω_{s} for the unperturbed bucket. Consider, for example, the case of equal amplitudes and constant energy separation. The synchrotron frequency in a stationary bucket is

$$f_{\mathrm{s}} = \sqrt{h|\eta|eV/2\pi\beta^2 E}/\tau \tag{3}$$

and the bucket half-height is

$$H_{\mathrm{B}} = \beta\sqrt{2eVE/\pi h|\eta|} \quad . \tag{4}$$

The symbols in the equations and the rest of this note are defined in Table 1. The relation between frequency and energy separation is

$$\frac{\Delta f}{f} = -\eta\frac{\Delta p}{p} = -\eta\frac{\Delta E}{\beta^2 E} \quad . \tag{5}$$

Combining these three relations one finds that the energy separation in bucket height units is simply related to the frequency separation in synchrotron frequency units:

$$\alpha \stackrel{def}{=} \frac{\Delta f}{f_{\mathrm{s}}} = 2\frac{\Delta E}{H_{\mathrm{B}}} \quad . \tag{6}$$

For $\alpha = 2$, the hypothetical independent buckets overlap 50 % in energy, and the single particle motion is chaotic everywhere within them. The case $\alpha = 4$ gives tangent boundaries for the hypothetical buckets. This value gives a lower limit for stable motion[5], but tracking calculations show that there is nonetheless rather rapid effective emittance growth. Emittance growth is not entirely absent at much larger separations, but practically it is acceptable to hold bunch trains separated by $4H_{\mathrm{B}}$ on centers for several synchrotron periods. In this case there is the space for a complete empty bucket between the upper and lower hypothetical buckets.

The capture of two bunches into a single bucket will produce gross emittance dilution unless the bunches can be brought much closer together than $4H_{\mathrm{B}}$ on centers. There are obvious advantages to bringing them together as fast as practicable. First of all, although chaos happens, chaotic motion does take time. Secondly, when accelerating at higher synchronous phase, the rf voltage is higher; hence the beam loading is relatively less. The following discussion of choice of rf parameters refers to symmetric acceleration and deceleration toward a mean energy, a case which minimizes the number of independent parameters and simplifies visualizing the scenario. The accelerator and beam parameters used are tabulated in Table 2.

Assume that bunches of $6 \cdot 10^{10}$ protons from the Booster can be as small as 0.1 eVs. The twenty MI cavities produce a maximum voltage of approximately 3 MV at injection frequency. Slip stacking will be carried out with one cavity at each frequency to provide a controllable low voltage at constant energy separation. The maximum acceleration rate with 150 kV is about 7 GeV/s at a synchronous phase of 31° and synchrotron oscillation period of

Table 1: Meaning of symbols in equations and text

Symbol	Meaning
$V_{\rm rf}$	rf voltage [MV]
$V_\circ$	rf voltage amplitude of single system [MV]
$\omega_\circ$	rf angular frequency on central orbit [Hz]
t	time [s]
δ	rf angular frequency offset from $\omega_\circ$ [Hz]
$f_{\rm s}$	synchrotron oscillation frequency [Hz]
h	rf harmonic number
e	elementary particle charge (> 0) [C]
β	relativistic velocity v/c
γ	relativistic energy $E/m_\circ c^2$
η	time slip factor $\gamma_T^{-2} - \gamma^{-2}$
E	total energy for synchronous particle [MeV]
τ	beam circulation period [s]
$H_{\rm B}$	half height of stationary bucket [MeV]
f	rf frequency [Hz]
p	momentum [MeV/c]
$S_{\rm B}$	stationary bucket area [eVs]
$\tau_{\rm s}$	period of synchrotron oscillation [s]
$\phi_{\rm s}$	synchronous phase of rf [deg]

Table 2: Accelerator and beam parameters for Main Injector slip stacking

Parameter	Symbol	Value	Units
mean reference orbit radius	$R_\circ$	528.30	m
synchronous energy	E_s	8938.28	MeV
transition energy/$m_\circ c^2$	γ_T	18.6	
rf peak voltage, each system	$V_\circ$	0.15	MV
rf voltage at closest approach, each system	$V_\circ$	0.085	MV
rf harmonic	h	588	
shunt resistence of 20 rf cavities	$R_{\rm shunt}$	$2 \cdot 10^6$	Ohm
loaded Q of cavities without feedback	Q	$2 \cdot 10^3$	
synchrotron tune (150 kV)	ν_s	$3 \cdot 10^{-3}$	
bucket height (150 kV)	H_B	12.9	MeV

3.3 ms. If the acceleration curve is parabolic, the average acceleration rate is 3.5 GeV/s. In going from $\phi_\mathrm{s} = 0$ to $\phi_\mathrm{s} = 31^\circ$, the bucket area changes from 0.32 to 0.1 eVs. Defining an approximate adiabaticity parameter

$$\bar{\alpha} = \frac{\Delta S_\mathrm{B}/S_\mathrm{B}}{\Delta t/\min\tau_\mathrm{s}} \ , \tag{7}$$

one finds that $\bar{\alpha} = 0.1$ for a 23 ms acceleration time, a time in which each bunch train would change energy by 80 MeV. Thus, there is considerable freedom to choose energy separation and acceleration time. A reasonable maximum energy separation would be about 80 MeV total, $\Delta E/E = 0.9$ %. At this separation, the batches slide about 1.6 batch lengths per Booster cycle. For the total separation of 50 MeV, which provides just one batch length of slip per cycle, there is only $3.3H_\mathrm{B}$ of energy separation; one might in this case choose a somewhat lower voltage, like 100 kV, for slipping at constant separation and ramp the voltage to the full 150 kV during acceleration. The case for which tracking results will be presented is that of 80 MeV initial separation and symmetric acceleration and deceleration of 40 MeV.

The principal issue is what nominal bunch center energies at the time of phase alignment result in the lowest effective emittance for the combined bunch. To reduce this to a one parameter search, the qualitative arguments in the preceding paragraph are used to set the initial separation, rf voltage, and acceleration rate. An additional constraint inferred from the minimum separation condition for stationary buckets is that the nominal bucket boundaries should not overlap. This constraint has been checked by tracking trials in some moving bucket cases. Testing the change in the optimum with respect to small changes in other parameters can indicate whether it is reasonably close to the multi-parameter optimum or if it is dangerously sensitive to the other parameters. Clearly the probability of approaching the single-particle optimum performance in the more realistic multi-particle case is substantially greater if the single-particle optimum is reasonably stable.

Developing Satisfactory Design Parameters

The first step in the modelling has been to run trial cycles in reverse for test particles located at the intended end points for bunch centers. This allows checking the parameter curves and establishes starting values for the phases of bunches to be brought together. The next step is to track forward with matched bunches in addition to single particles at the bucket centers. These runs stop automatically when the bucket-center particles are aligned in phase. The final distribution is captured in rf at the central frequency and the area of a 95 % containment contour is calculated. Several values of the capture voltage are tried to establish the minimum size for the containment contour, that is to say the voltage for the matched bucket.

Fig. 1 shows the phase space distribution of stacked bunches and the 95 % containment contour for the most successful trial. The coordinates are energy in MeV relative to the energy on the central orbit and rf phase divided by the harmonic number in degrees. There is a signature of success in this figure which trials have shown is related to minimum emittance, namely some particles in each bunch have been brought practically to $E = 0$ but none have crossed the axis. Various sets of parameters have produced results rather close to this, but any in which particles would cross the axis lead to very disrupted distributions.

There has been no systematic effort to optimize the single particle dynamics results with respect to all of the parameters. Figure 1 suggests that curves which deliver the bunches tangent to the axis with a voltage on each system not much greater than needed for the containing bucket are likely to be satisfactory. Furthermore, the fastest practical final approach appears to help also. This is about as far as the single particle dynamics can go. In anticipation of beam loading problems, however, it is helpful to select curves which do not reduce the cavity voltages unnecessarily. There is another suggestive feature of the distribution in Fig. 1. Notice that the symmetric acceleration-deceleration process has resulted in a complementary yin-yang shape for the components. This shape may reduce slightly the amount of empty phase space compared to what can be achieved bringing a moving bunch up to one at fixed energy. If it is important practically to hold one batch at fixed energy, the matching may need some detailed changes to minimize the penalty for the unsymmetric distribution.

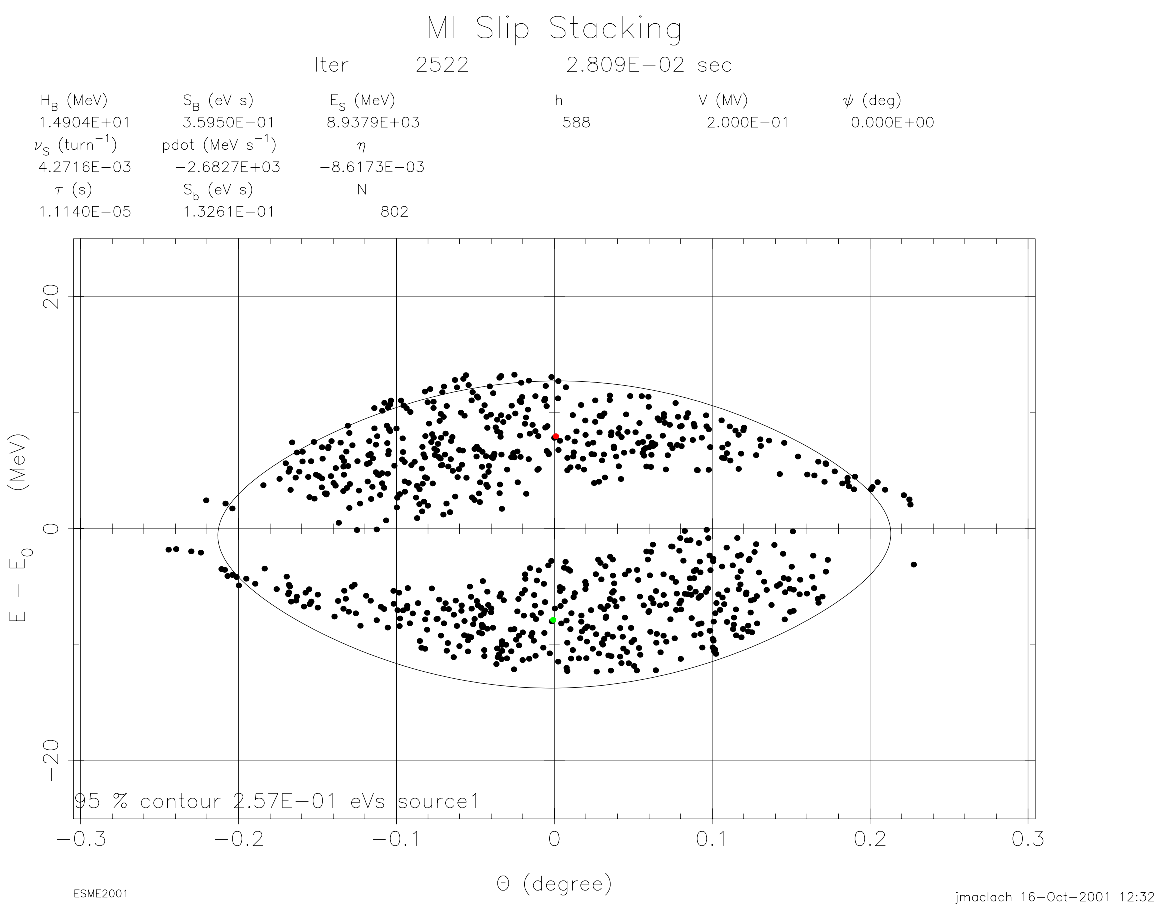

Figure 1: Single particle dynamics result for the phase space distribution for two bunches of 0.1 eVs combined by slip stacking. Machine and beam parameters are detailed in Table 2. Axes are rf phase divided by harmonic number (abscissa) and energy [MeV] with the origin on the central orbit (ordinate). 95 % of the bunch is within a matched contour of 0.26 eVs.

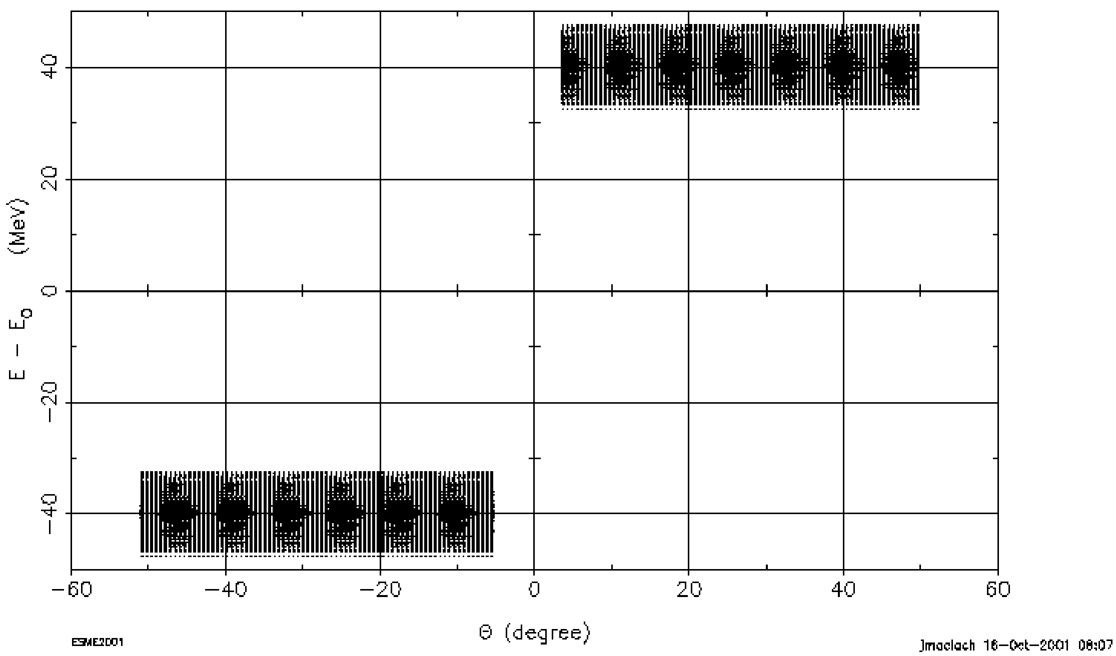

Figure 2: The initial distribution consisting of two 83 bunch batches separated in energy by 80 MeV. It is used in all of the following illustrations of the effect of the beam loading on slip stacking results.

Beam Current Effects

Two full-intensity Booster batches will contain about 10^{13} protons, constituting an average current of 140 mA. However, the peak current at the rf fundamental can be expected to fluctuate between zero and an ampere or so at a frequency which ranges from about 4 kHz to 670 Hz during the stacking. The beam space charge and interaction with vacuum chamber longitudinal impedances can be expected to be appreciable at this beam current, but far more critical is the control of the rf voltage to the ten kV level in the presence of MV levels of beam excitation. The initial distribution for all of the following results is shown in Fig. 2; two batches of 83 bunches 0.1 eVs each are separated by 80 MeV on centers and just shy of starting to overlap. If the A and B cavity groups are tuned for the separate batches, the beam loading voltage produced by all twenty cavities is 600 kV for the initial condition. A simplistic answer to how much suppression of this beam loading is required can be obtained by repeating the tracking calculation with the total $R_{\rm shunt}$ reduced by an arbitrary trial factor. This has been done with perfectly conducting wall impedance but no other longitudinal impedances. The result shown in Fig. 3 is comparable to that in Fig. 1 except that the charge of 10^{13} protons is taken into account with a factor 25 (28 dB) suppression of the beam loading. The effects of the beam current show up in the bunch shapes and therefore in the optimum capture voltage, but not in the emittance of the combined distribution.

What more can be said about the required beam loading compensation without having the specific characteristics of the feedback and/or feedforward systems? If there is ideal feedback around each cavity, the effect should be to de-Q the cavities by the factor $1/(G-1)$, but $R_{\rm shunt}/Q$ should remain unchanged. Fig. 4 shows the result of de-Q'ing by 25 (*not* 24). Obviously, 28 dB of feedback is not identical to uniform elimination of 96 % of the excitation. The reason the de-Q'ing leads to greater disruption than across the board attenuation is that more harmonics of the beam circulation frequency fall within the broadened resonance. Fig. 5, the corresponding result for 40 dB of feedback, is very satisfactory and is closely similar to Fig. 3 for the factor 25 overall attenuation. However, 40 dB of feedback over several circulation harmonics is not easy to come by. Some combination of correction at the rf fundamental plus less strong suppression of neighboring circulation frequency harmonics and feedforward correction may be required. Fig. 6 shows the time dependence of the beam loading voltage for this case. It varied between about 17 kV and 32

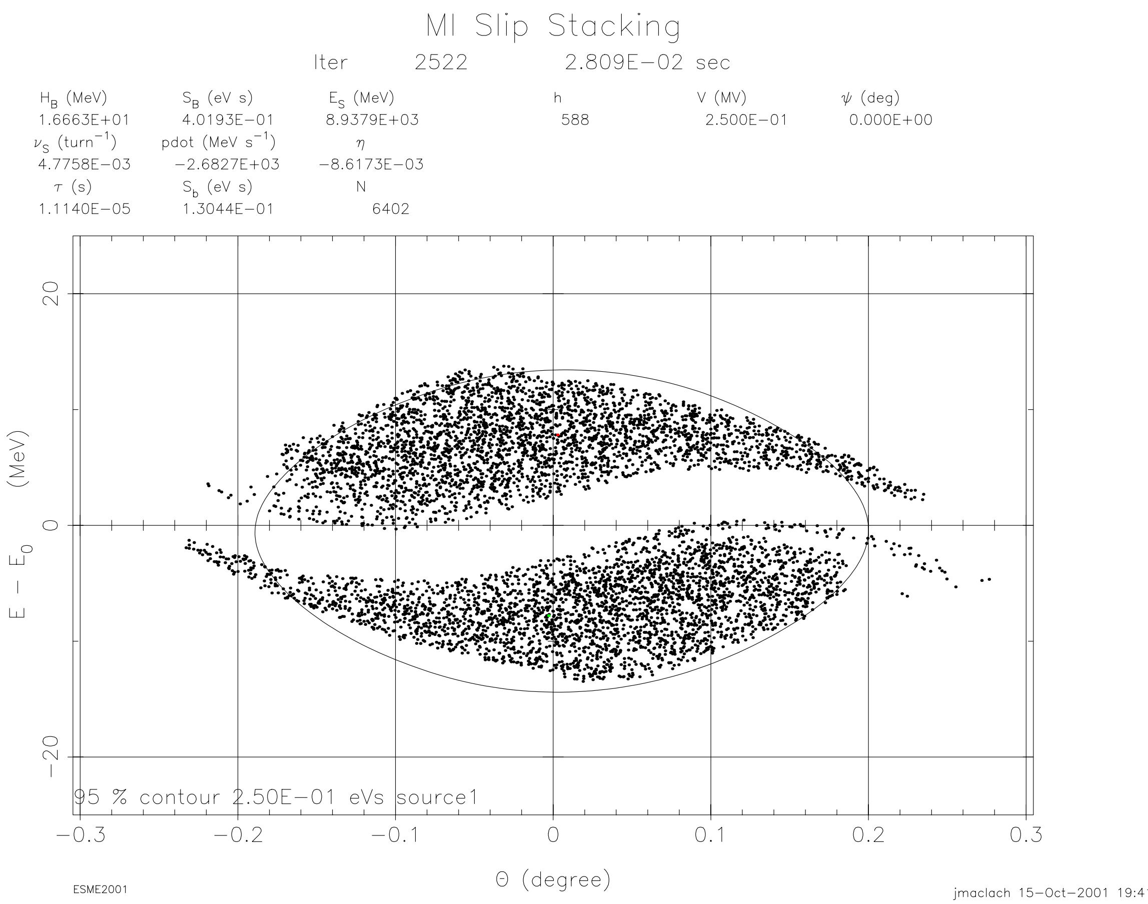

Figure 3: Comparable to Fig. 1 except that the beam loading voltage is reduced to 1/25 of the value it has naturally, corresponding to a highly idealized correction of 28 dB. A matched contour of 0.25 eVs contains 95 % of the bunch.

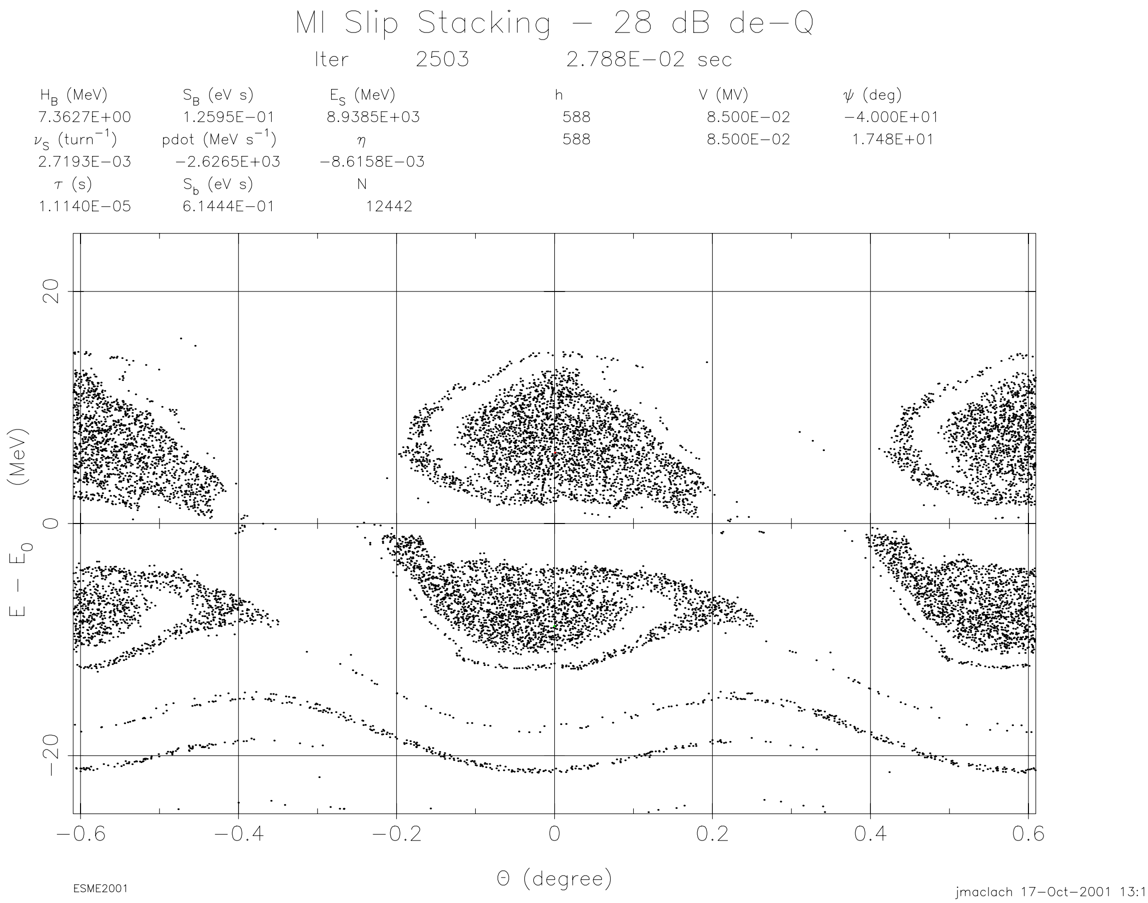

Figure 4: Comparable to Figs. 1 and 3, except that all 20 of the MI rf cavities have been de-Q'ed to correspond to an ideal feedback system with a gain of 26.

kV over most of the 28 ms process but dropped off somewhat in the final 8 ms. The rf voltage from each system was reduced linearly from 150 kV to 85 kV during the 28 ms; it fell below 100 kV at approximately 20 ms.

This note is not directed to engineering design of the feedback system. One can demonstrate by modeling that the beam loading can be reduced by about 20 dB by feedforward correction which delivers the bunch charges to the accelerating gaps with the correct timing but more or less arbitrary pulse form. If a feedback system can operate at the 20 dB level in an entirely independent fashion, the net correction would be sufficient. That independence, however, remains to be demonstrated. Both feedforward and feedback with one turn delay correct the beam loading from a particular turn with information derived from the previous turn. The objective of each system is to deliver to the gaps the current that cancels the beam current. However, the feedback error signal is derived from the rf voltage on the gaps whereas the feedforward signal is derived from a beam current pickup. When a particular system is adequately specified, the reaction of that system to a given beam current spectrum can be calculated directly. The calculated residual beam loading voltage on the cavity can be applied to the particles in a tracking calculation to see how the beam will behave for that system. If system performance can be represented in terms of the degree of suppression of particular circulation frequency harmonics in the gap voltage, it is very straightforward to demonstrate the resulting effect on the beam.

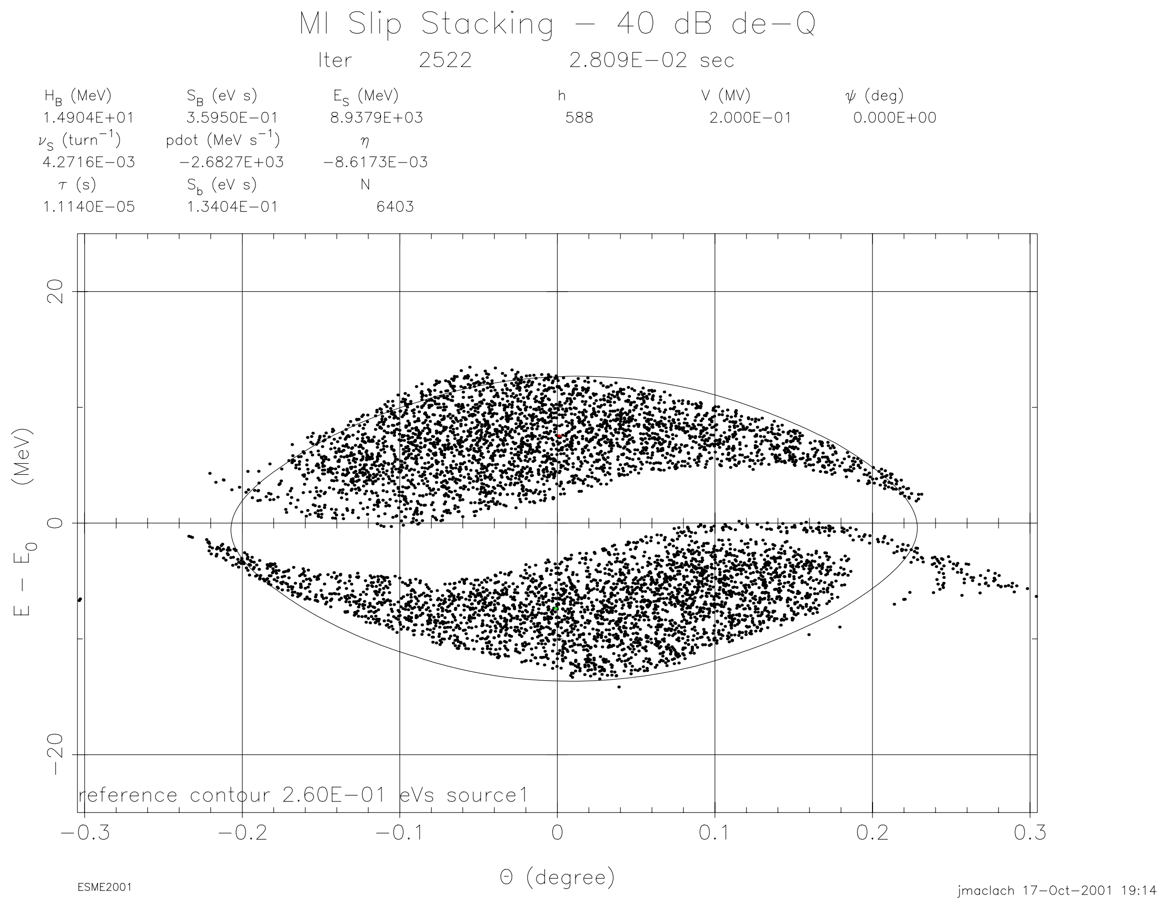

Figure 5: Comparable to Figs. 1,3, and 4, except that the MI cavities are de-Q'ed corresponding to ideal feedback with gain of 101. A matched contour of 0.26 eVs contains 95 % of the bunch.

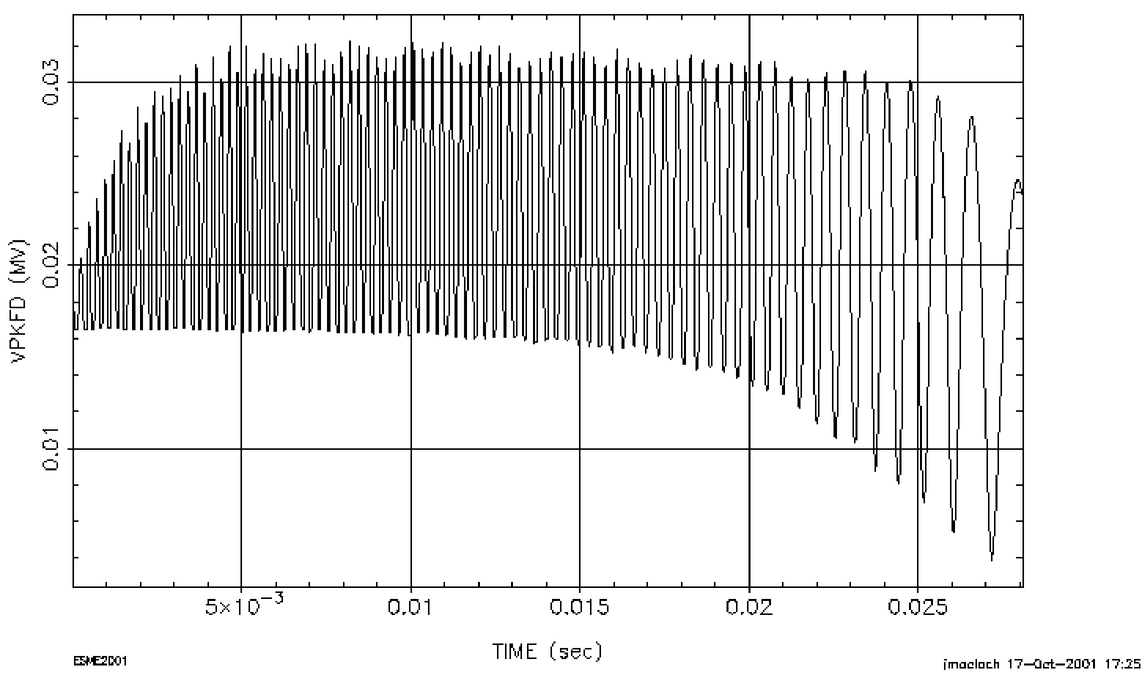

Figure 6: The beam loading voltage [MV] *vs*. time [s] during slip stacking with 40 dB feedback compensation; from the calculation which gave Fig. 5 for the combined bunch.

Summary

The dynamics of slip stacking have been examined for the case of combining two Booster batches in the Main Injector. Single particle dynamics were used to establish a practicable final energy separation for the bunches at the time of phase alignment. The requirements for beam loading compensation have been established in a somewhat idealized approach to feedback correction. There exist techniques (simulation code) for more complete treatment of the effectiveness of compensation once frequency domain properties of the compensating systems are known. The case of more than two batches was not treated, although relevant observations from the two batch case are not notably negative.

References

[1] C. Ankenbrandt, “Slip Stacking: A New Method of Momentum Stacking”, Fermilab internal note FN-532 (1981)

[2] D. Boussard and Y. Mizumachi, “Production of Beams with High Line-Density by Azimuthal Combination of Bunches in a Synchrotron”, IEEE Trans. Nucl. Sci., vol./ **NS-26**, No. 3 (June 1979)

[3] Shekhar Shukla, John Marriner, and James Griffin, “Slip Stacking in the Fermilab Main Injector”, in *Snowmass 96, New Directions for High Energy Physics,* Snowmass CO 25 June – 12 July 1996

[4] J. A. MacLachlan and J. F. Ostiguy, “User’s Guide to ESME 2001”, FERMILAB-TM-2132 (November 2000) Latest information at http://www-ap.fnal.gov/ESME

[5] F. E. Mills, “Stability of Phase Oscillations Under Two Applied Frequencies”, BNL Int. Rept. AADD 176 (1971)