Fermi National Accelerator Laboratory

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Jets and Associated Missing Energy

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Introduction

In considering the design of a calorimeter it is useful to have a lower bound defined by irreducible physical processes. This bound can be compared to those imposed by design choices over which one has some control. For example, how thick should a calorimeter be? A scale is set by comparing to the case of an ideal calorimeter of very great thickness. This device will itself "leak" energy in a manner similar to that of a thin calorimeter where energy leaks out the back due to insufficient hadronic shower containment. The irreducible leakage is due to processes whereby a gluon jet splits into a pair of heavy quarks which subsequently decay semi-leptonically. The processes, $g \rightarrow Q\bar{Q}$ and $Q \rightarrow qe\nu$ lead to energy being carried off by the neutrino and thus missing energy in the event. This process is one in which a basic physical limitation is placed on the achievable accuracy for the measurement of the energy of gluon jets.

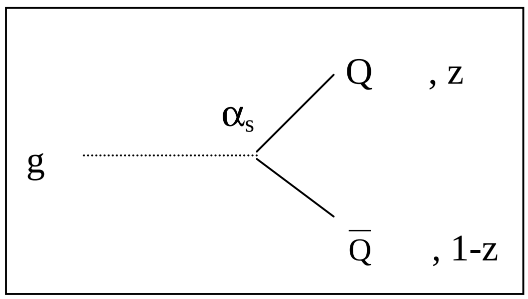

Figure 1: Lowest order splitting diagram for a gluon jet to virtually decay into a heavy quark-antiquark pair with a strength governed by the strong coupling constant. The variable z is the longitudinal fraction of the gluon momentum.

Gluon Jet Splitting into Heavy Quarks

In principle a full cascade of the gluon "shower" should be pursued down until the hadronization scale set by the parameter Λ which sets the scale of energy where the strong interaction become strong. At other energies the coupling constant can be found[1] by running it up from that scale to the scale Q.

$$\alpha_s (Q^2) = 12\,\pi/[33 - 2nf]\ln(Q^2/\Lambda^2) \tag{1}$$

In Eq. 1 nf is the number of "active" flavors, or those with mass < $\sqrt{\alpha^2}$.

In this note, we consider only the leading order process because we are only concerned with large missing energy contributions. The probability that a gluon of transverse momentum Pt has a virtuality M and that it splits[2] into a $Q\bar{Q}$ pair is:

$$dP/dzdM^2 \sim (\alpha_s / 4\pi)\,(1/M^2)[\,z^2 + (1-z)^2\,] \tag{2}$$

In Eq. 2, z is the fraction of the momentum of the gluon parent taken by the heavy quark.

Recall that in isotropic S wave decays, the distribution of secondary energies is ~ uniform. Therefore, the splitting can be thought of crudely as the ~ isotropic decay of a gluon of mass M > 2 M_Q into a $Q\overline{Q}$ pair. The center of mass velocity, β, and the resulting limits on the laboratory momentum fraction z are:

$$\beta = \sqrt{1 - (2M_Q/M)^2} \tag{3}$$

$$z_{min} = (1-\beta)/2, \quad z_{max} = (1+\beta)/2$$

In the case where $M \rightarrow 2 M_Q$ the threshold behavior $\beta \rightarrow 0$, $z \rightarrow$ ½ obtains, while in the case where $M >> 2 M_Q$ the familiar zero mass quark result, $0 < z < 1$ is found. In this latter case, the quarks uniformly populate all momenta from zero to the full parent momentum, Pt.

The doubly differential probability can be integrated over all decay configurations to yield:

$$dP/dM^2 \sim (\alpha_s / 48\pi)(1/M^2)[(1+\beta)^3 - (1-\beta)^3] \tag{4}$$

$$\rightarrow (\alpha_s / 6\pi)(1/M^2)$$

$$\rightarrow (\alpha_s / 8\pi)(1/M^2)\,\beta$$

Note that the virtuality M is distributed as 1/M. This factor favors low gluon "mass" and hence leads to suppression of the splitting probability. In the case where $\beta \rightarrow 0$, the threshold behavior that $dP/dM \sim \beta$ is observed, while if $\beta \rightarrow 1$, the zero mass quark result is found. In this latter case, a second integration yields the total probability to split into a $Q\overline{Q}$ pair.

$$P \sim (\alpha_s (2M_Q)^2 / 6\pi)[\ln(P_t / 2M_Q)^2] \tag{5}$$

In what follows, a numerical integration of Eq.4 is used to give a more accurate representation of the threshold suppression of the total probability. Typically, this results in a two fold reduction in the estimated probability compared to the results obtained using Eq.5. The results of this integration are shown in Fig.2 as a function of the transverse momentum of the gluon jet. Typically, there is a 6% chance for a gluon to split into a cc pair, and a 4% chance to split into a bb pair.

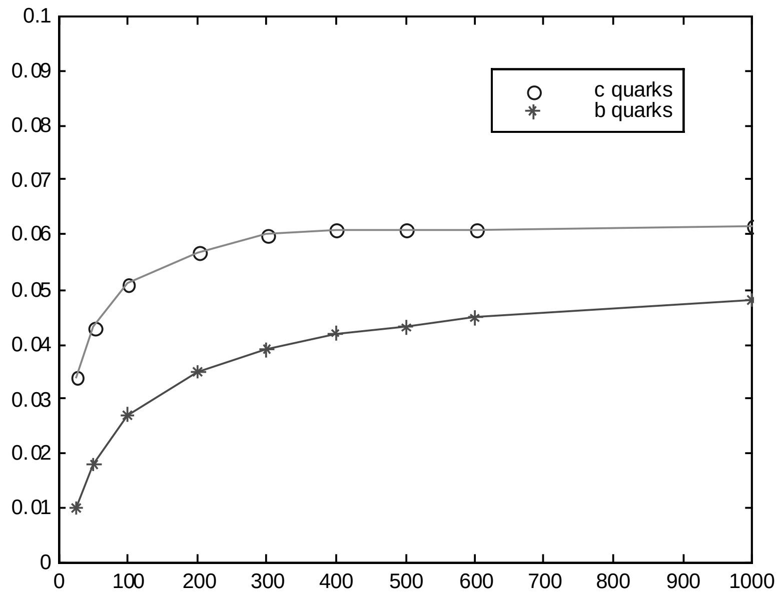

Figure 2: The probability for a gluon jet to split into a c or b pair of quarks as a function of the Pt of the gluon jet.

The decay mode Q → qeν leads to missing energy and no telltale muon to indicate the presence of the neutrino. For c decays, we average D^+ and D^0 and assign a 12 % branching ratio, while for b we take the B average and assign a 5% branching ratio.

For the decay V-A matrix element weighting[3] is used in the approximation of zero mass leptons. It is important to use the correct matrix elements because the decay is, for example, b → c W, W →eν. Thus the c take off most of the energy because of the quasi two body nature of the decay. For example, a 100 GeV b yields a 28 GeV e and ν on average, while the c takes off 43 GeV on average. The (V-A) weight is applied to the uniform Dalitz plot, dxdy. The center of mass maximum momenta are used to scale the lepton and neutrino energies E_l and E_ν. Mo is the quark mass in Eq.6. The decay rate is Γ and the familiar limit for zero mass final state quarks is also given in Eq. 6.

$$x = E_l/E_{lmax} \qquad (6)$$

$$y = E_\nu/ E_{\nu max}$$

$$d\Gamma/dxdy \sim 2x\,[Mo/ E_{lmax} - y - x(E_{lmax} /E_{\nu max})] - Mo /E_{lmax}(1 - y)$$

$$\rightarrow 2x\,[2 - y - x] + 2(y - 1)$$

For each gluon jet the probability to find a heavy quark times the semi-leptonic branching ratio was evaluated. If a decay was chosen, the full V-A weighting was applied and the neutrino energy taken off was recorded and subtracted from the gluon energy.

Estimate of Gluon Missing Et

Very roughly one can estimate the effect of gluon splitting on missing energy. The c have a 6% probability and a 12% branching ratio, while the b have a 4% probability and a 5% branching ration. Hence, we expect 1.8% decays which take off 14% of the energy on average. Therefore, in a 500 GeV jet, we expect that the probability to have 70 GeV missing energy is roughly 0.018 due to this splitting process.

The results of the Monte Carlo program described above are given in Fig.3. The leading order splitting probabilities are evaluated by numerical integration. The distributions of M and z are taken as shown in Eq. 2. If a splitting occurs the 3 body semi-leptonic decay is performed with V-A weighting as prescribed above. The missing energy is taken to be the neutrino energy. There are no resolution effects assumed due to the calorimetry.

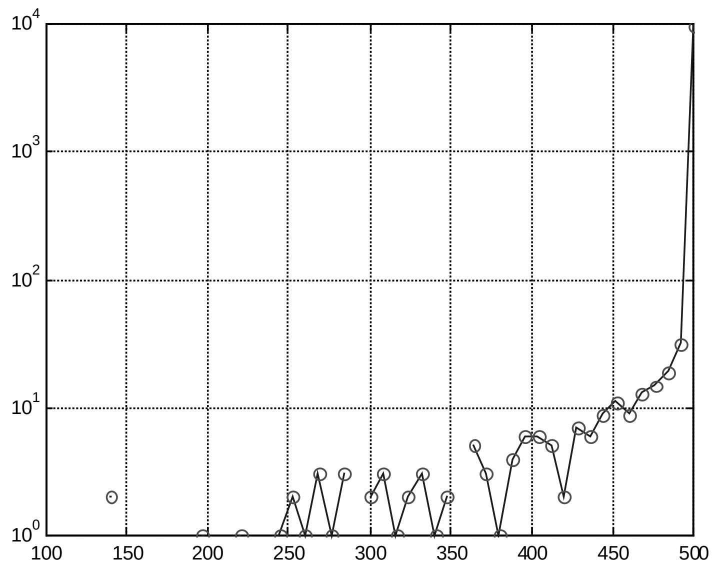

Figure 3: Distribution of visible jet energy for 10,000 gluon jets of 500 GeV

The data given in Fig. 3 are shown with a different presentation in Fig. 4. The number of jets out of 10000 with a missing energy of at least Et is shown. Note that the probability to observe 70 GeV missing energy is ~ 0.007, where we “guess – estimated” 0.018 above. Thus, the Monte Carlo work is in rough agreement with a back of the envelope estimation.

The results given in Fig. 4 are intrinsic to the mutability of the objects being measured. The size of the effects shown here can serve as a benchmark against which to assess the problem of leakage due to insufficient depth for containment or other instrumental biases.

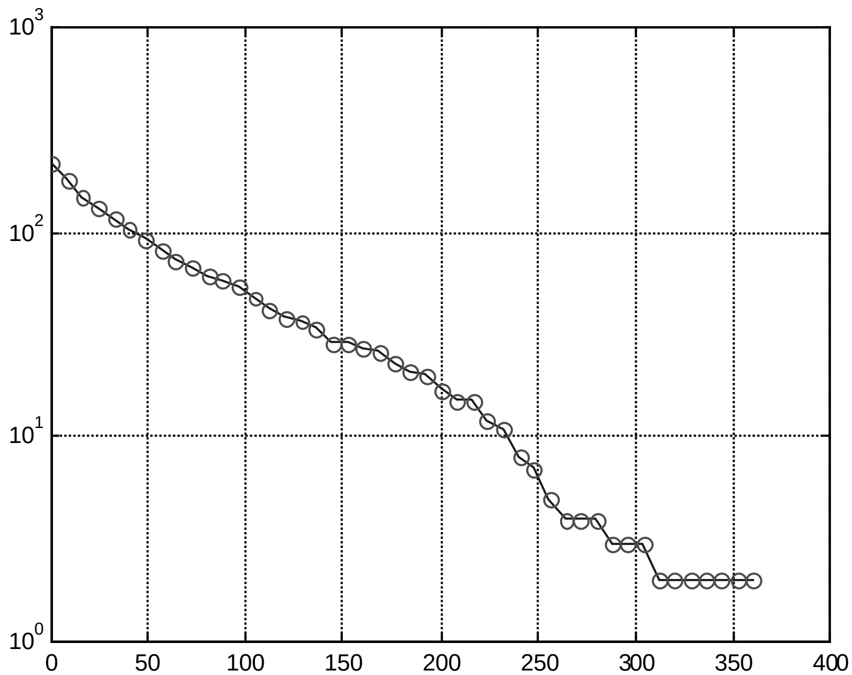

Figure 4: Number of events with missing energy greater than or equal to Et out of a total of 10,000 gluon jets of 500 GeV. There is a 1% probability of at least 50 GeV and a 0.1% chance to observe ~ 250 GeV.

Decays in Flight, in Showers, in Jets

In addition to gluon splitting, there is the issue of jet leakage due to decays in flight or pion decays in the hadronic showers themselves. The problem requires quite a bit of care to assess completely. We content ourselves here with back of the envelope calculations to make a first look at the potential severity of the problem.

The jet will fragment into an ensemble of particles, the leading one having <z> ~ 0.2 on average. Only that particle will have a chance to contribute substantially to missing jet energy. If there is a free path for tracking of ~ 1m and if the pion is the leading fragment of a jet of 500 GeV, then the decay in flight has a probability ~ (1m/7.6m)(0.14 GeV/100 GeV) or ~ 0.00018. Note that there is a telltale muon in the event, and it takes the bulk of the energy, 78 GeV as compared to the 21 GeV of the neutrino. Compared to the Monte Carlo estimates of Fig.4, this decay in flight appears to be negligible.

In order to roughly evaluate decays of pions in an hadronic shower, we construct a very simple model. Each new "generation" occurs at one absorption length increased depth in the shower. Each generation makes <n> particles, with neutral fraction fo = 1/3. The neutrals "freeze out" rapidly since the radiation length is much shorter than the absorption length and only the charged pions transport the energy of the shower to the next generation. The expression for the number of charged shower particles, nc, the energy of the shower particles, e, and the neutral energy eo, as a function of generation number, $\nu = z/\lambda$, where z is the calorimeter depth, are given in Eq.7. The incident hadron energy is E.

$$nc = (1 - fo)^{\nu - 1}\langle n\rangle^{\nu} \tag{7}$$

$$e = E / \langle n\rangle^{\nu}$$

$$eo / E = fo(1 - fo)^{\nu - 1}$$

The shower attains a maximum when pion multiplication is no longer kinematically possible, at $E_{th} \sim 2\, m_{\pi} = 0.28$ GeV. When the process $\pi + P \rightarrow \pi + \pi + P$ falls below threshold. That defines the generation number at shower maximum, ν_{max}.

$$e = E_{th} \text{ when } \nu = \nu_{max} \tag{8}$$

Using these expressions, we can evaluate the probability, P, of a decay at each depth in the hadronic shower. The charged pion lifetime is τ, and its mass is m.

$$P = nc\ (\lambda / c\tau)\ (m/e) \tag{9}$$

For example, using Eq. 7, 8 and 9 we can evaluate a 500 GeV gluon with $\langle n\rangle = 9$. At the first generation, e ~ 53 GeV and the total probability for these first generation jet fragments to decay in the calorimeter before interacting is ~ 0.0005. A glance at Fig.4 shows that this is not the dominant effect. We conclude that gluon splitting dominates the intrinsic energy leakage.

In order to attach these estimates to some real data[4] we avail ourselves of “punch-through” data for tracks exiting a very thick calorimeter. The data is given in Fig.5. It is clear that there is a hadronic component which falls off with a scale set by the absorption length. Deep in the shower there is also visible another component, which rises steeply with energy. We assert that this component is due to muons from the decay of shower pions.

The scale for the probability effect is from 0.001 to 0.01. Note that any particle which exits the calorimeter registers. For example a 5 GeV muon will ionize and penetrate the full calorimeter depth. Thus, these soft muons/neutrinos will not contribute substantially to the missing energy.

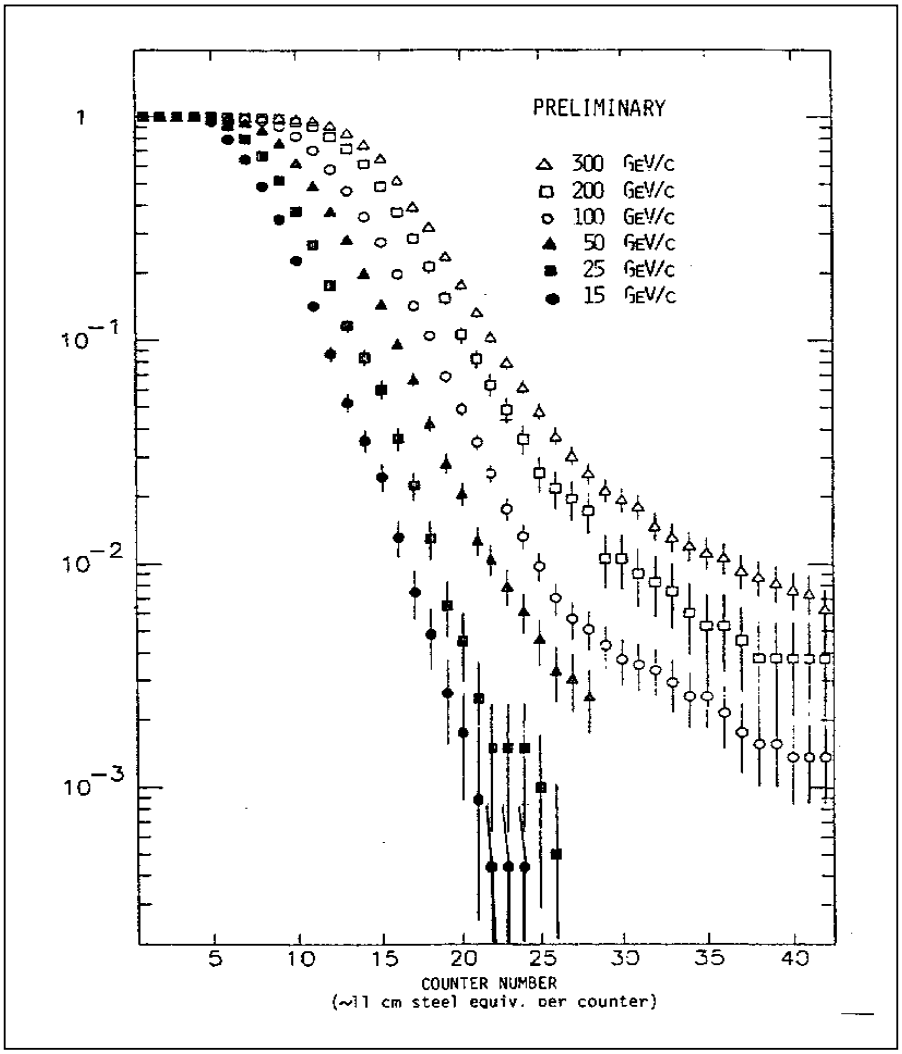

Figure 5: Punch-through probability for incident hadrons from 15 to 300 GeV as a function of depth in the calorimeter

Note that since there are $<n>^{\nu}$ fragments with a γ time dilation factor ~ $1/<n>^{\nu}$ we expect a rapid rise with incident energy. The expressions given in Eqs. 7-9 were used to find the decay probability for 5 GeV shower particles as a function of incident hadron energy. The 5 GeV energy was chosen since at that energy a muon decay will penetrate deep into the calorimeter. The results are shown in Fig.6.

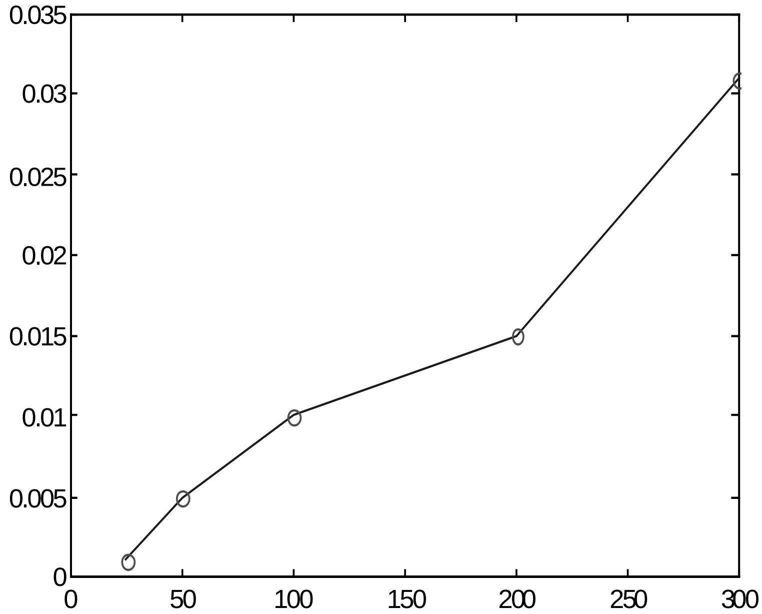

Figure 6: Shower pion decay probability for ~ 5 GeV pions as a function of the energy of the incident hadron.

Comparing the results of the simple model shown in Fig.6 to the data of Fig. 5, it appears that we can have some confidence in the order of magnitude of the results of the simple model. Therefore, we conclude with some assurance that the major effect in calorimeter leakage is gluon splitting into heavy quarks with subsequent semi-leptonic decays.

References

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