Fermi National Accelerator Laboratory

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# Jets, Radiation and Calorimetry

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# Jets, Radiation, and Calorimetry

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## Introduction:

The calorimetric measurement of quarks and gluons relies on the spatial localization of the jet of hadrons associated with the fundamental entities. In turn, this association is made by assuming that the jet fragments are localized in a region of kinematic variables close to the direction of the initial parton. Typically, a jet is defined by a cone in space delimited by R.

$$R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} \qquad (1)$$

In the expression for R, the pseudorapidity $\eta = \ln[\tan(\theta/2)]$ is an approximation to rapidity y for jet fragments with masses which are small with respect to the transverse momentum of the fragment. Note that 1 particle phase space is simply

$$d^4P\, \delta(P^2-M^2) \sim d\mathbf{P}/E \sim dy d\varphi dPt^2 \qquad (2)$$

Thus R is a natural variable to use to localize the jet fragments since they are assumed to have a limited transverse momentum, Pt, with respect to the parent parton.

## Radiation Kinematics:

This simple picture obtains in the absence of radiation. In the case where the parton can radiate, there is always a finite probability for a very wide-angle radiated particle. Consider the radiation of a massless particle by a parton of "virtuality", or squared mass, of to resulting in a particle of final virtuality t, where to > t. The kinematics is shown in Fig.1, where the initial parton energy is Eo; the final parton energy is E1. The radiated massless gluon is emitted with angle $\theta$ with respect to the final state parton and it has energy $\omega$.

The "splitting" is specified by $\omega$ = (1-z)Eo, E1 = zEo. Energy and momentum conservation leads to the relation, when z ~ 1, for the gluon emission angle

$$\theta^2 \sim (to-t)/[Eo^2(1-z)z] \qquad (3)$$

Therefore, the emission angle is known once the initial and final parton virtualities are known and the splitting momentum fraction, z, is specified. Note that the kinematics implies "angular ordering". The high virtuality partons emit at wider angles, in general, than those at reduced virtuality.

**Radiation Dynamics:**

The emission of a single gluon has been understood for a long time [1]. There is a finite probability for the emission of a significant fraction of the parton energy outside any reasonable cone radius R. For the process, e + e → e + e + g the fraction of events which have a fraction ε within a cone of half angle R is

$$F = 1 – [4\alpha_s(t) /3\pi][\ln(R)(3 + 4\ln(2\varepsilon) + \pi^2/3 – 5/2] \qquad (4)$$

Note that the QCD coupling constant is evaluated at the virtuality, t, of the initial parton and the only process considered is 2 → 3 scattering. A plot of F(t,R,ε) is shown in Fig.2. Note that F is substantially different from 1 for contained energy fractions > 0.8 and cones sizes < 0.5.

In fact, the problem is worse than that expected from single emission. The radiation of the parton has to be followed down a chain of final state virtualities, rather like an electron shower. The problem is sketched out in many texts on QCD [2]. We consider only the simple case of a quark splitting into a quark plus a gluon. The probability for that to occur is taken to be

$$P \sim [\alpha_S/2\pi][dt/t][4/3(1+z^2)/(1-z)] \qquad (5)$$

The probability for a given splitting is proportional to the QCD coupling constant, is weighted towards low t and z ~ 1. Thus we expect the emission to favor soft gluons. Note that the softer gluons, Eq.3, are emitted at wide angles, while the harder ones have small emission angles.

**Shower Development:**

A cutoff zc is defined for which the emitted gluon is too soft to be of consequence, zc < z < 1-zc. A sum in leading log approximation over multiple gluon emission leads to the probability that no emission occurs from virtuality to down to virtuality t.

$$P(t|to) \sim [\alpha_S(t)/\alpha_S(to)]^{-\gamma/4\pi b} \qquad (6)$$

$$\gamma = (1/2\pi)\int(1+z^2)/(1-z)dz$$

In Eq.6 b is the QCD value, $b = (1/48\pi^2)(33 - 2f)$, and the QCD mass scale cutoff where QCD becomes strong is $\Lambda$ where $\alpha_S(t) = 1/[4\pi b\ln(t/\Lambda^2)]$. We use the value for $\Lambda$ of ~ 0.25 GeV. This formulation correctly takes into account the growth of the coupling constant as the shower drops ever lower in virtuality. Note that P(t|to) is small if to – t is large. It is extremely unlikely to drop a long way in virtuality without emitting a gluon.

**Monte Carlo Model:**

A simple Monte Carlo can then be written which retains the main QCD features. In principle after the first emission both the emitted gluon and the remaining parton have virtualities and they should be followed as they evolve. However, as the virtualities are ~ zt and (1-z)t and as the splitting favors soft gluons, it is roughly true that the gluon is massless, z ~ 1 and only the quark need be followed as the shower develops.

The Monte Carlo uses a fixed value of $\Lambda$ and has as input the cutoff for soft gluons, zc ~ 0.01. The initial virtuality t is chosen to be $Pt^2$. A cutoff virtuality where perturbation theory no longer applies and jet fragmentation should be inserted is taken to be tc ~ 1 $GeV^2$ which is > $\Lambda^2$. The shower develops by first choosing a value for t using Eq.6. A simplifying assumption that $1 + z^2 \sim 2$ in the integral for $\gamma$ is made in order that t can be chosen analytically. Since z ~ 1 the approximation is justified.

If t < tc the shower ends. If not a value of z is chosen from zc to 1-zc weighted by the splitting function (again with $1 + z^2 \sim 2$). All the virtuality t is given to the final state quark. The emission angle R ~ θ follows from the values of t, to and z (Eq.3). The quark is followed through subsequent shower generations until if falls below virtuality tc.

**Monte Carlo Results:**

Some sample showers for 1 TeV jets are shown in Fig.3. All parton shower particles, quarks and gluons, are shown with their energy and emission angle ~ R. Note that the sum of all shower particle energies = 1 TeV. These randomly chosen four jets illustrate the fact that radiation may, indeed, send significant energy outside the cone radius R defining the jet since R cannot be made arbitrarily large in the presence of other jets, the underlying event, and other pileup events.

The contour for a full shower analogous to that for 2 → 3 only shown in Fig.2 is given in Fig.4. Obviously, the situation is worsened by the inclusion of all the steps in the parton shower. The basic shape of the two contours is the same. There is a long radiative tail in the energy distribution inside any reasonable cone size, say R < 0.7. Within a cone of that size there is a non-negligible fraction of events where enough energy is lost that the calorimetric measurement of energy does not define the error in the energy.

In addition, the energy, on average, within a cone of R = 0.7 is measurably less than the parton energy, due to radiation. For example, at R = 0.7 this simple Monte Carlo model indicates that half the events have an energy < 80% of the parton energy. Thus the mean mass would be expected to shift down by ~ 20%. This fact implies that jets must be “calibrated” in situ in order to set a reasonable mass scale for dijet resonance searches.

In a “complete” Monte Carlo study using PYTHIA [3], significant effects due largely to final state radiation were seen for dijets. A 1 TeV dijet resonance was found to have a 7% mass resolution with R = 0.7 without radiation but with jet fragmentation and calorimetric energy resolution applied. With radiation “turned on” the mean mass shifted down by ~ 10% and the width increased to 13.5%. These observations lead to the conclusion that radiation will place serious limitations on jet spectroscopy in the LHC experiments.

**References:**

[1] G. Sterman and S. Weinberg, Phys. Rev. Lett. 39, 23, 1436 (1977)

[2] V. Barger and R. Phillips, Collider Physics, Addison-Wesley (1987)

[3] K. Maeshima et al., Dijet Mass Resolution of the CMS Calorimeter, CMS Note (1988)

**Figure Captions:**

Figure 1: Emission kinematics for a parton of initial energy Eo, virtuality to Emit a gluon of energy $\omega$ at an angle $\theta$ with respect to final state parton of energy E1 and virtuality t.

Figure 2: Contours of the fraction of 100 GeV jets containing a fraction of the initial parton energy within a cone of radius R.

Figure 3: Four quark “showers” of 1 TeV energy. The scatter plot is the shower particle energy vs. the emission angle R.

Figure 4: Contours of the fraction of 1 TeV jets containing a fraction of the initial parton energy within a cone of radius R using a simple Monte Carlo shower model.

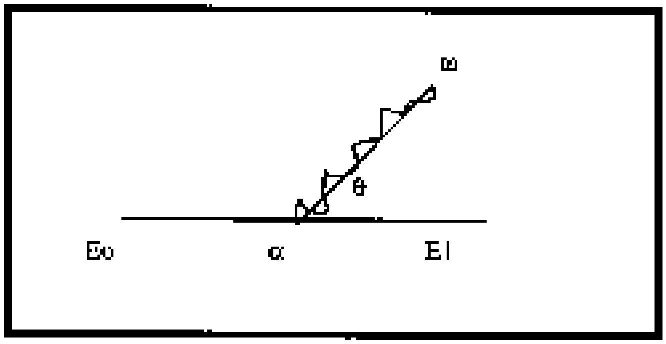
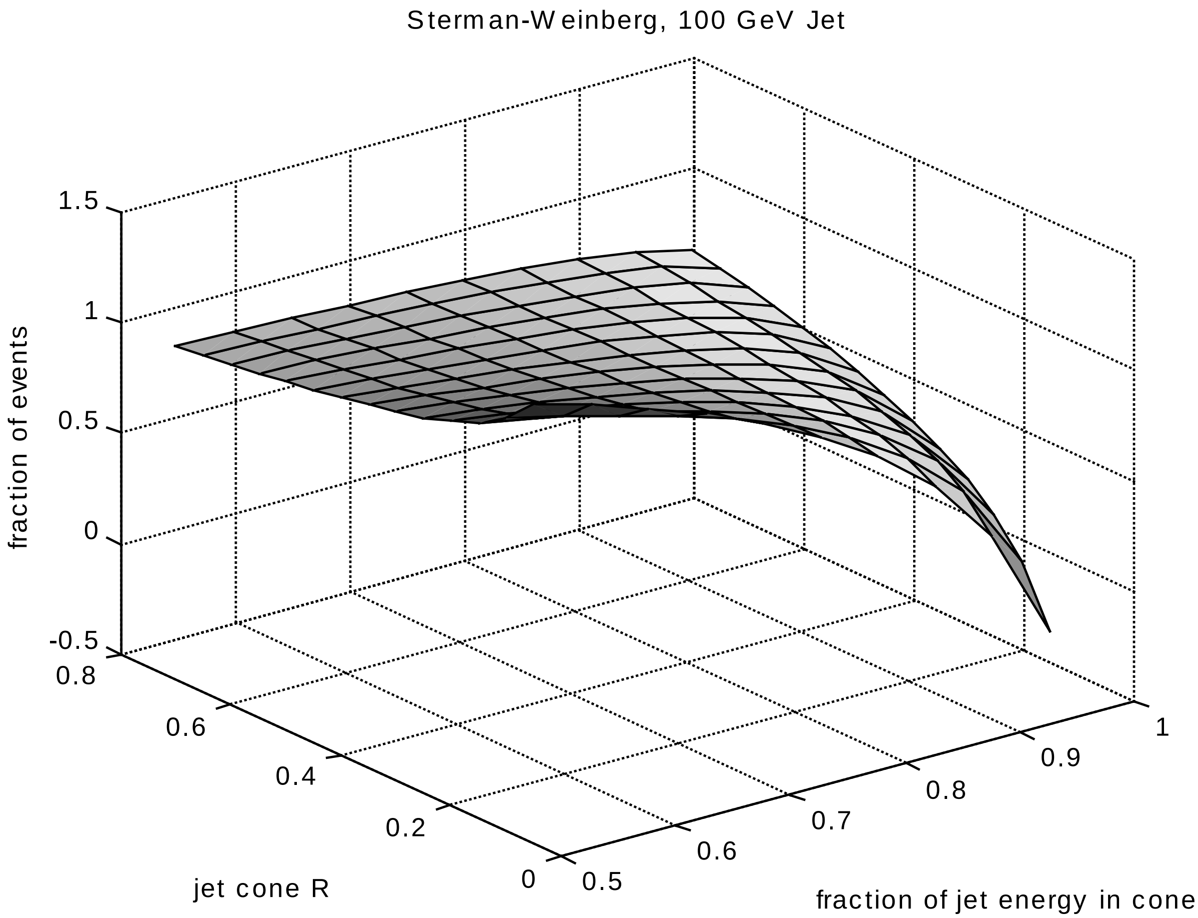



Figure 1



Sterman-Weinberg, 100 GeV Jet
1.5
1
0.5
0
-0.5
fraction of events
0.8
0.6
0.4
0.2
0
jet cone R
0.5
0.6
0.7
0.8
0.9
1
fraction of jet energy in cone


Figure 2

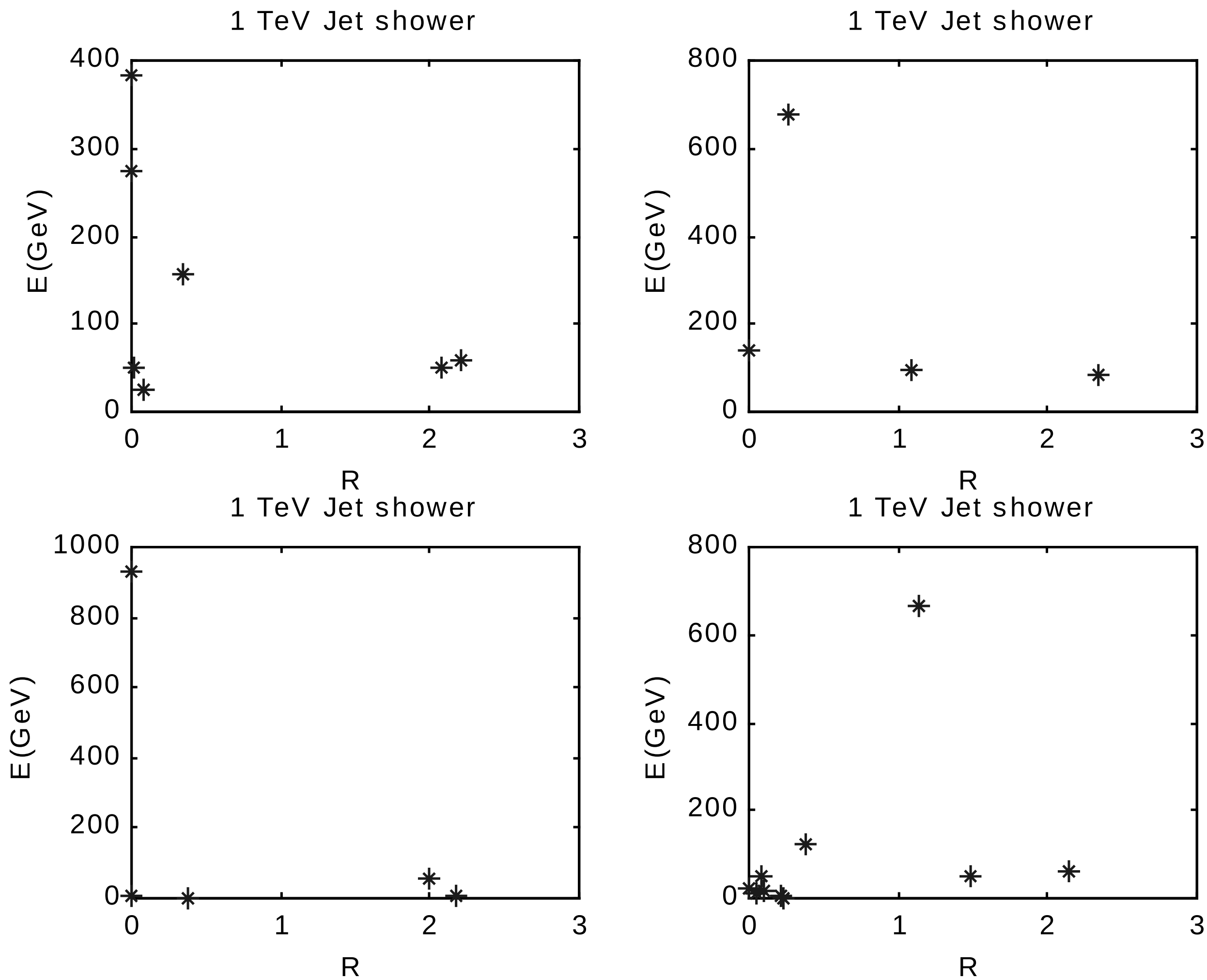
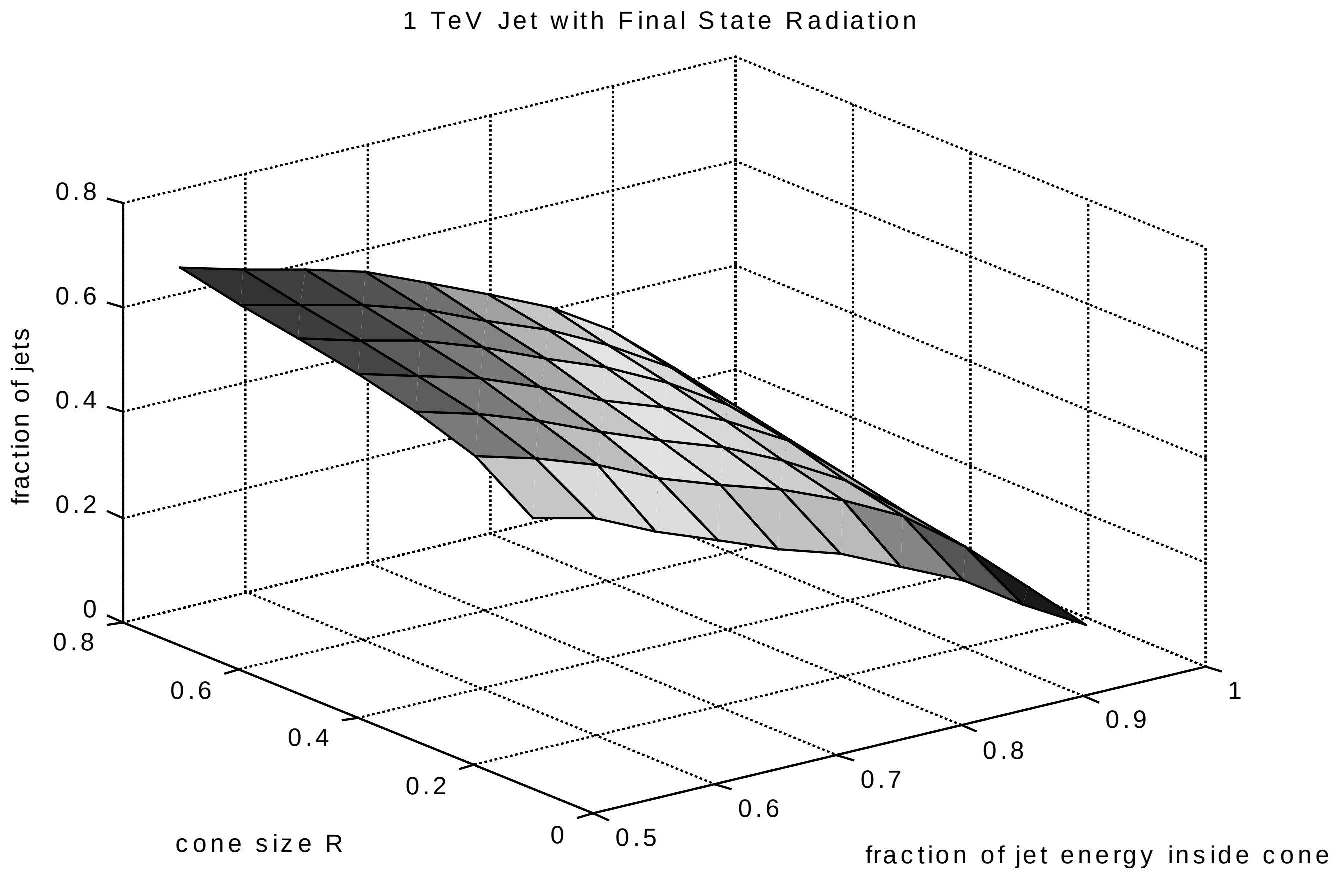

1 TeV Jet shower
E(GeV)
R
0
100
200
300
400
0
1
2
3
1 TeV Jet shower
E(GeV)
R
0
200
400
600
800
0
1
2
3
1 TeV Jet shower
E(GeV)
R
0
200
400
600
800
1000
0
1
2
3
1 TeV Jet shower
E(GeV)
R
0
200
400
600
800
0
1
2
3

Figure 3



1 TeV Jet with Final State Radiation
fraction of jets
0.8
0.6
0.4
0.2
0
0.8
0.6
0.4
0.2
0
cone size R
0.5
0.6
0.7
0.8
0.9
1
fraction of jet energy inside cone


Figure 4