Fermi National Accelerator Laboratory

FERMILAB-FN-563

Future Collider Runs in the Tevatron: Beam-Beam Simulation Results

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April 1991

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1 Introduction

Future collider runs at Fermilab will be fundamentally different from the first (86-87) and the second (88-89) runs. Starting with the third (1992) run, closed orbits of protons and antiprotons will be helically separated everywhere in the Tevatron except at interaction regions B0 and D0. The reason for beam separation is related to the problem of beam-beam interaction. In the 88-89 run, 6 antiproton bunches were made to collide head-on with 6 proton bunches at 12 crossing points symetrically distributed around the ring, resulting in a tune shift (and a tune spread) of 0.024. This beam-beam tune spread covered the area between the 5^{th} and the 7^{th} order resonances which have relatively large resonance widths in the Tevatron. Beam separation reduces the tune shift by eliminating unnecessary head-on collisions. The tune space between the 5^{th} and the 7^{th} order resonances can be filled again by increasing the beam brightness. Therefore, the purpose of beam separation is to increase luminosity.

1.1 Luminosity

The ultimate objective in collider operation is to maximize the integrated luminosity. So far most of the efforts have gone into improving the initial luminosity. To maximize the integrated luminosity one also has to improve the luminosity lifetime. Towards this end, a lot of work was done on controlling the emittance growth in the Tevatron[1]; there are also plans to employ such methods as bunched beam stochastic cooling[2] to reduce the transverse emittances of colliding proton and antiproton

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bunches. The initial luminosity is given by the expression

$$L = \frac{3\gamma f B N_p N_{\bar{p}}}{\beta^*(\varepsilon_p + \varepsilon_{\bar{p}})\pi} F(\sigma_z/\beta^*) \tag{1}$$

where γ is the relativistic factor, f is the revolution frequency, B is the number of bunches per beam, N_p and $N_{\bar{p}}$ are respectively the number of protons and antiprotons per bunch, β^* is the beta function at the interaction point (assumed equal for horizontal and vertical planes), ε_p and $\varepsilon_{\bar{p}}$ are the proton and antiproton 95 % normalized emittances respectively (here also assumed equal for horizontal and vertical planes), and F is a form factor associated with the ratio of the bunch length to beta function at the interaction point.

To improve the initial luminosity one can increase the beam intensities N_p, $N_{\bar{p}}$, and decrease β^* and the emittances. Achieving maximal luminosity improvement demands nothing short of an upgrade of all systems in the accelerator chain. Currently, the Fermilab Linac is being upgraded from 200 MeV to 400 MeV to reduce space-charge effects in the Booster, various other improvements for the Booster are under way, the Antiproton Source has a new stochastic cooling system, and there are plans to improve the $\bar{p}$ production rate. A new ring, the Main Injector, has been proposed to replace the Main Ring. The dynamic aperture of the Main Ring at injection energy of 8 GeV is smaller than the beam size, therefore it presents a bottleneck in the accelerator chain. The Main Injector should also boost the proton intensity to $33 - 50 \times 10^{10}$ particles per bunch in the Tevatron. The new low-beta quadrupoles around B0 and D0 will be capable of reducing the β^* to 0.25 m (current value 0.50 m).

1.2 Why helical?

Upgrade scenarios based only on beam brightness improvement and β^* reduction schemes will hit a dead end. There are two fundamental limits. The first is the so-called beam-beam limit. In hadron colliders, the beam-beam limit refers to the limited resonance-free area in the tune-resonance diagram. No matter where you choose the working point, if you keep improving the beam brightness (N_p/ε), the tune spread due to beam-beam interactions will sooner or later be so large that you will run out of workable tune space.

The second limit comes from intensity-dependent beam instabilities. This limit has not been reached in the Tevatron yet but one can foresee it happening in the near future. Therefore the orders of magnitude enhancement in luminosity cannot be facilitated by simply increasing N_p/ε and reducing β^*. The frontier in hadron colliders is multi-bunch operation. Orders of magnitude enhancement in luminosity can only be achieved by increasing the number of bunches.

Proton and antiproton beams can be separated in one plane (horizontal for instance) or in two planes (helical). In the 1-D separation scheme the flexibility is limited because bunch encounters must be avoided where the beam separation is small. In the helical separation there is a great deal of flexibility regarding beam-crossing points and the number of bunches. The number of bunches, is limited by the number of buckets in the Tevatron and the necessity for abort gaps. Other limits include detector capabilities, and the requirement that the distance from detector to separator should be free of bunch-bunch encounters. In the 1992 Collider Run, we will still operate with 6 bunches per beam. Run-IV will feature 36 bunches per beam, and the ultimate multi-bunch ($p\bar{p}$) operation foreseen[3] for the Tevatron proposes 108 bunches per beam. Possible bunch loading scenarios for the Tevatron are explained in Ref.[4].

1.3 New Problems

How much separation?

The helical separation scheme makes it possible to increase the initial luminosity greatly but also introduces a unique set of accelerator physics problems. The first question that comes to mind is: How much separation is sufficient (from the beam dynamics point of view)? This number has to be smaller than the limit set by the maximum field strength that can be achieved in the separators with a tolerable sparking rate (1 spark per week).

Feed-down effects:

Another issue is the feed-down effects. For example, the Tevatron lattice includes chromaticity correcting sextupoles which are centered around the old design orbit. When beams are separated they go through these sextupoles off-axis and experience a quadrupole field (hence the jargon "feed-down"). If left uncorrected the tunes of protons and antiprotons would move in opposite directions and may cross dangerous resonances. A separate circuit of sextupoles has been installed to correct this[5].

Are field errors important?

There is also the issue of field errors. Uniformity of the dipole field in bending magnets gets worse as the beam goes farther off axis. Separated beams in their new helical orbits will experience stronger systematic field errors which will drive certain resonances. Although these resonances are expected to be weak, the effects should be calculated for better understanding[6].

Beam-Beam Resonances :

Another important question is the strength of beam-beam resonances. The beam-beam interaction can be viewed as a special beam-line element containing many multipoles. If it is a head-on interaction there is no dipole term (all even poles vanish), and to first order it acts like a quadrupole for small amplitude particles. It differs from a regular quadrupole in that it is focusing in both planes whereas a regular quadrupole focuses in one plane and defocuses in the other. The beam-beam interaction also produces coupling. If it is a long-range interaction there will be a dipole term which will distort the closed orbit.

The multipole content of the beam-beam interaction will be more apparent for the large amplitude particles. Due to the terms higher than r^2 in the expansion of the beam-beam kick, the tune shift will depend on amplitude. In most cases amplitude dependence can be beneficial since it provides a mechanism for resonance detuning. On the other hand the same amplitude dependence can also cause a particle to fall into a resonance.

Beam-beam resonances by themselves are not strong enough (in the Tevatron) to cause resonance overlap but if there are side-bands due to various tune modulation mechanisms then the probability of resonance overlap is greatly enhanced. A quantitative assesment of this situation, particularly for the 12^{th} order resonance, has been given[7] by applying the analytic theory of tune-modulated beam-beam resonances to the Tevatron.

In this paper ...

In this paper we present a simulation analysis of beam-beam interactions in future Tevatron Collider Runs. We will:

(1) find the optimum lattice tunes (working point)

(2) calculate the tune shifts and spreads (and the tune density distributions)

(3) discover scaling laws and predict the beam-beam effects for the high luminosity collider operation

2 The Model

The simulation code HOBBI[8] implements a weak-strong model of the beam-beam interaction in which a time- invariant (transverse) gaussian proton distribution is assumed. A single antiproton is tracked around the ring and each time it encounters a proton bunch the Montague[9] form of the beam-beam kick is applied. The test particle (antiproton) is called the "probe" and the proton bunches are called the "source".

2.1 The beam-beam kick

The beam-beam kick due to a single interaction with a round beam is given by

$$\Delta x' = -\left(\frac{4\pi\xi}{\beta_x}\right)\left(2\frac{\sigma^2}{r^2}\right)\left(1 - e^{-\frac{r^2}{2\sigma^2}}\right)\cdot(x + d_x) \tag{2}$$

$$\Delta y' = -\left(\frac{4\pi\xi}{\beta_y}\right)\left(2\frac{\sigma^2}{r^2}\right)\left(1 - e^{-\frac{r^2}{2\sigma^2}}\right)\cdot(y + d_y) \tag{3}$$

with

$$r^2 = (x + d_x)^2 + (y + d_y)^2 \tag{4}$$

where x and y are the horizontal and vertical displacements of a probe particle from its own closed orbit, and d_x (d_y) is the horizontal (vertical) distance between the centers of the source bunch and the probe bunch. ξ is the so-called "linear tune shift" parameter which is given by

$$\xi = \frac{N r_p}{\pi(\varepsilon\beta\gamma)} \tag{5}$$

for a round beam. β_x (β_y) is the value of the amplitude-function at the interaction point, σ is the r.m.s. width of the source bunch, ε is the source bunch emittance, and

N is the source bunch intensity. r_p is the classical proton radius and finally, β, γ are the relativistic factors. At the beam energy of 1 TeV the numerical value is

$$\xi = 0.00733 \frac{N\ [10^{10}]}{\varepsilon\ [\pi\text{mm} - \text{mr}]} \tag{6}$$

2.2 HOBBI

The simulation code HOBBI[8] is a module containing many subroutines (functions) written in C++. HOBBI module also contains functions related to displaying, archiving and re-constructing the 4-D phase-space on an Evans & Sutherland (PS390) graphics terminal via AESOP [10] shell. This version of HOBBI is running on the VAX-ALMOND cluster. There is a "C" language version running on the UNIX-CARTOON cluster. HOBBI.C does not contain the AESOP related functions.

- The input to HOBBI is a file containing the lattice information at beam-crossing points, separator locations, and at arbitrary user-defined points around the ring. This file also contains the linear transfer matrices between the beam-crossing points and other information such as the separator kicks (in μrad), tunes, emittances, energy, etc. The HOBBI input file will be refered to as the "lattice" henceforth.
- HOBBI was originally designed to be an interactive program for exploratory orbit analysis in the presence of beam-beam interactions. All the beam parameters including tunes can be changed during an interactive session, providing the flexibility to explore the entire parameter space. Another useful feature of HOBBI is that it tracks either a test proton against (source) antiproton bunches or a test antiproton against (source) proton bunches. HOBBI can treat synchrotron oscillations, dispersion effects and tune modulation.
- The output of HOBBI is a 4-D array of normalized phase-space variables. Numerous other calculations are performed internally by HOBBI such as tune shifts, separations etc. These are declared as global variables so that other programs linking with the HOBBI module can access them. An application program must include (#include) the necessary header files. In the case of HOBBI.C these header files are named "hobbi.h" and "hobbi.structures".

2.3 Lattices used in this study

We used three different lattices in this study. The lattice described in file "6x1bd.dat" contains the collider configuration for the 1992 run. The input file "34x1bd.dat" describes the Collider Run with the Main Injector. The third file "34x1.dat" describes a situation where there are no head-on interactions (all 68 beam-crossings involve long-range interactions). The beam parameters contained in these files are listed in Table 1.

	6x1bd.dat	34x1bd.dat	34x1.dat
Energy [TeV]	0.9	1.0	0.150
Number of proton bunches	6	34	34
Number of low-beta insertions	2	2	0
Number of beam-crossings (for a given antiproton)	12	68	68
Number of head-on interactions	2	2	0
Number of long-range interactions	10	66	68
protons/bunch [$\times 10^{10}$] (nominal)	10	30	30
proton emittance [πmm-mr] (nominal)	15	30	30
antiproton emittance [πmm-mr] (nominal)	14	22	22
β^* (at B0,D0) [m]	0.5	0.5	

Table 1: These numbers represent the initial configurations as they exist in the files. Everything can be changed during a HOBBI session.

3 Colliding Beams Sequence

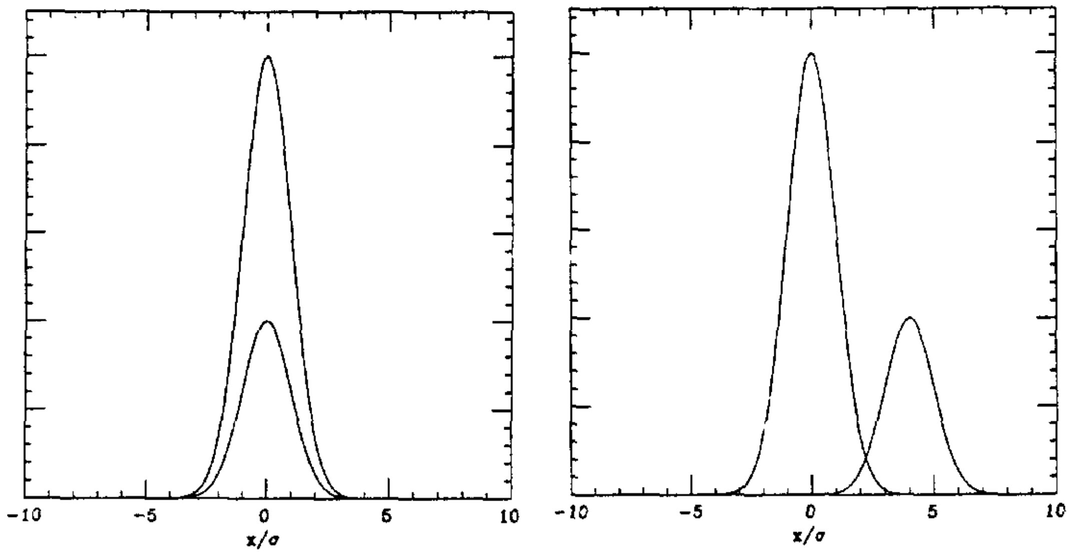
The motivation for developing separation criteria comes from the practical problem of keeping protons and antiprotons safely separated during the colliding beams sequence. The procedure for colliding beams in the helical separation scheme is a nontrivial one. It goes as follows:

1) Proton bunches are injected into the Tevatron at 150 GeV. After the injection bump is removed, proton bunches circulate on a "smoothed" orbit. The orbit smoothing is necessary for two reasons. First, if the closed-helix (orbit with separators off) deviates from the center of magnets by large amounts (10 mm) at certain places then the opened-helix (orbit with separators on) will sample nonuniform dipole fields at those places which can be detrimental to beam lifetime. Secondly, the feed-down circuit is designed with the assumption that the closed-helix is going through the centers of the magnets.

2) The proton helix is opened by powering one separator in each plane.

3) Antiprotons are injected onto the helix. The reason for injecting antiprotons directly onto the helix is to avoid head-on beam-beam interactions which would cause a significant tune shift in a multi-bunch operation. At this stage, all beam crossing points including B0 and D0 involve long-range interactions and the separator voltages should be such that the antiproton beam samples only weak nonlinear fields.

4) Protons and antiprotons are accelerated to 900 GeV on the helix. Separator voltages will not simply scale with energy because the beam size shrinks; hence separation rules are needed to design the "separator-ramp".



Head-on beam-beam interaction   Long-range beam-beam interaction

Figure 1: Transverse profiles for source and probe bunches.

5) After stabilizing at flat-top (900 GeV), beams will be squeezed to low-beta at B0 and D0. During the squeeze the separation of beams will necessarily decrease, particularly at B0 and D0, beam-beam collisions will involve nonlinear interactions before they settle down to head-on interactions. This transition through the nonlinear region requires a careful analysis and a simulation study. Motivated with this need we now turn to the discussion of the "separation parameter" .

4 Separation Parameter

Fig.(1) illustrate the difference between long-range and head-on beam-beam interactions. The distance between the centers is d and the conventional (dimensionless) separation parameter is given by $D \equiv d/\sigma_{source}$. Here "source" refers to the bunch with higher intensity; in the case of the Fermilab Collider it is the proton bunch. The "probe" refers to the weaker bunch (antiproton). This terminology is also suggestive of the weak-strong model where the source (strong) bunch creates the field and the probe (weak) bunch samples it. The source bunch distribution is not changed by the probe beam.

The other assumption in the model is that the beam-beam interaction takes place in such a short time that for all practical purposes it is a kick (only the direction of the probe particle changes, not its instantaneous position). The mathematical expressions for the horizontal and vertical kicks are given above (Eq.(2) and Eq.(3)). Here we want to illustrate the nonlinear nature of the beam-beam kick. Fig.(2) shows the kick experienced by an oppositely charged probe particle as it passes by a round gaussian source bunch. If the probe particle has the same charge as the source

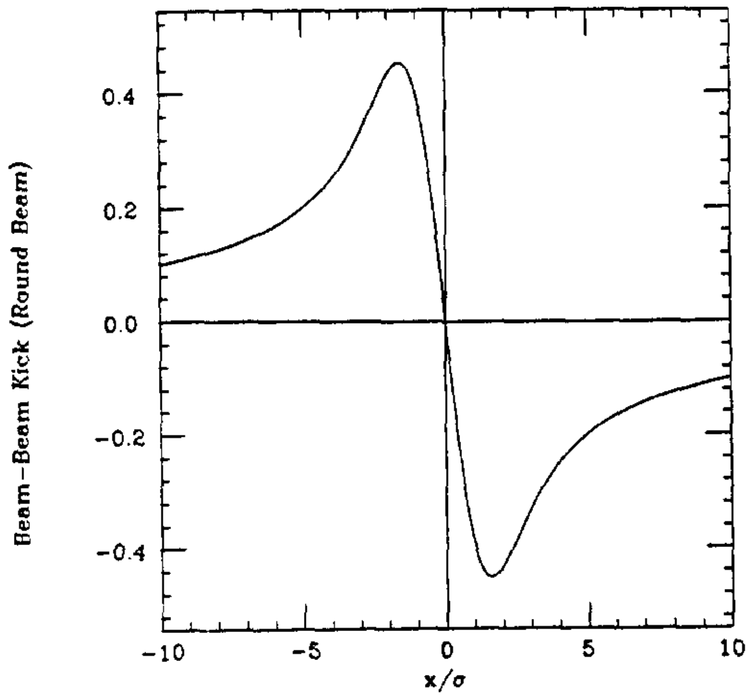


Figure 2: The kick experienced by an oppositely charged probe particle as it passes by a round gaussian source bunch.

bunch the kick curve will be mirror symmetric of the one in Fig.(2). Note that for $-1 < x/\sigma < 1$ the beam-beam kick is fairly linear. It is strongly nonlinear between $1 < x/\sigma < 2$ and $-2 < x/\sigma < -1$ and also exhibits nonlinearity for $|x/\sigma| > 2$ as the strength of the kick decreases asymtotically ($\sim 1/r$).

The strongly nonlinear region of the beam-beam kick is to be avoided. This gives us a simple separation criterion,

$$D > 5 \tag{7}$$

This criterion is also supported by the operational experience at the CERN Sp$\bar{p}$S Collider and by early simulation efforts[11] studying the phase-space structure of beam-beam resonances.

The $D > 5$ criterion is illustrated in Fig.(3). It can be extended to 2-D. The usual practice is to replace D with

$$D \equiv \sqrt{\left(\frac{d_x}{\sigma_x}\right)^2 + \left(\frac{d_y}{\sigma_y}\right)^2} \tag{8}$$

One of the objectives in this paper is to reduce the dimension of the parameter space by 2. With this objective in mind we use the following 2-D extension of D

$$D \equiv \sqrt{\frac{(d_x{}^2 + d_y{}^2)}{\sigma_x \sigma_y}} \tag{9}$$

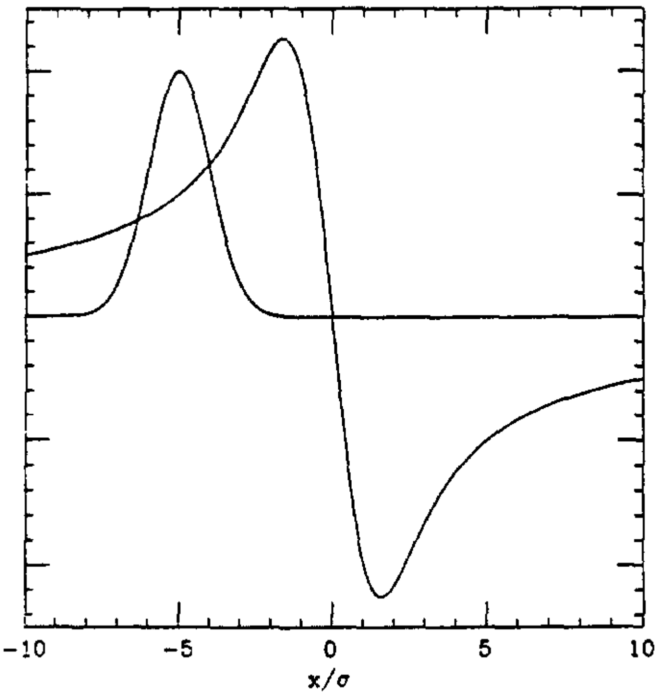


Figure 3: $D = 5$ separation

which has the effect of making the source beam rounder. For nearly round beams, such as the beams in hadron colliders, we propose Eq.(9) as the D parameter. For flat beams Eq.(8) should be used.

The above definitions Eq.(8) and Eq.(9) for D are obviously independent of the amplitude. We need a separation parameter which is also a function of the probe particle amplitude. We propose

$$S \equiv \sqrt{D^2 - 2DA + A^2} \; = \mid D - A \mid \tag{10}$$

where A is the normalized amplitude relative to the center of the probe bunch. The parameter S should be interpreted as the "effective amplitude" since the probe particle oscillates around a closed orbit which is separated from the source beam by D, the difference $|D - A|$ gives the "effective" amplitude relative to the source bunch. The instantaneous amplitude relative to the source bunch can be greater or smaller than S depending on the separation. We take the absolute value of the difference because a negative amplitude would not be physicaly meaningful.

In Eq.(10), D is the new 2-Dimensional parameter given in Eq.(9) and A is a parameter dependent on the probe particle amplitude

$$A \equiv \sqrt{\frac{(a_x^2 + a_y^2)}{\sigma_x \sigma_y}} \tag{11}$$

where a_x and a_y are the probe particle amplitudes (in physical units). The parameter A can be written in another way

$$A \equiv \sqrt{A_x^2 \left(\frac{\sigma^2_{x\_\mathrm{probe}}}{\sigma_x \sigma_y}\right) + A_y^2 \left(\frac{\sigma^2_{y\_\mathrm{probe}}}{\sigma_x \sigma_y}\right)} \tag{12}$$

where

$$A_x \equiv \frac{a_x}{\sigma_{x\_\mathrm{probe}}} \quad , \quad A_y \equiv \frac{a_y}{\sigma_{y\_\mathrm{probe}}}$$

If there are many beam-crossing points then we simply take the average of S over the beam-crossing points. Henceforth we will use S and $< S >$ interchangably.

4.1 Separation Rules from qualitative analysis

The advantage of using the parameter S is that one can develop sound rules for separation, given emittances and separator voltages, without going into simulation. The conclusions from this qualitative analysis will be checked with simulation in the following sections.

The strongly nonlinear regions of the beam-beam kick are to be avoided. This simple criterion gives us the separation rules in terms of S.

$$\begin{aligned} 1 < S < 2 &\quad \text{to be avoided} \\ S > 2 &\quad \text{acceptable} \\ 0 < S < 1 &\quad \text{acceptable} \end{aligned} \tag{13}$$

This suggests that $S > 2$ is the preferred situation. There are times and places when S cannot be made larger than 2, such as when the beam is squeezed to low-beta, and then the preferred situation is $0 < S < 1$. The low-beta squeeze steps can be designed with this criterion in mind. The parameter S will be different for different amplitudes. Here we list the preferred conditions for the Colliding Beams Sequence in terms of $S(A_x, A_y)$. These rules will be refined according to simulation results.

$$\begin{array}{rll} \text{Open} - \text{Helix at 150 GeV} & S(A_x = 3, A_y = 3) > 2 & \\ \text{During Ramp} & S(A_x = 3, A_y = 3) > 2.5 & \text{since tunes move a lot} \\ \text{Open} - \text{Helix at 900 GeV} & S(A_x = 3, A_y = 3) > 2 & \\ \text{Low} - \text{Beta Squeeze} & S_{\mathrm{B0}}(A_x = 3, A_y = 3) > 2 & \text{Initial squeeze steps} \\ & S_{\mathrm{D0}}(A_x = 3, A_y = 3) > 2 & \\ & S_{\mathrm{B0}}(A_x = 1, A_y = 1) < 1 & \text{Final squeeze steps} \\ & S_{\mathrm{D0}}(A_x = 1, A_y = 1) < 1 & \end{array} \tag{14}$$

5 Measuring the Nonlinearity

In any analysis it is important to find the natural parameter relevant to the particular problem being studied. When analytical methods are employed one tries the technique of "change of variables" to come up with a simpler or familiar set of equations. The analogous technique in simulation analysis is to devise "measures" such as "entropy","Hausdorff dimension", "Lyapunov exponent", "smear", etc. to isolate a particular aspect of the problem.

Our particular problem is the nature of long-range interactions. What is the contribution of long-range interactions to beam lifetime, to emittance growth, to background noise in the detectors? How much separation will be sufficient? What are the scaling laws relevant to long-range interactions? To be able to answer these questions one needs to develop a "measure" of badness. The "smear" [12] for example is a measure of nonlinearity which can be calculated for a particle with a given amplitude by tracking the particle around the ring for a few hundred turns. "Smear" is used as a criterion for long-term particle behaviour.

In this paper we develop another "measure" of nonlinearity with particular emphasis on long-range beam-beam interactions. With the help of this "measure" we devise separation rules and deduce scaling laws regarding long-range interactions.

5.1 χ-S and η-S Diagrams

In the following sections, as in this one, we will use HOBBI as the core of various simulation programs. These special purpose programs are linked with the HOBBI object module. The program we use in this section is called HOBBIX. It plots χ versus S for a mesh of amplitudes where χ is S averaged over turns.

$$\chi = < S >_{\mathbf{turns}} \tag{15}$$

HOBBIX also plots

$$\eta \equiv (\chi - S)/S \tag{16}$$

η is a measure of nonlinearity since χ would be equal to S in a linear system. Fig.(4) and Fig.(5) demonstrate the power of χ-S and η-S diagrams. As we predicted in section 4.1, the region of $1 < S < 2$ exhibits a high degree of nonlinearity. If there were no beam-beam interactions, the χ-S curve would have been a 45° straight line and the η-S curve would have been identically zero for all S.

To isolate the effects of long-range interactions, in Fig.(6) and Fig.(7) we plot χ-S and η-S for two different configurations (1992 Collider Run, Collider Run with the Main Injector) and for three different conditions a) no beam-beam interactions, b) only two head-on interactions at B0 and D0, c) all beam-beam interactions included. These figures show that the nonzero value of η is caused by long-range interactions only. Head-on interactions contribute very little to η and only when the probe particle amplitude is such that the resulting S value is between 1 and 2.

Another observation from Fig.(6) and Fig.(7) is that the η value for small amplitude (probe) particles is not zero when long-range interactions are turned on. This means that the small amplitude particles are suffering from nonlinearities. Using this clue and plotting η as a function of N_p, ϵ_p, and the number of long-range interactions, we will deduce separation rules and scaling laws in section 7.

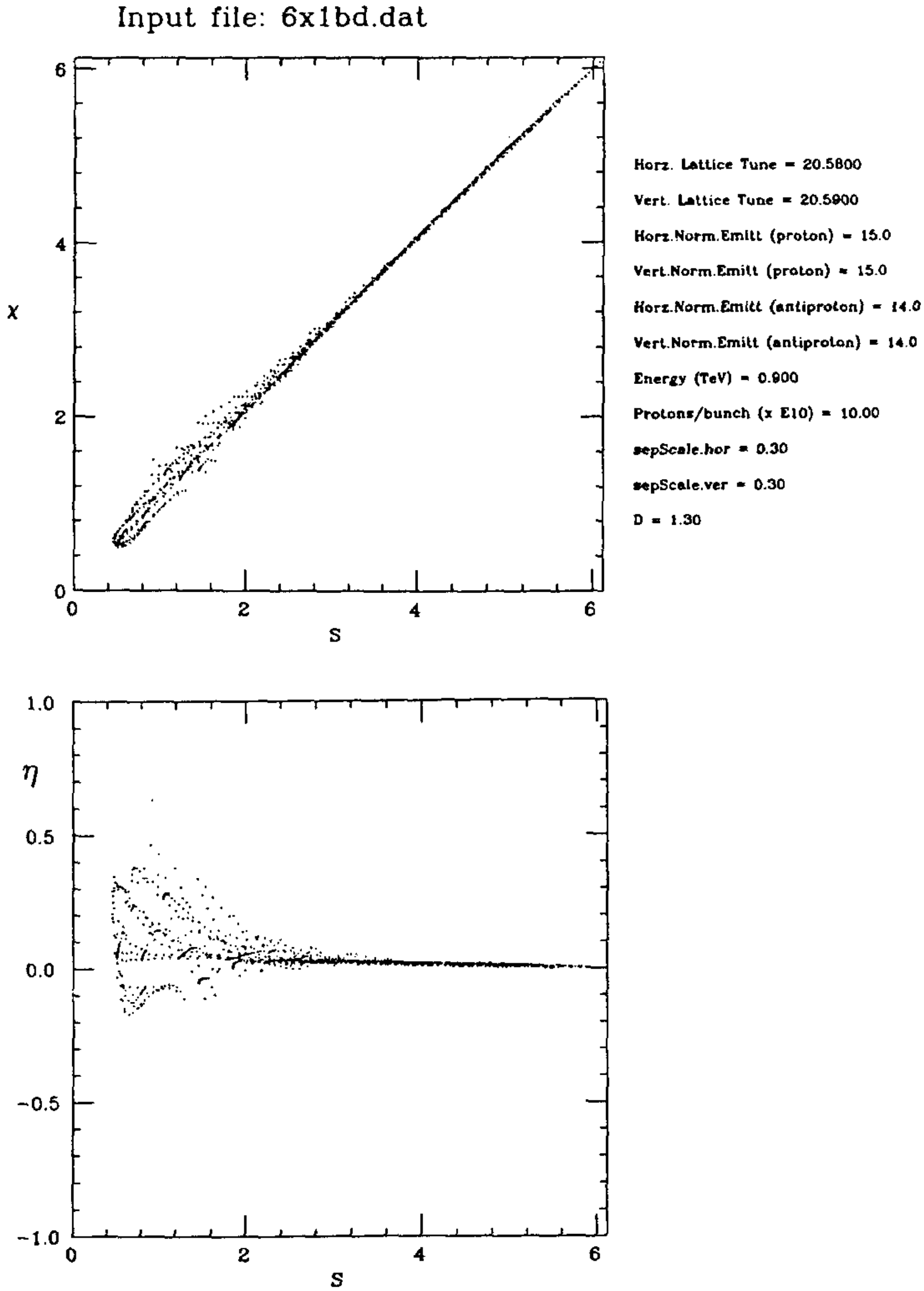


Figure 4: Diagrams showing the effect of nonlinearity for $1 < S < 2$. In a perfectly linear system these plots would be straight lines. Normalized deviation of S from its initial value as the particle circulates around the ring is caused by nonlinearity due to beam-beam interactions. The diagram has been generated from a mesh of amplitudes.

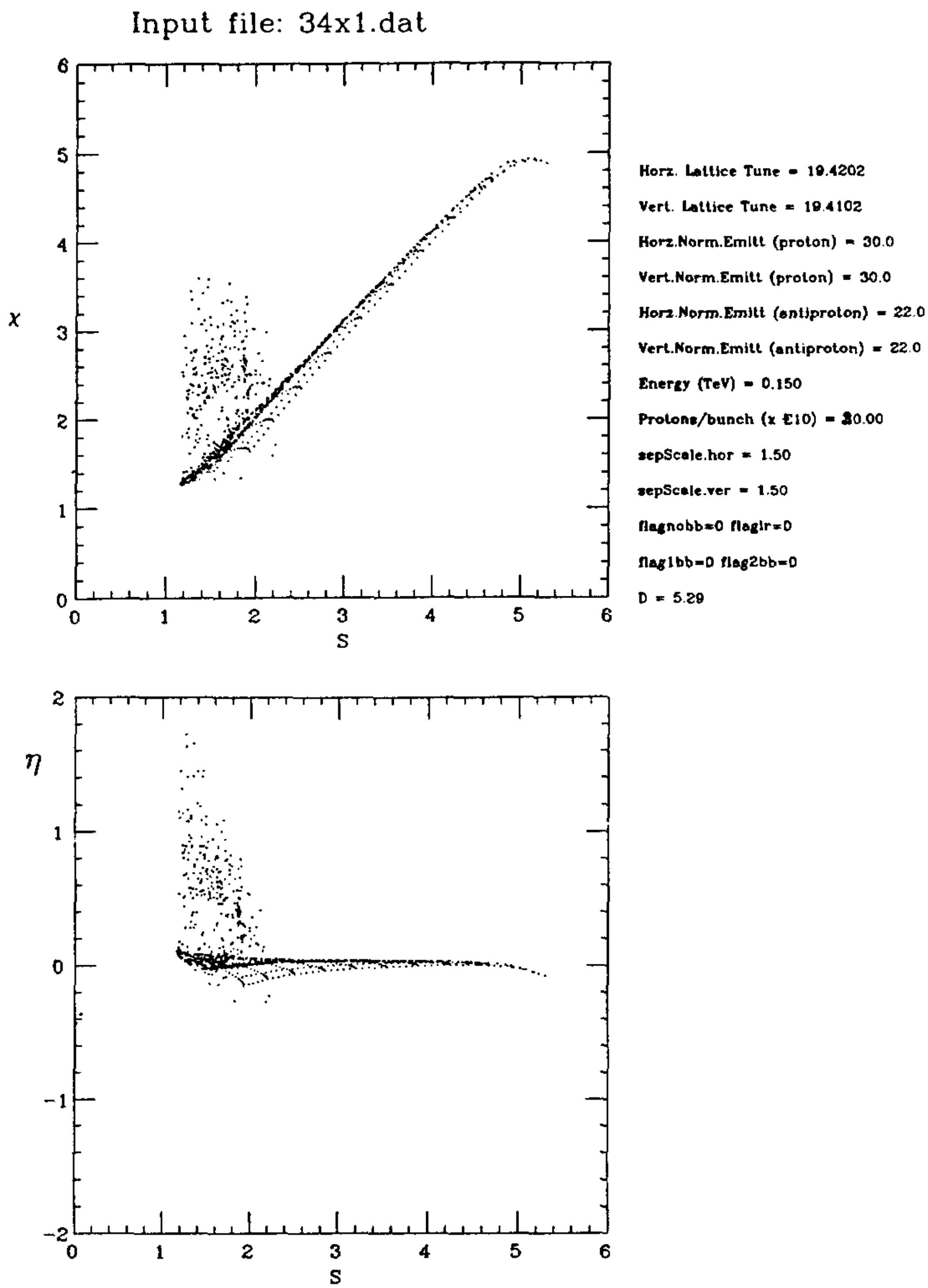


Figure 5: Another demonstration of the nonlinearity in the region $1 < S < 2$. These plots are for the 34x1.dat lattice which has only long-range interactions. Since the number of interactions per turn is larger than that of Fig.(4), the normalized deviation η is more pronounced when $1 < S < 2$.

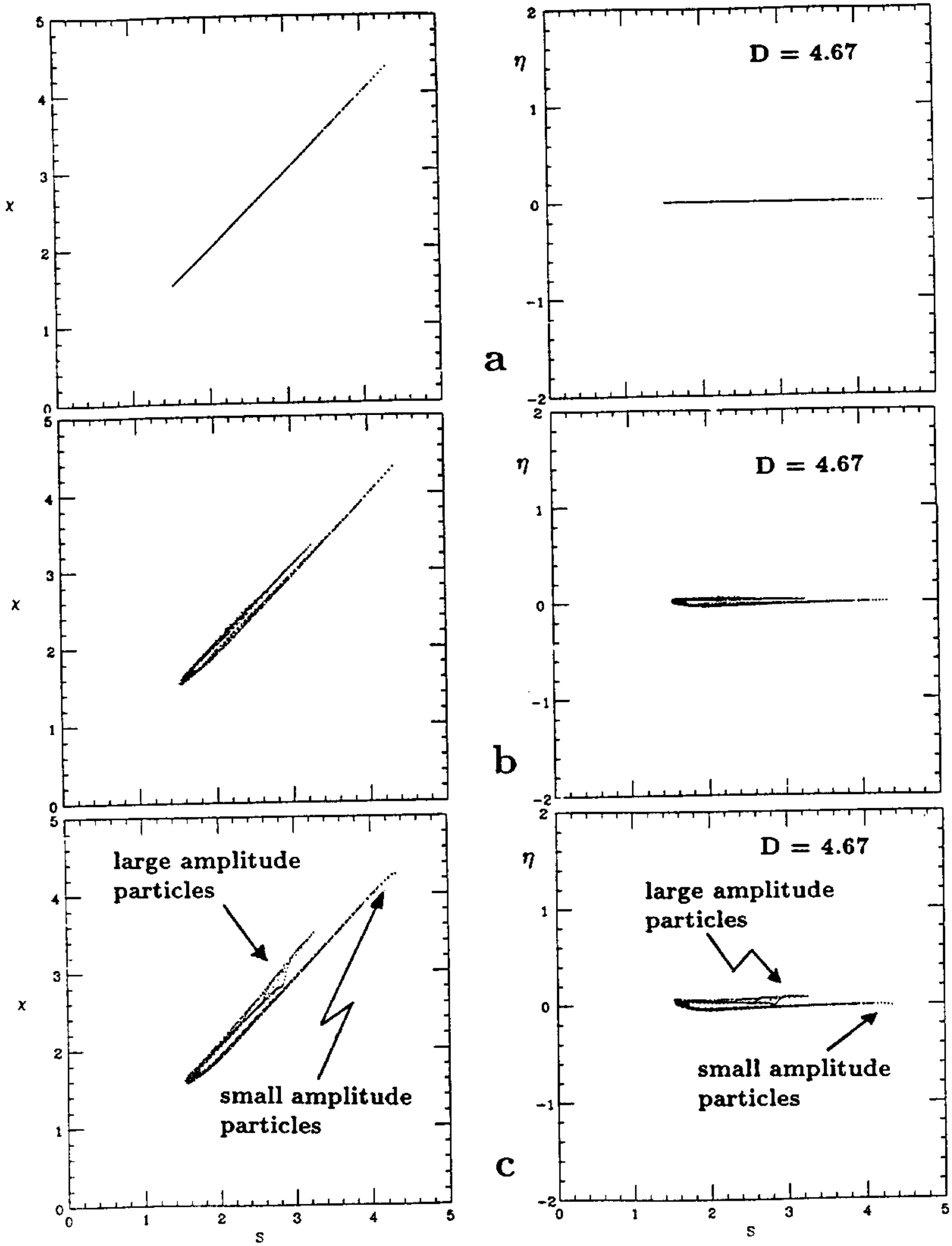


Figure 6: $\chi - S$ Diagrams for the 1992 Collider Run. a) no beam-beam interactions b) only head-on interactions c) 2 head-on, 10 long-range interactions. In the 1992 Collider Run, long-range interactions will not be an issue since the difference between b) and c) is small.

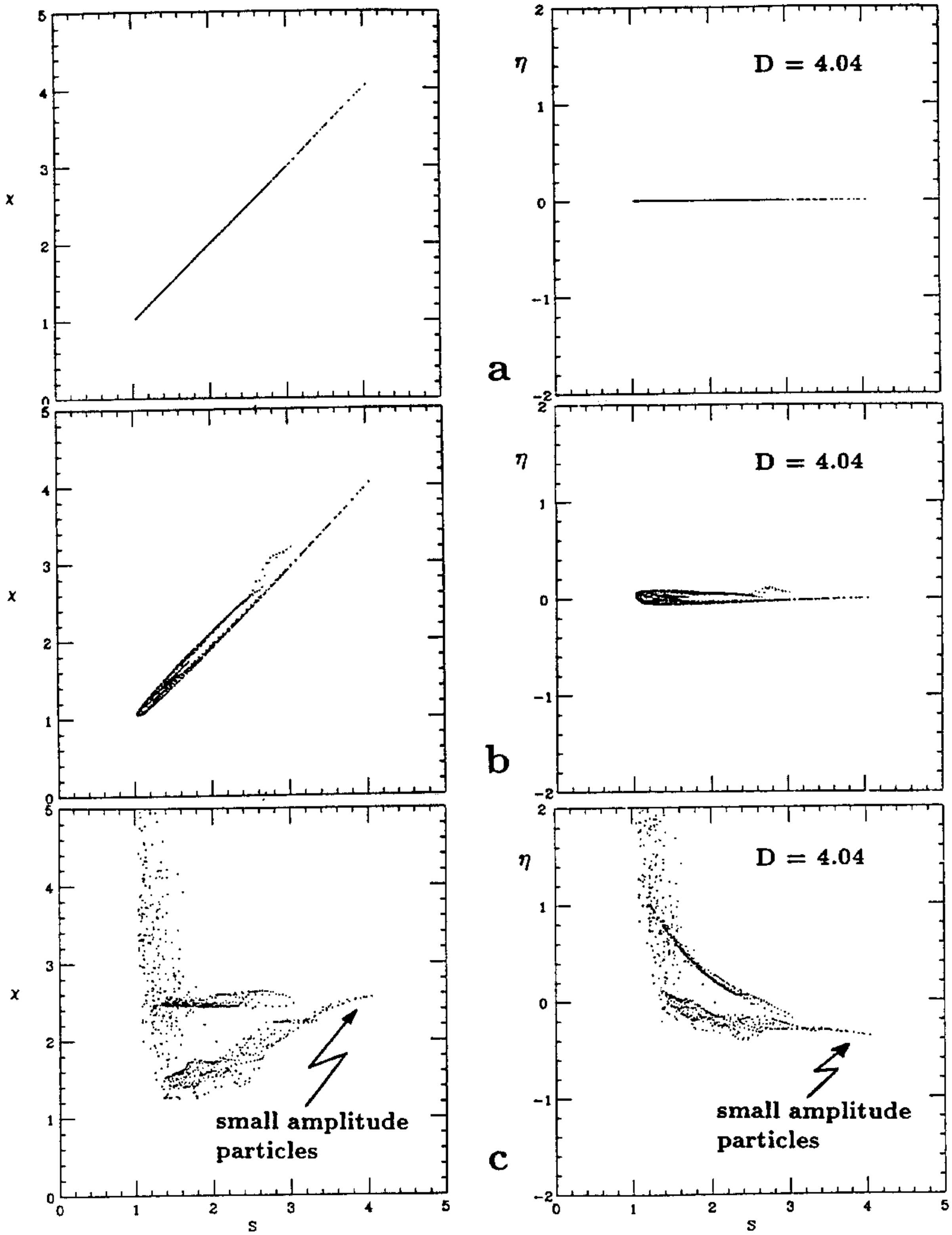


Figure 7: $\chi - S$ diagrams (left) and $\eta - S$ diagrams (right) for the Collider Run with the Main Injector. a) no beam-beam interactions b) only head-on interactions c) 2 head-on, 66 long-range interactions. Note that the nonzero value of $|\ \eta\ |$ is caused mostly by long-range interactions. Head-on interactions contribute very little.

6 Tune Shifts and Spreads

The tune shifts and spreads arising from beam-beam interactions can be calculated numerically or analytically. Tuneshift/spread from head-on interactions are well understood and given by [13], [14], [15].

$$\Delta\nu_x = \frac{1}{4\pi}\frac{N_p r_p}{\gamma}\frac{\beta_x}{\sigma_x\sigma_y}\int_0^1 \frac{dw}{\sqrt{\xi_x^3\xi_y}}[Z_0(\zeta_x) - Z_1(\zeta_x)]Z_0(\zeta_y) \tag{17}$$

$$\begin{aligned}
\xi_x &= 1 + (\frac{\sigma_x}{\sigma_y} - 1)w \\
\xi_y &= 1 + (\frac{\sigma_y}{\sigma_x} - 1)w \\
\zeta_x &= \frac{\beta_x I_x}{2\sigma_x\sigma_y}\frac{w}{\xi_x} \\
\zeta_y &= \frac{\beta_y I_y}{2\sigma_x\sigma_y}\frac{w}{\xi_y} \\
Z_n(\zeta) &= e^{-\zeta} I_n(\zeta)
\end{aligned} \tag{18}$$

where γ is the relativistic factor, r_p is the classical proton radius, I_n are Modified Bessel functions. I_x and I_y are the so-called action variables ($I_x = A_x^2/2$,$I_y = A_y^2/2$). A similar expression can be written for $\Delta\nu_y$ by interchanging x and y subscripts.

Analytical expressions for tune shifts arising from a long-range beam-beam interaction are more complicated. Expressions calculated from the multipole expansion of the long-range beam-beam kick are given in Ref.[14]. Integral expresions are given in Ref.[16].

6.1 Numerical Computation of Tune Shifts

Numerically, tune shifts can be calculated by two methods: 1) by finding the peak in the power spectrum (FFT) of the motion. 2) by calculating the tune shift from average phase advance. The second method is faster. The FFT method was used in Ref.[11] to calculate the beam-beam tune shifts for various amplitudes at different separations and source bunch intensities. The average phase advance method was used to study a particular configuration in the Tevatron where a single antiproton bunch collided with 34 proton bunches. All beam crossings involved long-range interactions. A tune shift footprint was generated from HOBBI for a mesh of antiproton amplitudes, and average tune shifts from this footprint were compared to measured values [17]. There was good agreement. The experimental errors however were large due to difficulties related to the low intensity of the antiproton bunch. More experimental studies are needed to test the tune shift prediction of HOBBI.

Here, we show the tune shift footprints for the future collider runs. Fig.(8) and Fig.(9) depict the tune shift/spread in the 1992 Collider Run and the Collider Run with the Main Injector respectively.

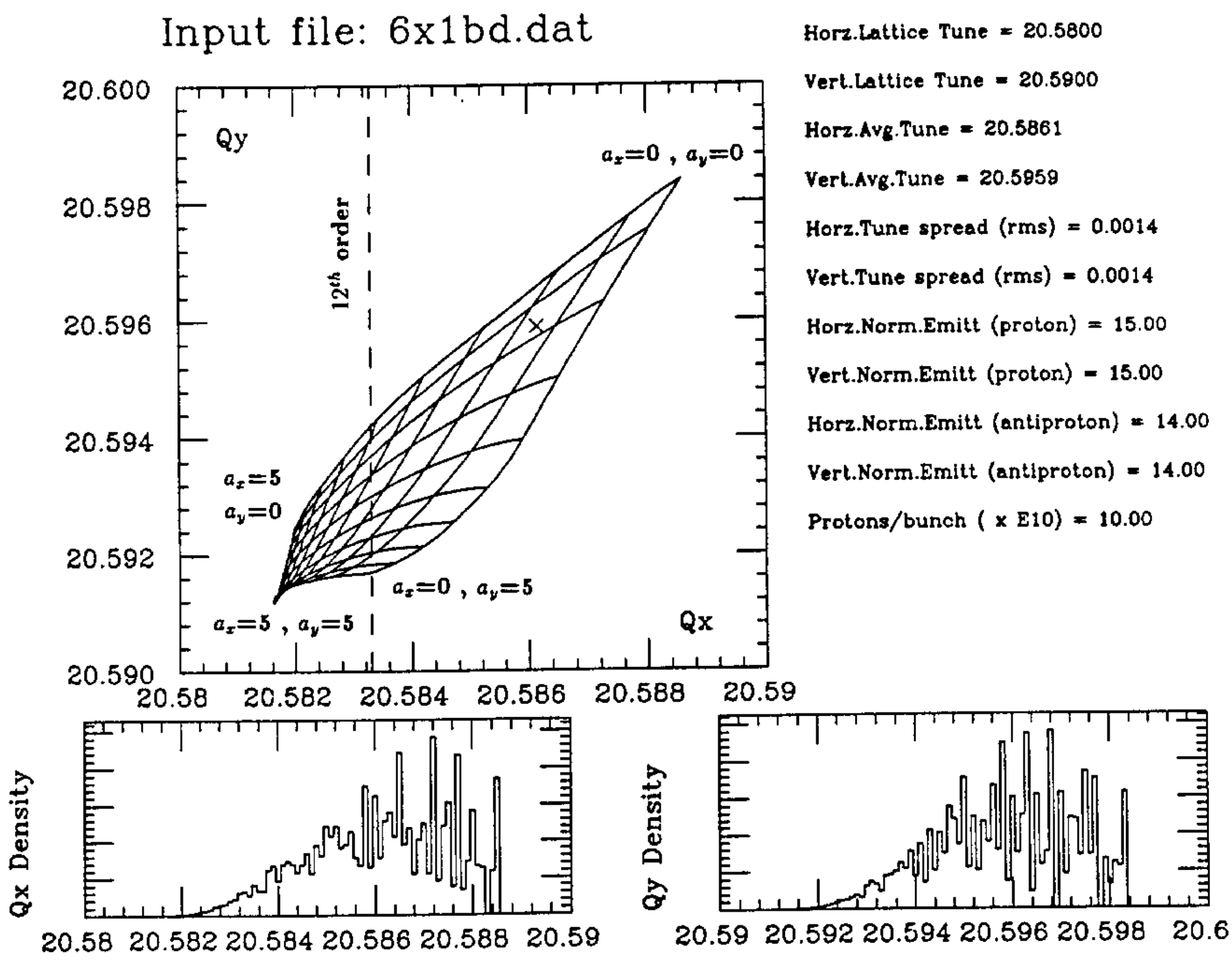


Figure 8: Tuneshift-footprint for the 1992 Collider Run.

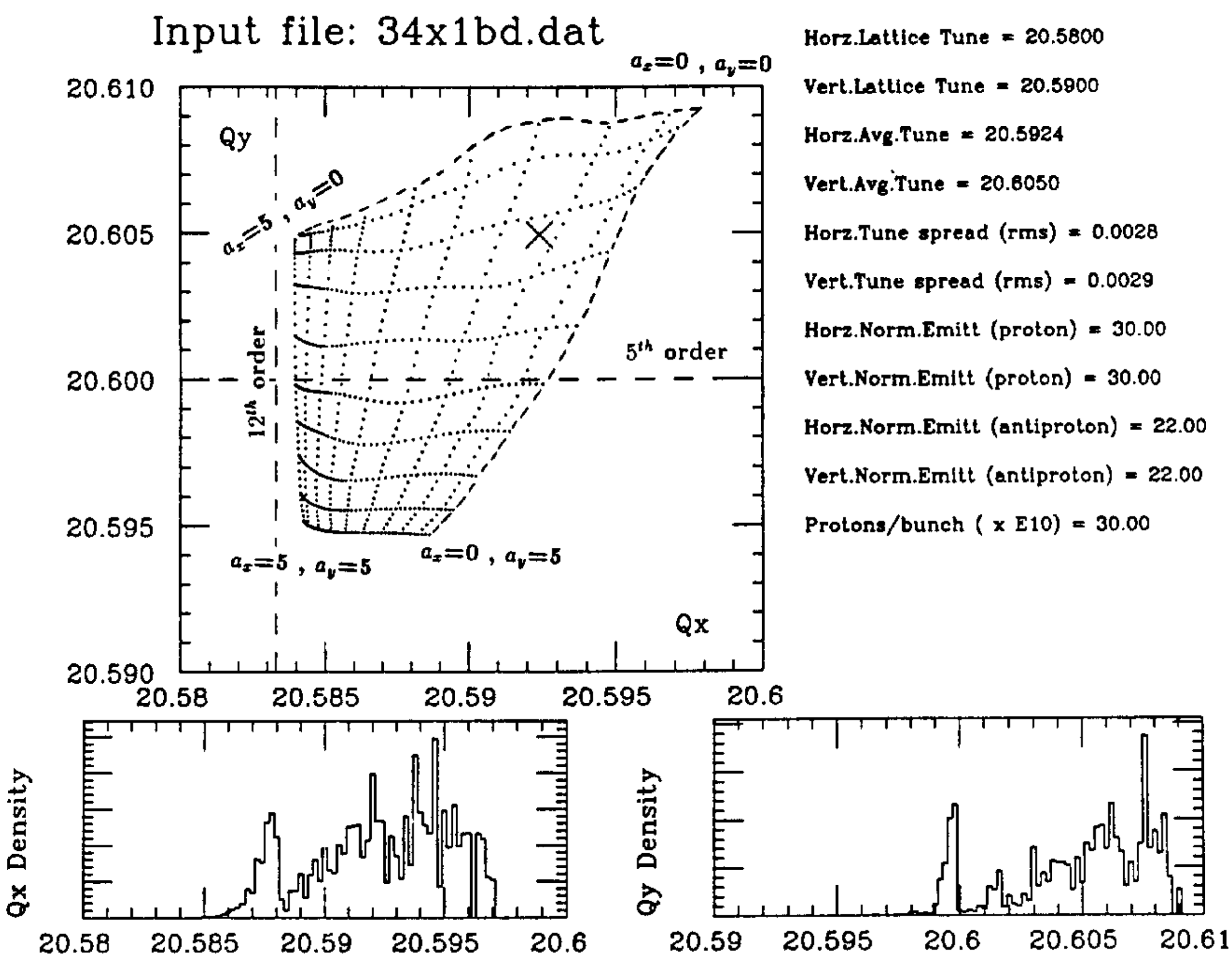


Figure 9: Tuneshift-footprint for the Collider Run with the Main Injector. Lattice tunes (working point) are not yet optimized. Note the effects of the 5^{th} order resonance.

In general, the average tuneshift from long-range beam-beam interactions is small due to two reasons: 1) Strength of the beam-beam kick scales as $1/r$ 2) Some long-range interactions may contribute negative tune shift. The tune spread due to long-range interactions however can be large as shown in Fig.(10)

The tuneshift-footprint area due to long-range interactions can be reduced by increasing the average separation D. Fig.(10) and Fig.(11) illustrate this idea.

6.2 Selection of the Working Point

The lattice tunes can be adjusted using the tune correction quadrupoles in the Tevatron. One needs to find the optimum horizontal and vertical tunes. The trick here is to avoid the low-order resonances in the tune space. A relatively large resonance-free area exists at the usual working point between the 5^{th} and the 7^{th} order resonances. The tuneshift-footprint should fit in this area and there must be a safe distance between the small-amplitude tunes and the 5^{th},7^{th} order resonances. The $12^{t}h$ order resonance cannot be avoided but it was shown [18] that it is harmless in the Tevatron.

To determine the optimum tunes we used the tuneshift-footprint diagrams and the η-S diagrams. Fig.(12) and Fig.(13) compare the lattice tunes as given in HOBBI input files to the suggested tunes. Note the correlation between the position of the footprint in tune space and the value of η for small amplitude antiprotons.

In the case of the 1992 Collider Run, ($\nu_x = 20.5750$, $\nu_y = 20.5850$) is suggested as the working point; Fig.(12) shows clearly that it alleviates the effects of nonlinearities for the large amplitude particles. In the case of the Collider Run with the Main Injector, given the separator voltages and the proton emittance we have an average separation of $D = 4.04$. This separation yields a large tuneshift-footprint area, too large to fit the space between the 7^{th}, the 5^{th}, and the coupling resonances. The optimum positioning of the footprint was obtained by many simulation trials. The conclusion is that the lattice tunes ($\nu_x = 20.5730$, $\nu = 20.5810$) will produce acceptable results but small amplitude tunes will be too close to the 5^{th}. Therefore a better solution is to reduce the footprint area by increasing the separator voltages while keeping the ($\nu_x = 20.5730$, $\nu_y = 20.5810$) combination as the working point. There is one more issue that needs to be discussed. Since there will be 68 beam-beam interactions, and the bunch intensity is large, the coupling effect of the beam-beam interaction will be more visible. Therefore in the Main Injector era one should pay special attention to decoupling the Tevatron.

7 Scaling Laws for Long-Range Interactions

The purpose of orbit separation is to increase the luminosity by eliminating the unnecessary head-on interactions. Orbit separation brings the problem of long-range interactions. When the number of bunches per beam is small the long-range interactions do not pose any threat. In the case of multi-bunch operation the strength of nonlinearity will increase. The "η-measure" developed in previous sections can be

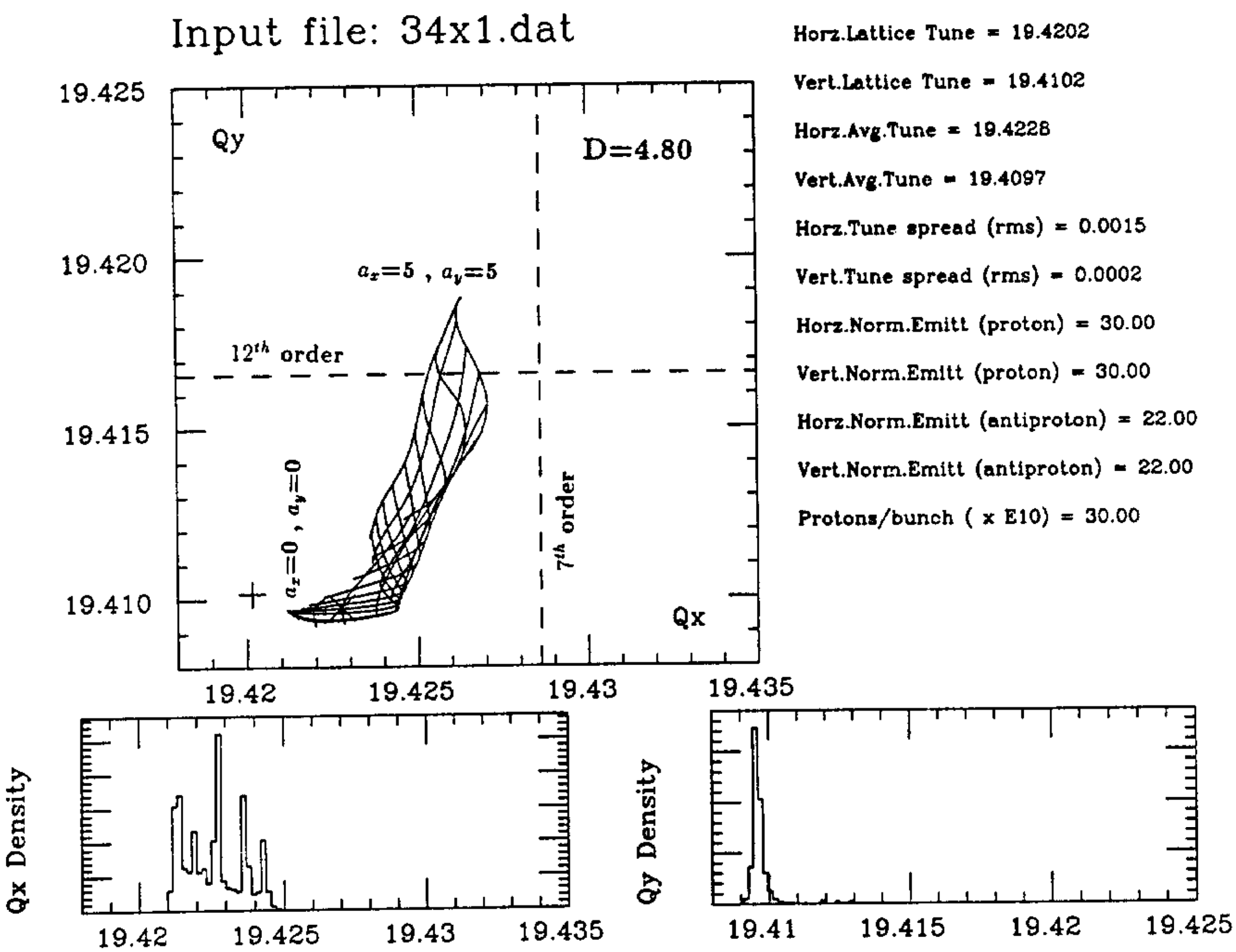


Figure 10: Tuneshift-footprint for the 34x1.dat lattice. All beam-crossings involve long-range interactions. The overall footprint area (tune spread) due to long-range interactions cannot be ignored.

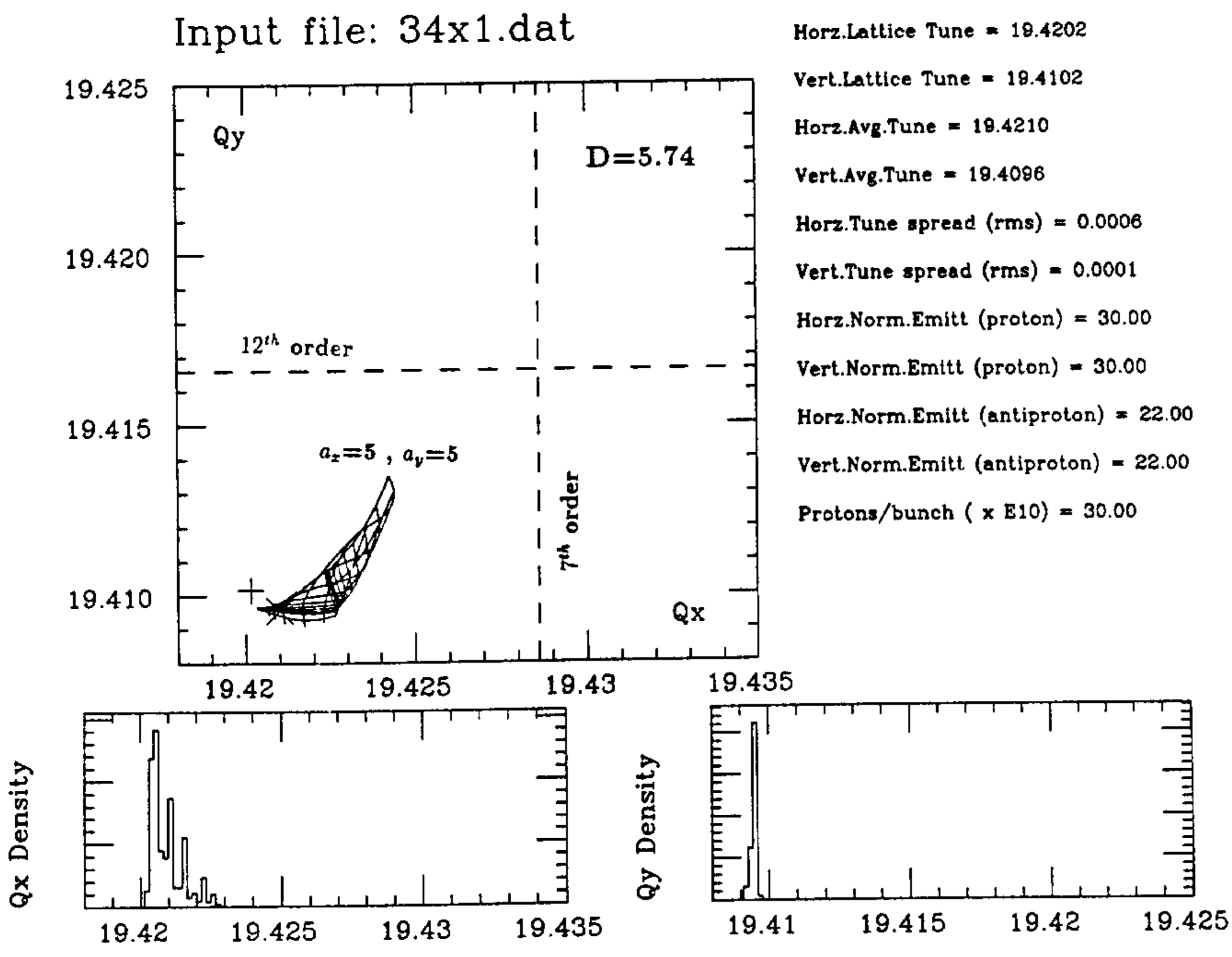


Figure 11: Tuneshift-footprint for the 34x1.dat lattice when D is increased from 4.80 to 5.74. The size of the footprint area is reduced by a factor of 3. The average tune shift is also reduced by a factor of 3.

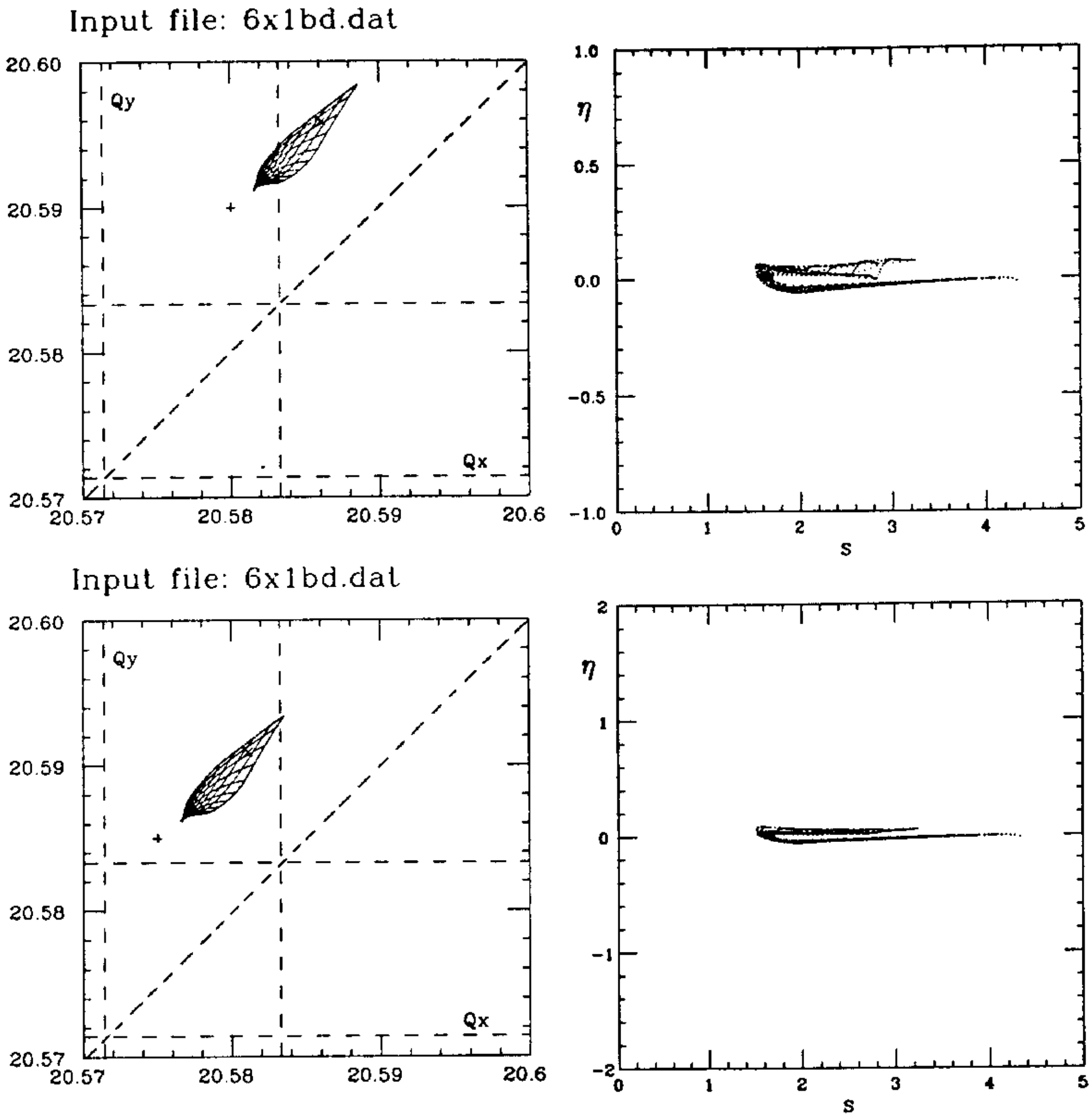


Figure 12: Comparison of working points for the 1992 Collider Run. The working point is indicated by a ' + ' sign on the tuneshift-footprint diagrams. ($\nu_x = 20.5750$, $\nu = 20.5850$) is recommended as the optimum working point.

Input file: 34x1bd.dat

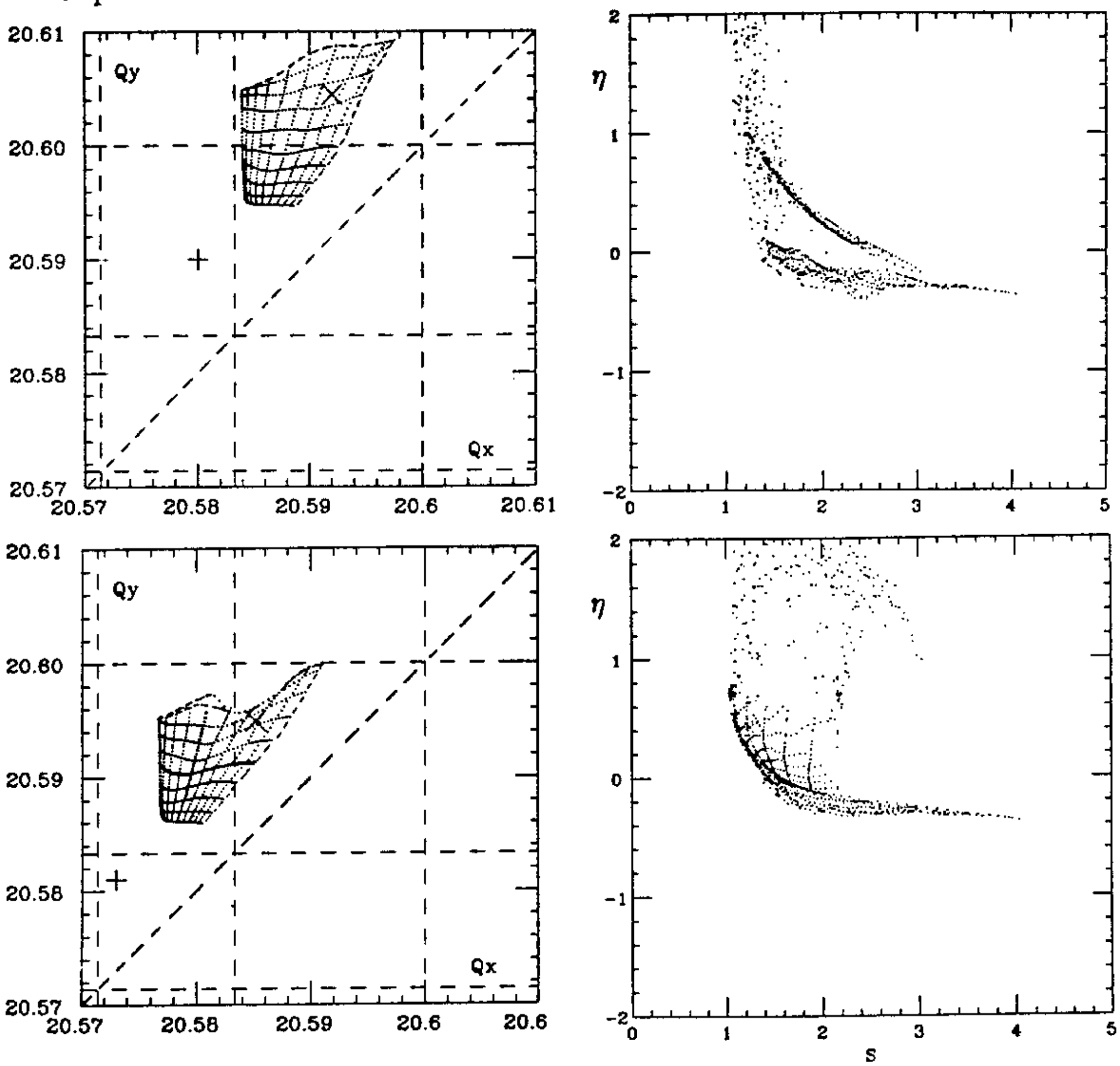


Figure 13: Comparison of working points for the Collider Run with the Main Injector. The working point in the upper diagram is not recommended due to the proximity of the 5^{th} order resonance. We suggest ($\nu_x = 20.5730$, $\nu = 20.5810$) as the optimum working point. In this case, however, the tuneshift-footprint is too close to the coupling resonance. Given the size of the footprint this is unavoidable. Reducing the footprint area by increasing the separator voltages would help the situation.

used to quantify these nonlinear effects.

In this section we investigate the dependence of η (for a given S) on bunch intensity, emittance and the number of long-range interactions. In Fig.(14) we show η-S plots for the 34x1bd.dat lattice with different bunch intensities. All the other parameters are kept the same. Here, we will concentrate on the small amplitude particles since they are the ones that contribute most to the luminosity. It can be seen in Fig.(14) that the absolute value of η for small amplitude particles increases with N_p. Fig.(15) shows that this is a linear relationship.

We also generated similar plots by keeping all beam parameters but emittance constant and plotted $\eta(S = 3)$ versus emittance in Fig.(16). The error bars in this graph indicate the thickness of the η-S plots at $S = 3$. Since particle amplitude is roughly found by $A = |D - S|$, in this case, $S = 3$ corresponds to amplitude $A = 1$ because $D = 4$. The conclusion from Fig.(16) is that the η for a given S does not depend on the emittance to first order. This conclusion seems counter-intuitive but closer thought reveals that the parameter S was indeed the right choice because it successfully reduces the dimension of the parameter-space by 2. In other words, one no longer deals with particle amplitude A, separation D or emittance ε but just one parameter S. The size of error bars, however, increase with emittance, indicating a second or higher order effect.

Finally, we studied how the number of long-range interactions affect the strength of nonlinearity. As expected it is a linear relationship as shown in Fig.(17).

The conclusion from Fig.(15), Fig.(16) and Fig.(17) is that the strength of nonlinearity scales as

$$| \eta | \propto N_p N_{lr} \tag{19}$$

where N_{lr} is the number of long-range interactions and N_p is the proton bunch intensity. Head-on interactions do not contribute to $\eta(S \approx D)$ (for small amplitude particles).

7.1 Tolerable Nonlinearity Criterion

In order to determine the proportionality constant in Eq.(19) one needs to know the value of η at which the nonlinearity is so strong that it causes beam loss. One could investigate this limit by studying the long term behaviour of particles in simulation. This approach would be more relevant for large amplitude particles and is currently being pursued. Since the scaling law, Eq.(19), was deduced for small amplitude particles we should find the η-limit when $S \approx D$. It is very fortunate that in the case of small amplitude particles, the η-limit can be established directly from operational experience, by relating the "scaling law" to the beam-beam tune shift limit.

The tune shift for small amplitude antiprotons from a single head-on beam-beam interaction is given by

$$\xi = \frac{N_p r_p}{\pi(\varepsilon_p \beta\gamma)} \tag{20}$$

for a round beam. ε_p is the proton emittance, N_p is the proton bunch intensity. r_p is the classical proton radius and β, γ are the relativistic factors. At the beam energy

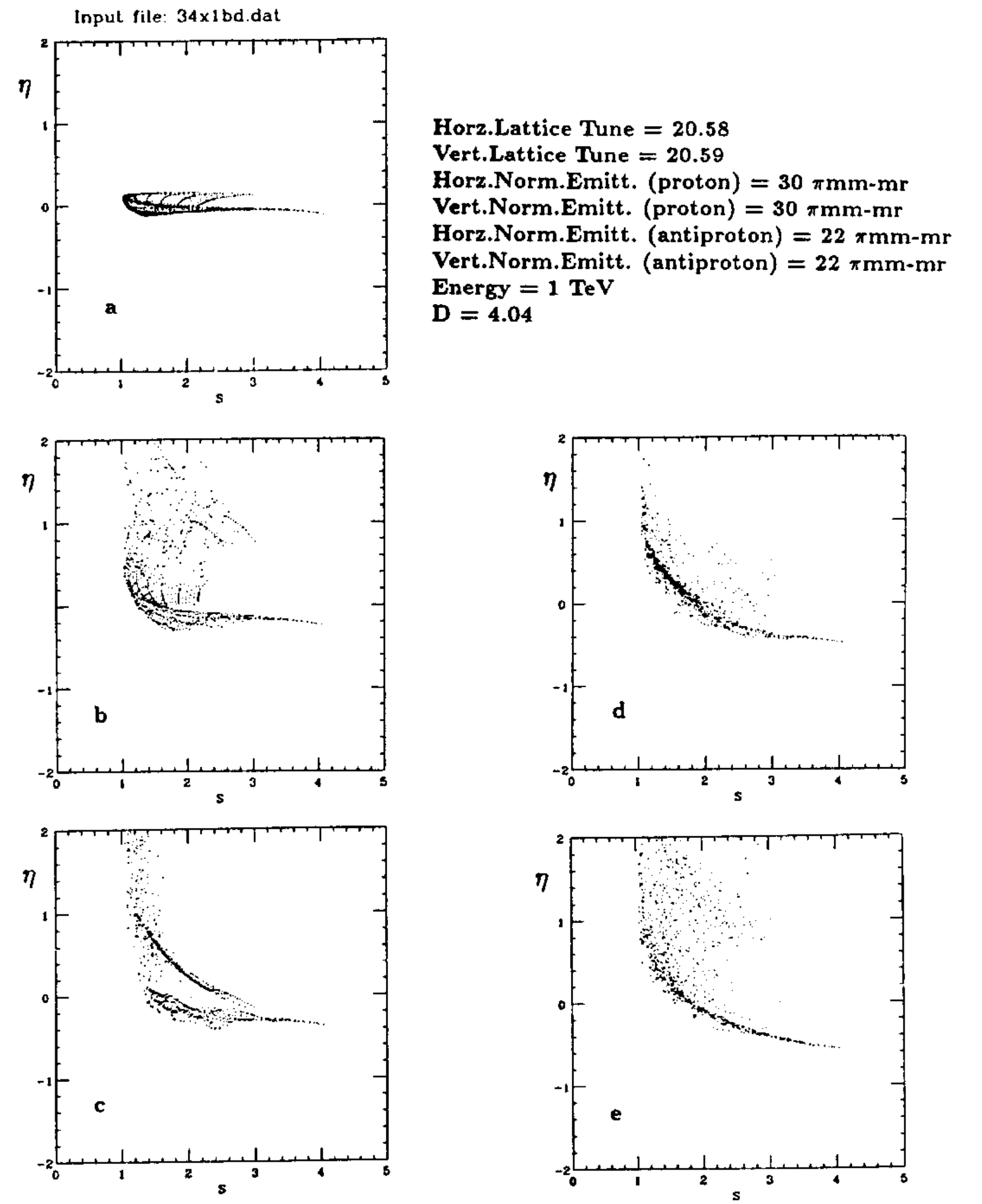


Figure 14: $\eta - S$ Diagrams for the Collider Run with the Main Injector with different bunch intensities. a) $N_p = 10 \times 10^{10}$ b) $N_p = 20 \times 10^{10}$ c) $N_p = 30 \times 10^{10}$ d) $N_p = 40 \times 10^{10}$ e) $N_p = 50 \times 10^{10}$. The structure in c) is caused by the 5^{th} order resonance. Note that the absolute value of η for small amplitude particles is increasing with N_p

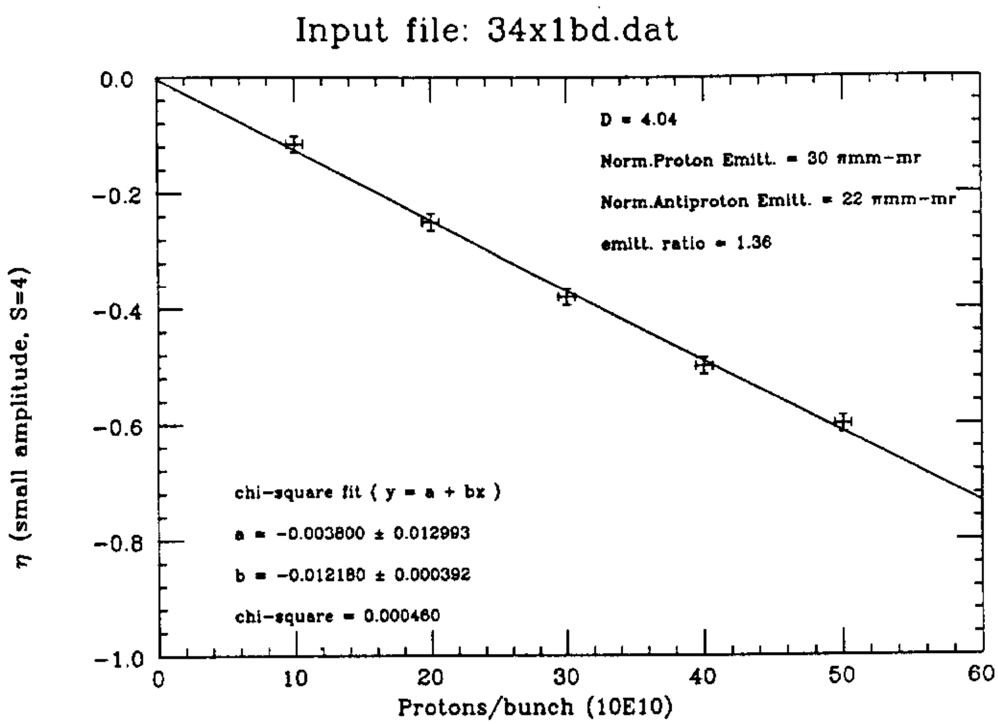


Figure 15: The strength of nonlinearity increases linearly with bunch intensity.

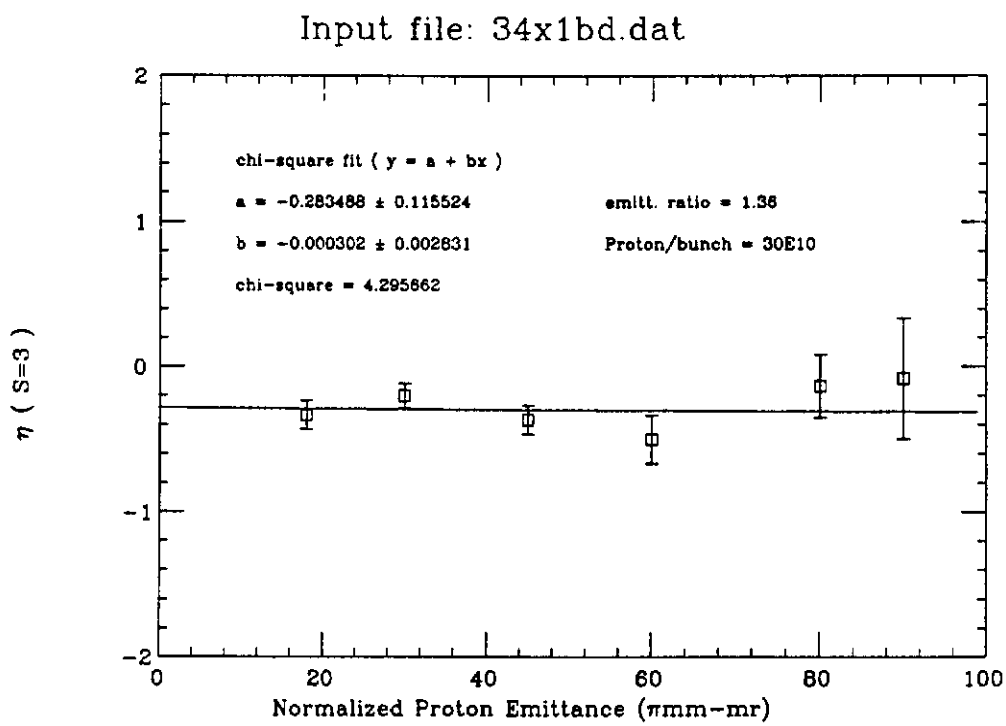


Figure 16: The strength of nonlinearity does not depend on the emittance to first order. The size of error bars, however, increase with emittance, indicating a second or higher order effect.

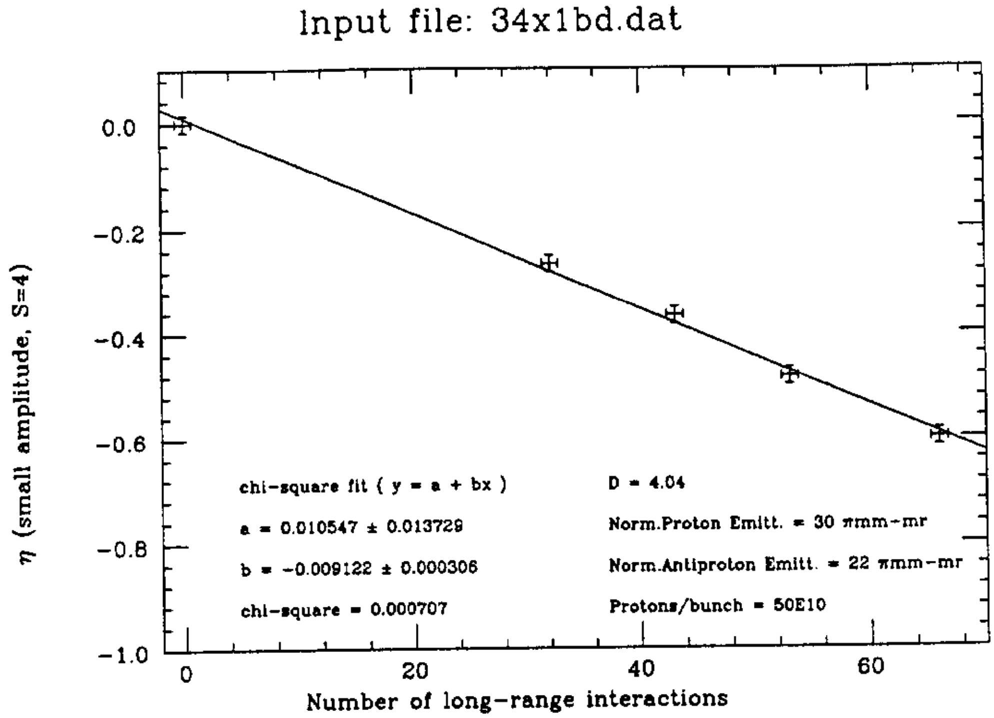


Figure 17: The strength of nonlinearity increases linearly with the number of long-range interactions.

of 1 TeV the numerical value is

$$\xi = 0.00733 \frac{N_p \; [10^{10}]}{\epsilon_p \; [\pi \mathrm{mm} - \mathrm{mr}]} \tag{21}$$

As we mentioned in the introduction, the tune space between the 5^{th} and the 7^{th} order resonances sets a limit on the total tune shift/spread. Theoretically this limit is 0.02857 but in practice it is smaller because of the resonance widths. The operational experience at the CERN Sp$\bar{p}$S and the Tevatron Collider has shown that the the beam-beam tune shift/spread limit is

$$\text{Tuneshift/spread} \; < \; 0.024 \tag{22}$$

This can be rewritten as

$$0.00733 \frac{N_p \; [10^{10}]}{\epsilon_p \; [\pi \mathrm{mm} - \mathrm{mr}]} N_{ho} R \; < \; 0.024 \tag{23}$$

where N_{ho} is the number of head-on interactions, $R \equiv (\Delta\nu_{\mathrm{total}}/\Delta\nu_{\mathrm{head-on\ only}})$ is the ratio that shows the tune shift contribution from long-range interactions. R can be found from tuneshift-footprint diagrams.

In the Tevatron Collider the number of head-on collisions will always be 2 (CDF and B0 detectors). Using $R = 1.23$ (obtained from Fig.(13)), and the nominal parameter for the Collider Run with the Main Injector (CRMI), $\epsilon_p = 30$ πmm-mr, we come up with an upper limit on proton bunch intensity

$$N_{p,\,\mathrm{CRMI}} < 40 \times 10^{10} \tag{24}$$

It is found graphically from Fig.(15) that $|\eta| \cong 0.5$ when $N_p = 40 \times 10^{10}$. Therefore the beam-beam η-limit is

$$|\eta(S \approx D)| < 0.5 \tag{25}$$

We shall call Eq.(25) the "tolerable nonlinearity criterion". This can also be viewed as the connection between the head-on and the long-range beam-beam interactions. In other words, a beam stability criterion based on the head-on beam-beam experience is related to another criterion which applies to long-range interactions.

7.2 Configuration Limit

In Fig.(15), Fig.(17), the tolerable nonlinearity limit is reached when $N_p = 42 \times 10^{10}$ while $N_{lr} = 66$ or when $N_p = 50 \times 10^{10}$ while $N_{lr} = 56$. The $N_p N_{lr}$ product is 2.772×10^{13} and 2.800×10^{13} respectively which suggests that there is a proportionality constant K

$$K = 1.8 \times 10^{-14} \tag{26}$$

which can be used to write the "configuration limit"

$$K N_p N_{lr} < 0.5 \tag{27}$$

Here, it is important to point out that K is not a universal constant. It is related to the average separation D. The larger the D the smaller the K. The form of the relationship between K and D will be pursued in a later publication.

In the 1992 Collider Run, $N_p = 10 \times 10^{10}$, $N_{lr} = 10$, thus the product $K N_p N_{lr} = 0.018$ will be much smaller than 0.5 . In CRMI, with $N_p = 30 \times 10^{10}$, $N_{lr} = 66$ the product $K N_p N_{lr} = 0.3564$ will still be less than 0.5. In CRMI, the bunch intensity can be increased up to $N_p = 40 \times 10^{10}$ while keeping $K N_p N_{lr} \leq 0.5$

7.3 Separation Rules from Simulation

The "tolerable nonlinearity criterion" can be written as separation rules.

$$\begin{aligned} \text{Open} - \text{Helix at 150 GeV} \quad & \eta(S \approx D) < 0.5 \\ \text{During Ramp} \quad & \eta(S \approx D) < 0.5 \\ \text{Open} - \text{Helix at 900/1000 GeV} \quad & \eta(S \approx D) < 0.5 \\ \text{Low} - \text{Beta Squeeze} \quad & \eta(S \approx D) < 0.5 \end{aligned} \tag{28}$$

In the light of these separation rules, we can comment on the overall beam behaviour in future collider runs. From Fig.(6), Fig.(7) and Eq.(28) we conclude that during the 1992 Collider Run, separation will not be an issue. Even when $S = 1.5$ there will be no noticeble beam loss. During the Collider Run with the Main Injector only the particles in the transverse tails of the antiproton bunches will suffer from nonlinearities and may be lost as a result. These tail antiprotons could contribute to the background noise levels at the CDF and D0 detectors unless something is done to remove them (i.e. scraped).

8 Pushing the Limits

The configuration limit $KN_pN_{lr} < 0.5$ does not say anything about the emittance. By substituting $N_p = (0.5/KN_{lr})$ in Eq.(23) and rearranging we come up with a constraint on proton emittance.

$$\varepsilon_p > 848\ R\frac{N_{ho}}{N_{lr}} \quad [\pi\text{mm} - \text{mr}] \quad \text{if}\ \ KN_pN_{lr} = 0.5 \tag{29}$$

In CRMI, $N_{lr} = 66$, $N_{ho} = 2$, $R = 1.23$ and if we push the proton bunch intensity limit to $N_p = 40 \times 10^{10}$ then the proton emittance has to be larger than 32 πmm-mr. We would like to point out that this constraint is based on the assumption that the tuneshift-footprint has to fit the area between the 7^{th} and the 5^{th} order resonances. If one moves the working point near the integer [19] then one has the freedom to reduce the proton emittance to improve luminosity unless other problems intervene.

These limits can be used to estimate the maximum initial luminosity in the Collider Run with the Main Injector.

$$L_{\max,\ \text{CRMI}} = 6.6 \times 10^{24} \frac{N_{\bar{p}}}{(32 + \varepsilon_{\bar{p}}\ [\pi\text{mm} - \text{mr}])}\ \frac{1}{\beta^*[cm]} \quad \text{cm}^{-2}\text{sec}^{-1} \tag{30}$$

Eq.(30) assumes that with the Main Injector we will have the capability to produce proton bunch intensities between $30 \times 10^{10} < N_p < 50 \times 10^{10}$ and therefore push it to its "configuration limit" of 40×10^{10} . The values of revolution frequency $f = 47700$ Hz and $\gamma = 1066$ were inserted.

All the information about beam dynamics and resultant limits are contained in the constants 6.6×10^{24} and 32. The rest of the parameters can be considered free (to first order).

After reaching the beam-dynamics limits on proton bunch brightness (N_p/ε_p) and N_{lr} one can still push the antiproton bunch brightness and reduce the β^* to 0.25 m. We caution, however, that pushing β^* does not help if β^* is smaller than the bunch length. Pushing $N_{\bar{p}}$ will hit other limits such as instabilities. Also, weak-strong model does not apply if $N_p = N_{\bar{p}}$.

9 Conclusions

In this paper we have studied the beam-beam interactions in future Tevatron Collider Runs. We have used the simulation code HOBBI as the central module and developed special purpose programs to generate $\chi - S$, $\eta - S$ and tuneshift-footprint diagrams. We used the weak-strong model which is valid for all the future collider runs in the Tevatron (assuming that the antiproton bunch intensity will never equal the proton bunch intensity).

We have advocated the use of a new separation parameter, S, in the discussion of beam-dynamics of long-range interactions. Using this parameter we have developed a "tolerable nonlinearity criterion" for small amplitude particles. This criterion can be expressed in terms of the parameter $\eta \equiv (\chi - S)/S$ where χ is the average S calculated by tracking the particle around the ring many turns.

- <u>Tolerable Nonlinearity Criterion</u>

$$|\,\eta(S \approx D)\,| \;<\; 0.5$$

We have generated tuneshift-footprint diagrams and calculated the tune spread in future collider runs. Based on these results we recommend the following tunes as the working point:

- <u>Working point for the 1992 Collider Run</u>

$$\begin{aligned} \nu_x &= 20.5750 \\ \nu_y &= 20.5850 \end{aligned}$$

- <u>Working point for the Collider Run with the Main Injector</u>

$$\begin{aligned} \nu_x &= 20.5730 \\ \nu_y &= 20.5810 \end{aligned}$$

We have advocated the use of η as a measure of nonlinearity, much in the spirit of "smear". We have found that the strength of nonlinearity, η, is proportional to the product $N_p N_{lr}$ and does not depend on proton emittance to first order. This is the scaling law of long-range interactions (since the nonzero value of η is caused mostly by long-range interactions)

- <u>Scaling Law for long-range interactions</u>

$$|\,\eta\,| \;\propto\; N_p N_{lr}$$

The scaling law can be written with a specific proportionality constant K which depends on D only. For $D = 4$, the constant is found to be $K = 1.8 \times 10^{-14}$.

- <u>Configuration limit</u>

$$K N_p N_{lr} \;<\; 0.5$$

which puts limits on the proton bunch intensity and the number of bunches per beam.

We have also found that when the proton intensity or the number of bunches per beam is pushed to its limit, the proton emittance has to be larger than the

- Lower bound on proton emittance

$$\varepsilon_p \quad > \quad 848 \; R \frac{N_{ho}}{N_{lr}} \quad [\pi\mathrm{mm}-\mathrm{mr}] \quad \text{if} \quad K N_p N_{lr} = 0.5$$

Using the nominal parameters for CRMI, and assuming that N_p can be pushed to its configuration limit of 40×10^{10}, the lower bound on the proton emittance is found to be 32 πmm-mr.

We concluded by estimating the maximum luminosity that can be achieved in CRMI,

- Maximum initial luminosity in CRMI

$$L_{\mathrm{max,\ CRMI}} \quad = \quad 6.6 \times 10^{24} \frac{N_{\bar{p}}}{(32 + \varepsilon_{\bar{p}}\ [\pi\mathrm{mm}-\mathrm{mr}])} \; \frac{1}{\beta^*[cm]} \quad \mathrm{cm}^{-2}\mathrm{sec}^{-1}$$

Using the nominal parameters of $N_{\bar{p}} = 3.7 \times 10^{10}$, $\beta^* = 50$ cm, the maximum initial luminosity will be $L_{\mathrm{max,\ CRMI}} = 9 \times 10^{31}\ \mathrm{cm}^{-2}\mathrm{sec}^{-1}$.

10 Acknowledgements

I thank L.Michelotti for getting me started in beam-beam interactions, and for providing the software tools such as HELICAL, AESOP and many others. I thank S.Peggs for his suggestion of using the average phase advance method in the computation of tune shifts and for providing a "Sun Sparc-Station" which speeded up the simulation runs considerably. I thank D.Finley for allowing me to finish this paper during a busy period in departmental activities. I thank M.Harrison and C.Ankenbrandt for a critical reading of this paper. My thanks are also extended to G.Goderre, G.Jackson and D.Herrup for valuable discussions.

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