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Lengthy Disturbances and Copper-to-Superconductor Ratio

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I. INTRODUCTION

In a superconducting cable strand, the superconductor is in the form of fine filaments embedded in a matrix of copper. A large amount of copper is definitely desired because it can conduct away the the heat and carry the current in the event of a heat deposition, so that a quench can be prevented. However, a sufficient amount of superconductor is necessary to carry the high dipole current. A higher amount is desired so that the point of operation is far away from the thermodynamic surface that separate the superconducting phase from the normal phase. Unfortunately, there must be a trade off between the amount of superconductor and copper, because there is only limited volume in the superconducting dipole magnet to house the superconducting cable. As a result, the copper-to-superconductor ratio becomes a crucial parameter, which must be carefully chosen in order to optimize the stability of the cable.

In a previous paper,[1] we studied this ratio for a disturbance evolving from a point deposition of energy using the method of minimum propagating zone (MPZ). The Superconducting Super Collider dipole (the C358A Cross Section) has been used as an example. The contribution of flux jumping, which may be of importance, has not been taken into account. We find that, for a cable current of 6.5 kA exposing to a maximum magnetic flux intensity of ~ 7 T, the optimized copper-to-superconductor ratio is $r \sim 0.59$, which is almost independent of the purity of copper and the surface heat transfer coefficient h of the cable provided that $h < 10000$ w/m^{2}K and a constant-temperature heat bath surrounding the cable is maintained. Unfortunately, this ratio is too much less than the ratio $r = 1.3$ used in the present fabrication of the cable strand. At a bath temperature of 4.35 K, the current will completely fill the superconductor filaments when the copper-to-superconductor ratio is $r \sim 1.8$. We see from Fig. 1 that the energy content of the MPZ vanishes at $r = 0$ and $r = 1.8$, where no coil degradation has been assumed. Thus, independent of whatever theory we can think of, it is very unlikely that the maximum of this energy curve will be at $r = 1.3$, if the disturbance does evolve from a point deposition of energy. If a realistic coil degradation of 5% is included, the highest allawable copper-to-superconductor ratio drops to only ~ 1.65 making an optimum ratio of $r = 1.3$ far more unlikely.

However, disturbances may not originate from a point of infinitesimal size. For example, the energy released due to coil movements, coils rubbing each other, flux jumps, et cetera, can deposit heat on a *finite* length of cable like a few mm or longer. In this paper, we try to look at disturbances that are lengthy and see whether the optimized copper-to-superconductor ratio will be altered.

II. COMPUTATION

We take as the initial temperature profile in the z direction along a cable strand,

$$\theta(z) = \begin{cases} \theta_p & |z| \leq L \\ \theta_p \exp\left(-\frac{x^2}{2\sigma^2}\right) & |z| > L \ , \end{cases} \tag{2.1}$$

where L is the half length of the disturbance which falls off with a Gaussian tail. Throughout this study, we take $\sigma = 0.1$ mm. The temperature evolution is governed by the time-dependent heat-flow equation

$$C_{\text{eff}}(\theta)A\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left[k(\theta)(1-\lambda)A\frac{\partial\theta}{\partial z}\right] + AG(\theta) - PH(\theta) \ , \tag{2.2}$$

where A is the cross sectional area of the strand and P the perimetric circumference, C_{eff} is the effective volume heat capacity, $k(\theta)$ the heat conductivity of copper, $G(\theta)$ the power generation per unit volume when the strand becomes partly or completely normal, $H(\theta)$ is the rate of surface cooling per unit length. Lastly, λ is the fraction of superconductor in the strand, which is related to the copper-to-superconductor ratio r by

$$\lambda = \frac{1}{1+r} \ . \tag{2.3}$$

All these parameters will be defined in detail in the Appendix and their values used in this study will be given there.

We pick a cable current of $I_{\text{op}} = 6.5$ kA and a maximum magnetic field of $B_M = 7.0$ T. With the phase-separating thermodynamic surface measured by Morgan,[2] the critical temperature θ_g when the superconductor reaches current saturation can be computed easily as a function of copper-to-superconductor ratio r. For illustration, let us assume a linear model (which is not used in this study), in which the critical current density is linearly proportional to temperature,

$$\frac{I_c}{\lambda SA} = -\alpha(\theta - \theta_c) \ , \tag{2.4}$$

where α is a positive constant, θ_c is the critical temperature above which no superconducting phase is possible at the particular magnetic flux density, I_c is the critical current in the cable consisting of $S = 23$ strands each with a cross sectional area A. Then θ_g is given by equating I_c to I_{op},

$$\theta_g = \theta_c - \frac{I_{\text{op}}}{\lambda\alpha SA} \ , \tag{2.5}$$

or, with Eq. (2.3),

$$\theta_g = \left(\theta_c - \frac{I_{\text{op}}}{\alpha SA}\right) - \frac{I_{\text{op}}}{\alpha SA} r \ . \tag{2.6}$$

Since θ_g cannot be less than the bath temperature θ_0, the maximum r allowed can be obtained from Eq. (2.6) by equating θ_g to θ_0. When the temperature is below θ_g, there is no heat generation and the cable strand is completely stable. Therefore, for a quench to occur, part of the heated zone must have temperature above θ_g.

III. RESULT ANALYSIS

The time-dependent heat-flow equation (2.2) is solved numerically using the initial profile of Eq. (2.1). We keep the half length L constant and vary the peak temperature θ_p. When θ_p is too big, of course a quench will occur. This situation is shown in Fig. 2(a) with copper-to-superconductor ratio $r = 1.0$, half length $L = 5$ cm, surface heat transfer coefficient $h = 5000.0$ w/m^{2}K, and $\theta_p = 4.91$ K. If θ_p is low enough, the heated region will subside as shown in Fig. 2(b). For each copper-to-superconducting ratio, we find the maximum θ_p that can just avoid a quench. The results are plotted in Fig. 3(a) for half lengths $L = 5$, 2, 1, and 0.5 mm. Because the computation is numerical, a finite length of cable strand of 8 cm has been assumed. No coil degradation has been included.

When the length of the heated zone is big, heat removal by conduction can take place only at the two ends of the zone, which may not be able to compete with the heat removal at the strand surface. Therefore, for a *long* heated zone, stability of the strand does not depend so much on the thermal conductivity and electrical resistivity of the copper, but is very sensitive to the surface heat transfer coefficient h and the copper-to-superconductor ratio. Since we do not rely so much on the copper for heat conduction, it will be much better to have more superconductor and less copper so that the point of operation will be farther away from the thermodynamic surface. This conclusion is verified by Fig. 3(a). We see that for $L = 5$, 2, and 1 mm, a higher maximum θ_p is favored by a smaller copper-to-superconductor ratio. When the half length falls to $L = 0.5$ mm, heat conductivity along copper dominates over surface heat removal, and the peak-temperature curve shows a maximum at $r \sim 0.65$.

We plot in Fig. 3(b) the energies of the initial temperature profiles of Fig. 3(a). Similar to the plots of the peak temperatures, a small copper-to-superconductor ratio is preferred for long heated zones of $L = 5$, 2, and 1 mm. For the shorter heated zone $L = 0.5$ mm, the peak is very broad and the best ratio is $r \sim 0.5$. Note that the energy curves in Fig. 3(b) are different from those in Fig. 1. Here, each curve starts from a nonzero energy at $r = 0$. This is because we fix the length of the heated zone and raise the peak temperature until an instability occurs. It is clear that the peak temperature θ_p has to be higher than θ_g in order to have heat generation, and with such a θ_p, the energy is therefore nonzero. The energy curves in Fig. 1 correspond to the minimum initial energy required to start a quench. Such a requirement can be reached at $r = 0$ when the peak temperature of the initial profile is finite but the width is very small. As a result, these energy curves starts from zero at $r = 0$.

We plot in Figs. 4(a) and (b) peak-temperature and energy similar to Figs. 3(a) and (b) but at maximum magnetic flux density $B_M = 6.0$ T. Qualitatively, the two situations are very similar. Here, the allowable copper-to-superconductor ratio extends to $r \sim 2.7$. Because the critical current densities are higher at a lower magnetic flux density, higher excursions of the peak temperatures and energies are observed.

IV. CONCLUSION

Our results show that for a heated region that is initially lengthy, a lower copper-to-superconductor ratio is preferred to regain stability. This is because heat transfer at the strand's surface wins over heat conduction through the copper. As a result, it is preferable to have more superconductor and less copper so that the point of operation will be farther away from the normal phase of the superconductor. Figure 1 also shows that it is unlikely to have $r \sim 1.3$ for a disturbance originating from point-energy deposition regardless of the theory. This leads us to postulate that not all the copper in the present superconducting cable is conducting thermally or electrically like normal copper. Therefore, there should be an *effective* copper-to-superconductor ratio in the strand which is much less than the actual ratio, and our next effort should be the computation of this effective ratio.

APPENDIX

1. Specific heats

In computing the heat required to set up a heated zone, we take at $\theta_r = 4.2$ K

$$\begin{aligned} \text{volume specific heat of Cu } C_{cu} &= 1.6 \times 10^3 \text{ j/m}^3\text{K} \\ \text{volume specific heat of NbTi } C_{sc} &= 6.8 \times 10^3 \text{ j/m}^3\text{K} \end{aligned}$$

and assume that they vary according to θ^3 at cryogenic temperatures. For the copper-NbTi complex, we can therefore define an effective specific heat

$$C_{\text{eff}}(\theta) = [\lambda C_{sc} + (1-\lambda)C_{cu}]\left(\frac{\theta}{\theta_r}\right)^3 . \tag{A.1}$$

2. Thermal and electrical conductivities of copper

The electric resistivity ρ_{cu} and thermal conductivity k of copper at cryogenic temperatures are related to a fair approximation by the Wiedemann-Franz law[3]

$$k\rho_{cu} = L_0\theta , \tag{A.2}$$

where the Lorentz number $L_0 = \pi^2 k_B^2/3e^2 = 2.45 \times 10^{-8}$ wΩK^{-2} with k_B the Boltzmann's constant and e the electronic charge. Below ~ 10 K, the electric resistivity of copper in a magnetic flux density B (in teslas) is[4]

$$\rho_{cu}(\theta, B) = \left(0.0032\,B + \frac{1}{\text{RRR}}\right) \times 1.7 \times 10^{-8}\ \Omega\text{m} , \tag{A.3}$$

where the residual resistivity ratio (RRR), a measure of copper purity, is taken as 100. Thus, the thermal conductivity is affected by magnetic flux as well as purity, and is linear in θ at low temperature.

3. Power generation and surface cooling

When the superconductor in the cable becomes partly normal or completely normal, heat is generated. The power generation per unit volume is given by

$$G(\theta) = \begin{cases} \dfrac{\lambda^2 \rho_{cu} j_{\text{op}}^2}{1-\lambda}\dfrac{\theta-\theta_g}{\theta_c-\theta_g} & \text{when } \theta_c > \theta > \theta_g \\[2ex] \dfrac{\lambda^2 \rho_{cu} j_{\text{op}}^2}{1-\lambda} & \text{when } \theta \geq \theta_c , \end{cases} \tag{A.4}$$

where the operating current density $j_{\rm op}$ is related to the operating current $I_{\rm op}$ by

$$j_{\rm op} = \frac{I_{\rm op}}{\lambda S A} \ . \tag{A.5}$$

In above, $S = 23$ is the number of strand in the cable and $A = 5.127 \times 10^{-7}$ m^2 is the cross sectional area of a strand.

The rate at which heat is transferred per unit length at the strand's surface is given by $PH(\theta)$, where P is the perimetric circumference of the strand. We assume that the cooling is proportional to the temperature difference between the cable and the surrounding bath, or

$$H(\theta) = h(\theta - \theta_0) \ , \tag{A.6}$$

where h is called the heat transfer coefficient and is assumed to be time and temperature independent. For cooling by nucleate pool boiling of He, $h \sim 5 \times 10^4$ wm^{-2}K^{-1}.

For a single strand, $P = 2\pi a$, where a is the radius of the strand. But there is only about 5% of helium inside a composite cable, and this helium does not flow very freely between the strands. As a result, it may not be suitable to talk about the surface cooling of one strand alone. We may take the cable as a whole, which contains $S = 32$ strands arranged in two rows. Area of cable is $A \sim S\pi a^2$, whereas the perimetric circumference is $P \sim 4a(S/2 + 2)$. Therefore, roughly $P/A \sim 2/\pi a$. Each turn of the cable is piled with the broad side one upon the other so that only the narrow sides, total length $8a$, are in contact with liquid helium. Then, $P/A \sim 8/S\pi a$. We are not so sure which value of P/A should be used. In the computation, however, we take $P/A = 2/a$. Results for other values of P/A can be obtained by scaling h.

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4. see for example, A. Chao, SSC Central Design Group Report No. SSC-N-434, 1987.

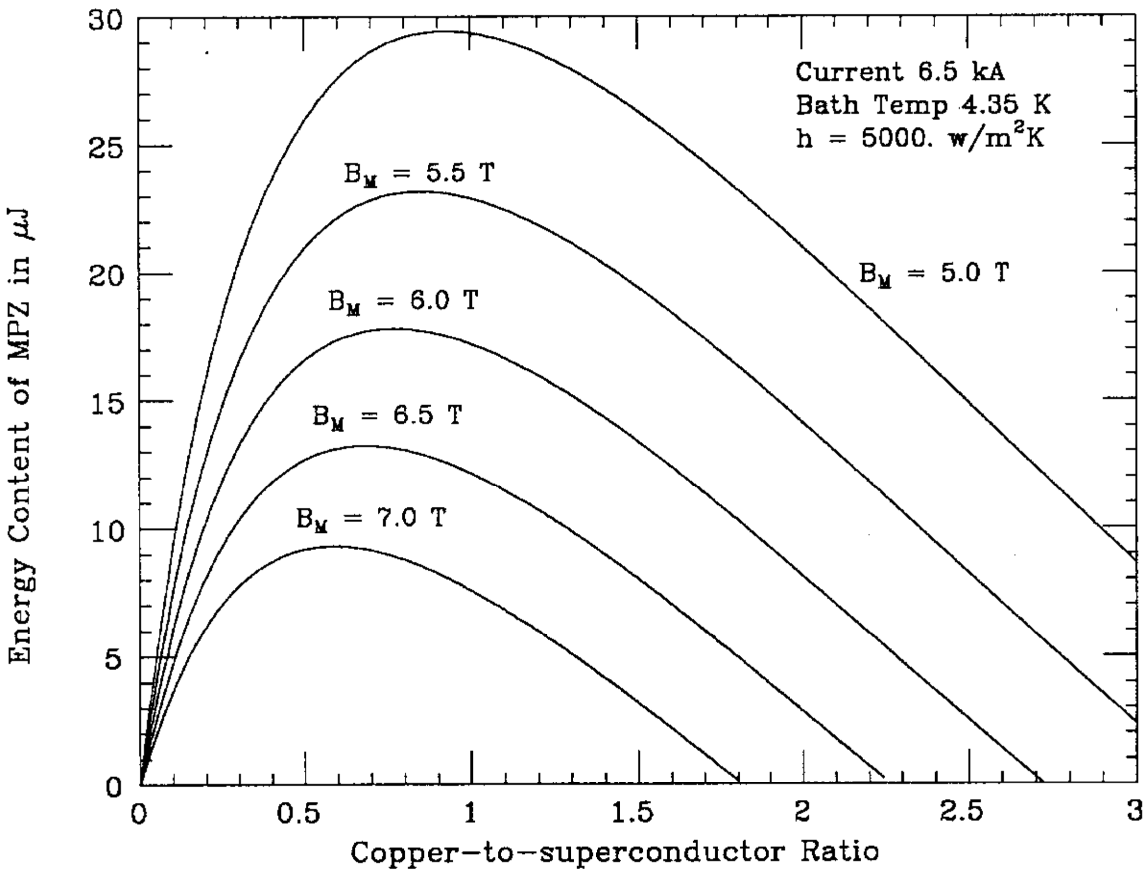

Fig. 1. Maximum nonquenching energy of point deposition is plotted as a function of copper-to-superconductor ratio r for different maximum magnetic flux density B_M. The cable carries a current of 6.5 kA at a bath temperature of 4.35 K with a surface heat transfer coefficient of 5000 w/m^{2}K. At $B_M = 7$ T, it is unlikely that the optimum copper-to-superconductor ratio can be made $\sim$ 1.3 by any simple mechanism. If a realistic coil degradation of 5% is included, the 7 T curve extends to $r = 1.65$ only, and the possibility of having $\sim$ 1.3 as the optimum ratio becomes more remote.

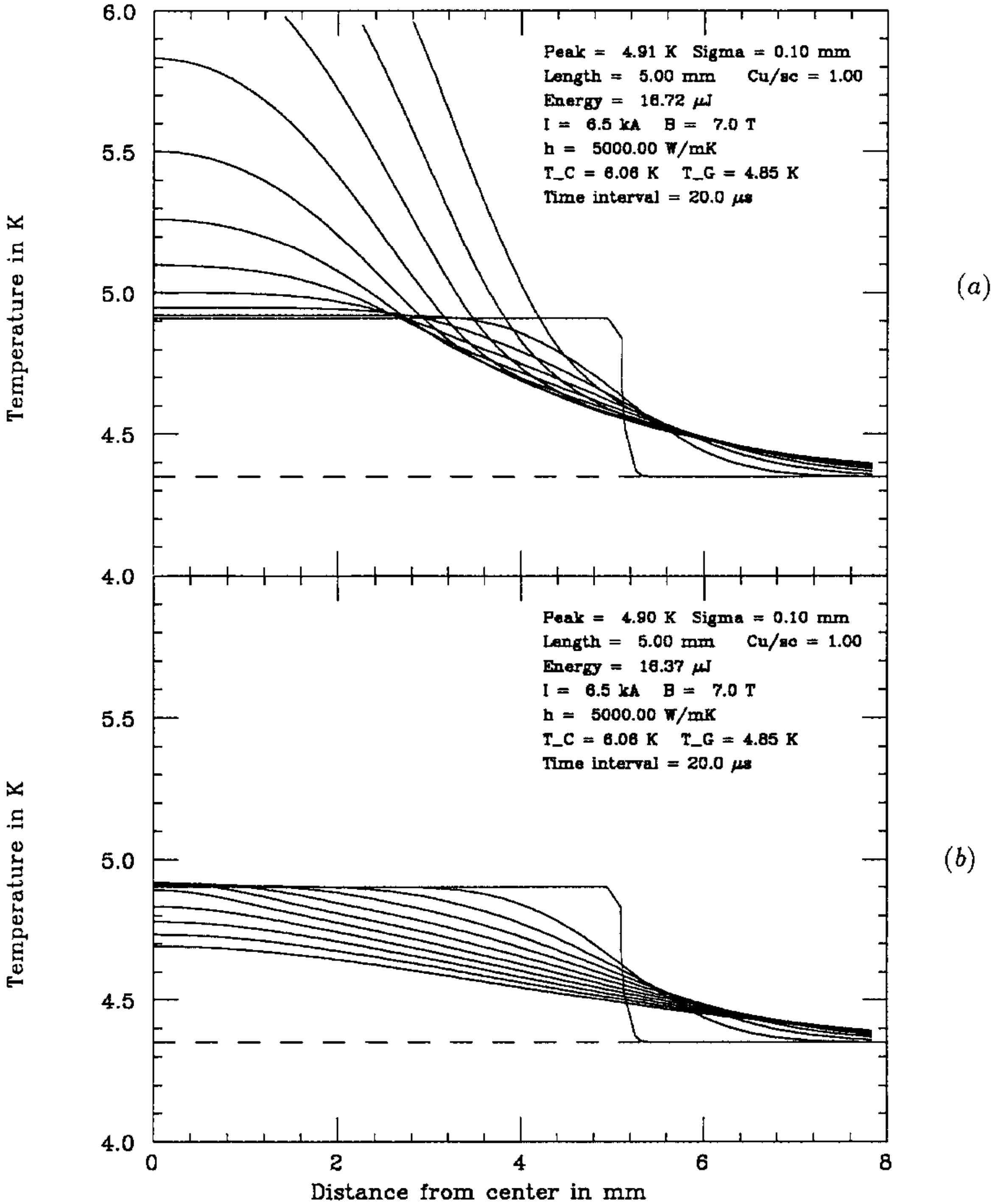

Fig. 2. Time evolution of a lengthy disturbance. (a) At $\theta_p = 4.91$ K, the disturbance spreads out to a quench. (b) At $\theta_p = 4.90$ K, the disturbance subsides.

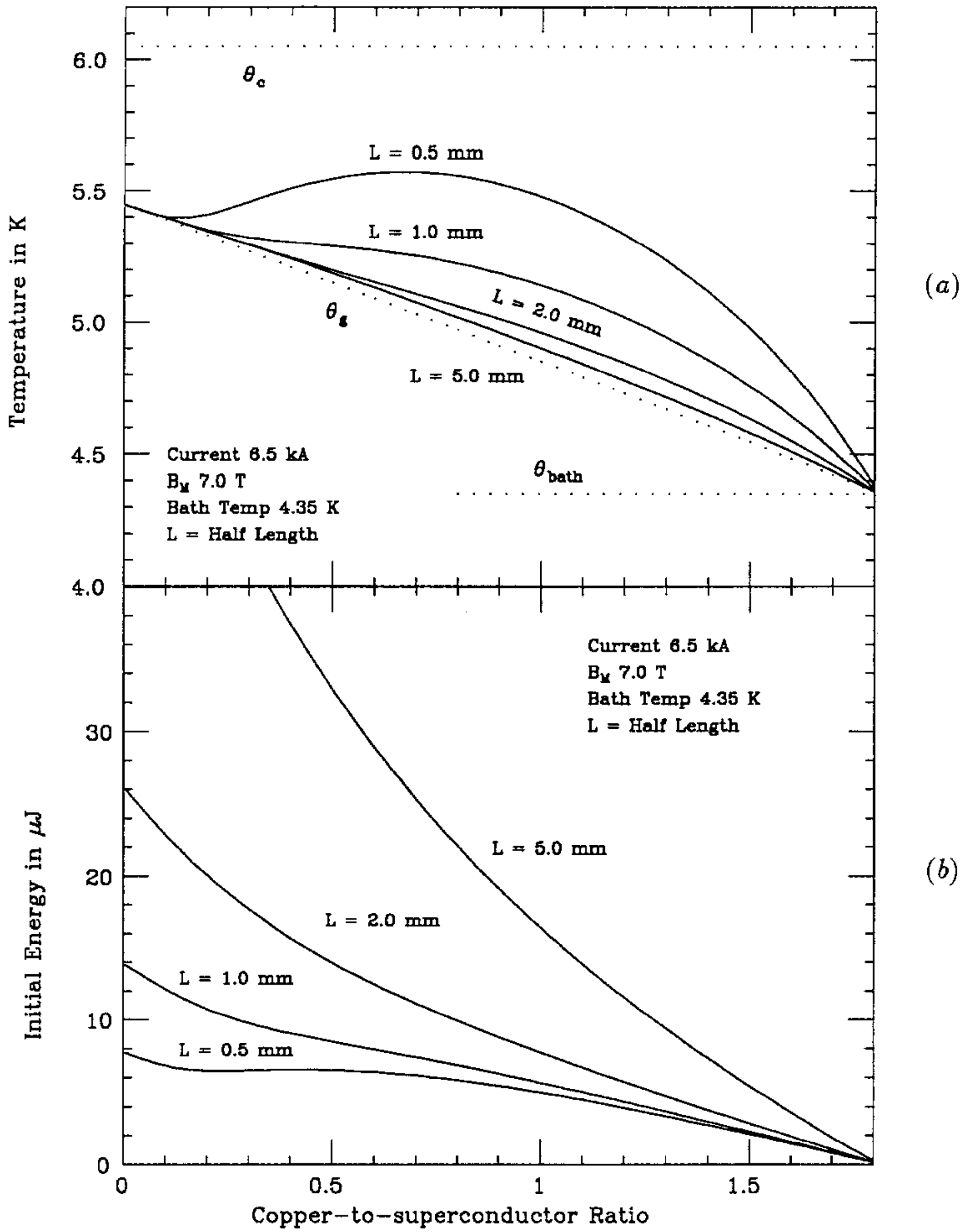

Fig. 3. (a) Fixing the half length of a disturbance at $L = 0.5$, 1, 2, or 5 mm, the maximum peak temperature allowable to avoid a quench is plotted as a function of copper-to-superconductor ratio. (b) The corresponding initial energy of the disturbance is plotted as a function of copper-to-superconductor ratio. The maximum magnetic flux density is kept at $B_M = 7$ T.

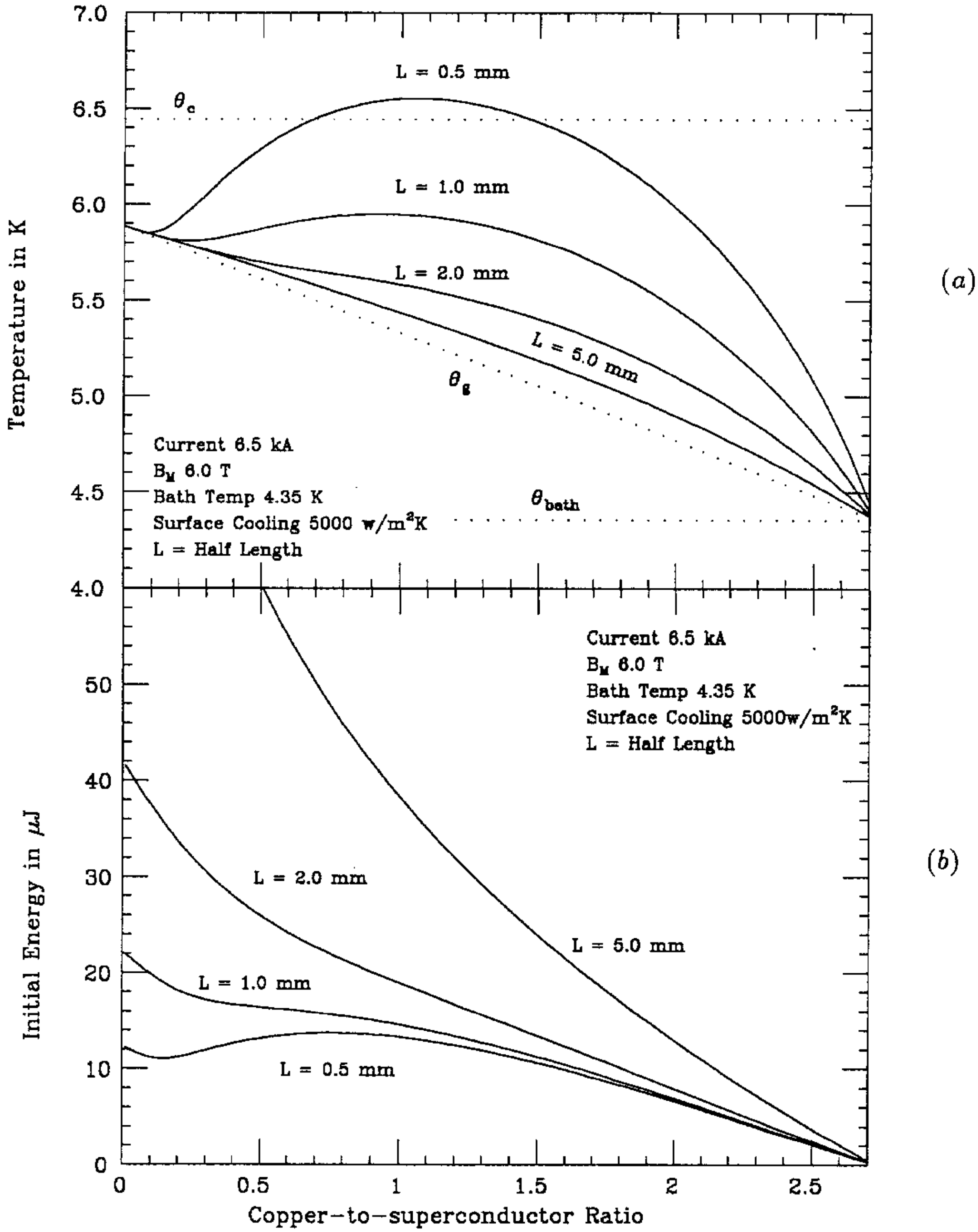

Fig. 4. Same as Fig. 2 but with maximum magnetic flux density at $B_M = 6$ T.