Fermi National Accelerator Laboratory

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Transverse Coupled-Bunch Instability in the Fermilab Main Ring

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I. INTRODUCTION

The transverse positions of all the bunches in the Fermilab Main Ring at 150 GeV recorded by the detector at E44 in September 1987 reveal a horizontal coupled-bunch growth starting 2.62 sec after injection (Fig. 1). The amplitude appears to increase by a factor of 1.7 in 1000 turns implying a growth rate of

$$\frac{1}{\tau} \approx \frac{1.7}{1000} \frac{1}{20.9 \times 10^{-6}} = 81 \text{ sec}^{-1} , \tag{1.1}$$

where $T_0 = 20.9$ μs is the revolution period around the ring. The growth of the horizontal emittance was also observed regardless of whether the horizontal damper is on or off as shown in Fig. 2.

In Section II, we try first to explain the periodic structures in Fig. 1, showing that the growth was due to a resistive wall instability. In Section III, we attpt to estimate the transverse impedance driving the growth and simulate the growth pattern of Fig. 1. In Section IV, the power of the damper required to damp out this instability is computed. The present damper is found sufficient to stabilize the growth. In Section V, we compute the growths of the emittance with damper on and off from Fig. 2. Our conclusion is that the damper might not have been working properly during the measurement.

II. TRANSVERSE COUPLE-BUNCH GROWTHS

Consider M equal bunches occupying all the rf buckets and they are performing transverse coupled-bunch motion of mode μ. At time t, the bunches have displacements:

$$\begin{array}{ll}
\text{1st bunch} & a_\mu e^{j\omega_\beta t} \\
\text{2nd bunch} & a_\mu e^{j[\omega_\beta t - 2\pi\mu/M]} \\
\cdots\cdots\cdots & \\
i\text{th bunch} & a_\mu e^{j[\omega_\beta t - 2\pi\mu(i-1)/M]} ,
\end{array} \tag{2.1}$$

where $\omega_\beta/2\pi$ is the betatron frequency and a_μ is the amplitude of oscillation of the μ-th mode. Note that a_μ can be complex.

If the first bunch passes through the detector E44 at time $t = t_0$, the ith bunch will pass through E44 at time $t = t_0 + (i-1)T_0/M$, where T_0 is the period of revolution. Thus the displacement of the ith bunch observed at E44 becomes

$$x_i = a_\mu e^{j\omega_\beta t_0} e^{j[\omega_\beta (i-1)T_0/M - 2\pi\mu(i-1)/M]} . \tag{2.2}$$

The time scale at E44 is in fact

$$\tau = \frac{(i-1)T_0}{M} , \tag{2.3}$$

because this is the time the displacement in Eq. (2.2) is observed if we set the clock to zero when the first bunch passes E44. In other words, what the detector at E44 records is

$$x(\tau) = a_\mu e^{j\omega_\beta t_0} e^{j[\omega_\beta - \mu\omega_0]\tau} ,$$

where $\omega_0/2\pi = 1/T_0$ is the revolution frequency. The above can be simplified to

$$x(\tau) = a_\mu e^{j\omega_\beta t_0} e^{j(\nu - \mu)\omega_0 \tau} , \tag{2.4}$$

where $\nu = \omega_\beta/\omega_0$ is the betatron tune.

For M equal bunches there are M modes of coupling. Thus, what we observe at E44 is

$$x(\tau) = \sum_{\mu=0}^{M-1} a_\mu e^{j(\nu - \mu)\omega_0 \tau} , \tag{2.5}$$

where we have absorbed $\exp(j\omega_\beta t_0)$ into a_μ for convenience.

Observation of Fig. 1 shows roughly 5.8 oscillations in 10 divisions or 200 μs. The period of revolution around the Main Ring is $T_0 = 20.9$ μs. This amounts to 0.606 oscillations per turn. Thus, in the sum of Eq. (2.5), the term with $\mu - \nu = 0.606$ has been excited much more than the others. The tune was $\nu \approx 19.4$ in the measurement. Therefore the instability was due to mainly mode $\mu = 20$, and suggests that the driving force was the low frequency part of the wall resistivity.

III. WALL RESISTIVITY AND LAMBERTSONS

The growth rate of the μth mode driven by the transverse coupling impedance $Z_\perp$ is[1]

$$\frac{1}{\tau_\mu} = -\frac{MI_b c}{4\pi\nu E/e} \sum_{k=-\infty}^{\infty} \mathcal{R}e\, Z_\perp[(kM - \mu + \nu)\omega_0] , \tag{3.1}$$

where E is the energy of the beam particles and I_b the *average* current of a bunch. Besides a resonance, the resistive wall can contribute to Eq. (3.1) because the impedance goes to infinity at zero frequency. The ring has a betatron tune of $\nu = 19.4$.

Therefore the coupled-bunch mode that has the fastest growth is $\mu = 20$, corresponding to the spectral line with $k = 0$ or at $|\omega/2\pi| = 0.6\omega_0/2\pi = 28.7$ kHz. The next lines are $M = 1113$ units of ω_0 away and give negligible contribution.

The vacuum chamber of the Main Ring consists of beam pipes of different cross sections: 8136 ft of 1.5 in. × 5 in. and 8496 ft of 2 in. × 4 in. sections in the dipoles, 1536 ft of rhombic pipes (approximate circular radius 3.8 cm) for the quads, 624 ft of 6 in. circular pipes in part of the straight sections, and 188.75 ft of 5 in. circular pipe in the rf region. The pipe in the rf region is of copper and the rest is of stainless steel.

Assume a conductivity of $\sigma = 1.4 \times 10^6$ $(\Omega\text{-m})^{-1}$ for stainless steel and $\sigma = 5.8 \times 10^7$ $(\Omega\text{-m})^{-1}$. Taking the thickness of the pipes as $\Delta = 1.2$ mm, the skin depth becomes equal to the pipe thickness when the harmonic $n_c = 2.63$ for stainless steel and $n_c = 0.064$ for copper.

The real part of the transverse coupling impedance for a circular beam pipe of radius b and length ℓ is

$$\mathcal{R}e\, Z_\perp = \begin{cases} \dfrac{R\ell}{\pi\Delta\sigma b^3 n} & n < n_c \\[2ex] \dfrac{R\ell}{\pi\delta\sigma b^3 n} & n > n_c\ , \end{cases} \tag{3.2}$$

If the beam pipe is rectangular in cross section with width w and height h, the impedance is[2]

$$\mathcal{R}e\, Z_\perp = \begin{cases} \dfrac{R\ell}{\pi\Delta\sigma n}\left(\dfrac{2}{h}\right)^3 F & n < n_c \\[2ex] \dfrac{R\ell}{\pi\delta\sigma n}\left(\dfrac{2}{h}\right)^3 F & n > n_c\ . \end{cases} \tag{3.3}$$

In above F is a form factor, which equals $\sim \pi^2/24$ when $w/h \gtrsim 2$. Using Eqs. (3.2) and (3.3), we find for the total transverse impedance of the beam pipes has a real part

$$\mathcal{R}e\, Z_\perp = \begin{cases} \dfrac{41.9}{n}\ \text{M}\Omega/\text{m} & n < 2.63 \\[2ex] \dfrac{25.9}{\sqrt{n}}\ \text{M}\Omega/\text{m} & n > 2.63\ . \end{cases} \tag{3.4}$$

The contribution of the copper pipe in the rf region turns out to be small and has been neglected.

At low frequencies, the wall current flows through each laminations of the Lambertson magnets. The contribution is very similar to the that of the wall resistivity.

Each lamination has a thickness of $\Delta = 0.953$ mm. The material of the laminations has a conductivity of $\sigma = 5 \times 10^6$ (Ω-m)$^{-1}$ and relative magnetic permeability of $\mu_\ell \approx 100$. Because of the high magnetic permeability, the skin depth will be very much reduced. The critical harmonic is $n_c = 0.017$ when the skin depth equals the lamination thickness. Therefore, for our consideration here, the wall current always flows in one skin depth only.

There are two 120 in. Lambertsons at E0, two 204 in. Lambertsons at F0, one 90 in. Lambertson at A0, and two 181 in. Lambertsons at C0. In total the Lambertsons occupy a length of $\ell = 1016$ in.; or there are roughly $N \approx 2.71 \times 10^4$ laminations. The laminations are approximated as annular rings of inner and outer radii $b = 1$ in. and $b + d = 2$ in. respectively and are shorted at the outer end. The longitudinal coupling impedance is[3]

$$Z_{\|} = (1+j)\frac{N}{\pi\delta\sigma}\ln\left(1+\frac{d}{b}\right) = (1+j)11.6\sqrt{n}\ \Omega\ . \tag{3.5}$$

The transverse coupling impedance is given by

$$Z_{\perp} = \frac{2R}{\bar{b}^2}\frac{Z_{\|}}{n} \approx (1+j)\frac{16.0}{\sqrt{n}}\ \mathrm{M}\Omega/\mathrm{m}\ , \tag{3.6}$$

where an average radius of $\bar{b} = 1.5$ in. has been used. We see that the contribution of the Lambertsons is significant. However, since $Z_\perp \propto b^{-3}$, our computation may not be accurate and can serve as an estimate only.

With a total number of 1.5×10^{13} particles in $M = 1113$ bunches, the growth rates for modes $\mu = 20$ to 27 are computed using Eq. (3.1). The results are listed in Table I. The maximum growth rate obtained agrees with the observation in Fig. 1.

A simulation has been performed by including equal initial amplitudes for these eight modes. Assuming that there will be some damping due to tune spread or damper, we arbitrarily reduce the each growth rate by 15.2 sec^{-1}. The growths are tracked for 3010 turns according to Eq. (2.5) with the growths embedded in the a_μ's. The result is shown in Fig. 3, where the first line is for turn 1 to 10, the second line for turn 1001 to 1010, the third line for turn 2001 to 2010, and the fourth line for turn 3001 to 3010. We see that the picture is very similar to the observation Fig. 1. Although eight modes have been included, essentially we see the periodic structure of mode $\mu = 20$ only because of its larger growth rate, while the other modes contribute through distortions of the sinusoidal behavior.

IV. ESTIMATION OF DAMPER GAIN

When the betatron amplitude is growing at a rate of $1/\tau$, the change in angle per

Mode		Growth rate in $\sec^{-1}$		
μ	$\lvert\mu - \nu\rvert$	wall	Lambertsons	total
20	0.6	65.8	19.4	85.2
21	1.6	24.7	11.9	36.6
22	2.6	15.2	9.3	24.5
23	3.6	12.8	7.9	20.7
24	4.6	11.4	7.0	18.4
25	5.6	10.3	6.4	16.7
26	6.6	9.5	5.9	15.4
27	7.6	8.8	5.5	14.3

Table I: Coupled-bunch growth rates driven by resistive wall and Lambertsons.

turn $\Delta x'$ with respect to the ideal orbit is given by

$$\frac{1}{\tau} = \frac{1}{x}\frac{dx}{dt} = \frac{\Delta x'}{x}\frac{\bar{\beta}}{T_0} , \tag{4.1}$$

where $\bar{\beta}$ is the average beta-function of the Main Ring, or

$$\left.\frac{\Delta x'}{x}\right|_{\mathrm{growth}} = \frac{2\pi R}{\bar{\beta} c}\frac{1}{\tau} . \tag{4.2}$$

The horizontal damper consists of two vertical plates of length $\ell = 1$ m separated by a horizontal distance of $d = 8$ cm. The damping is performed by applying a voltage difference G across the plates per unit of horizontal displacement of the bunch. This G is also known as the gain of the damper.

If the bunch is displaced horizontally by a distance x from the center of the beam pipe, the electric field set up between the damper plates is

$$\mathcal{E} = \frac{2Gx}{d} , \tag{4.3}$$

so that the bunch suffers a kick with a change in angle $\Delta x'$ given by

$$\frac{1}{2}\,\Delta x'\ell = \frac{1}{2}\,\frac{e\mathcal{E}}{\gamma m}\frac{\ell^2}{c^2}\ , \tag{4.4}$$

where m is the rest mass of proton and $E = \gamma mc^2$ its total energy. Or

$$\left.\frac{\Delta x'}{x}\right|_{\text{damper}} = \frac{2G\ell}{d(E/e)}\ . \tag{4.5}$$

Therefore, to damp out the growth, we must have

$$\frac{2G\ell}{dE/e} > \frac{2\pi R}{\bar{\beta}c}\frac{1}{\tau}\ , \tag{4.6}$$

or

$$G > \frac{\pi RDE/e}{\bar{\beta}c\ell}\frac{1}{\tau}\ . \tag{4.7}$$

With $R/\bar{\beta} \approx \nu$, to damp out a growth of 85.2 sec^{-1} at 150 GeV, we need a damper gain of at least $G = 0.21$ kV/mm.

A measurement of the damper gain gives $G \approx 2$ kV/mm and the maximum damper voltage available is 2.3 kV. For mode $\mu = 20$, the pattern repeats every $1113/20 = 55.7$ bunches. Thus the pickup must have a resolution better than $\sim 55.7/(2 \times 52)$ μs = 530 ms, where a rf frequency of 52 MHz has been used. In other words, the bandwidth must be bigger than 0.30 MHz. The present damper has a bandwidth from 2 to 4 MHz and a rise time of ~ 300 ns. These figures seem to be contradicting. But in any case, the time resolution of the damper is adequate. Thus, it appears that the damper should be capable of damping out the growth due to wall resistivity.

V. FURTHER ESTIMATIONS OF GROWTH RATES

Growth of horizontal emittance was observed in Fig. 2a with horizontal damper off and in Fig. 2b with horizontal damper on. The observations were made at E44 starting from 2.59 sec. In Fig. 2a, we see that the growth starts at ~ 2.64 sec. We try to compute the growth rate for each 10 ms period up to 2.74 sec and find an average growth rate of $1/\tau \approx 124$ sec^{-1}. When the damper is on, the growth starts later at 2.67 sec. The average growth rate computed for the next 5 consecutive 10 ms periods is ~ 139 sec^{-1}. So there is not much difference between having the damper on or of, although the damper did delay the start of the growth by ~ 30 ms. Therefore, it appears that the damper might not have been operating properly.

The horizontal damper is designed to damp out transverse couple-bunch instability at injection, where the growth rate is biggest according to Eq. (3.1). Theoretically, the factor of energy in the denominators of both sides of Eq. (4.6) cancel. This implies that a damper gain that is sufficient to damp out a growth at low energies should be able to damp out the same growth at higher energies. Unfortunately, this is not true in practice and the actual power of the damper may become weaker when the energy increases. This may explain why the observed growths had not been damped.

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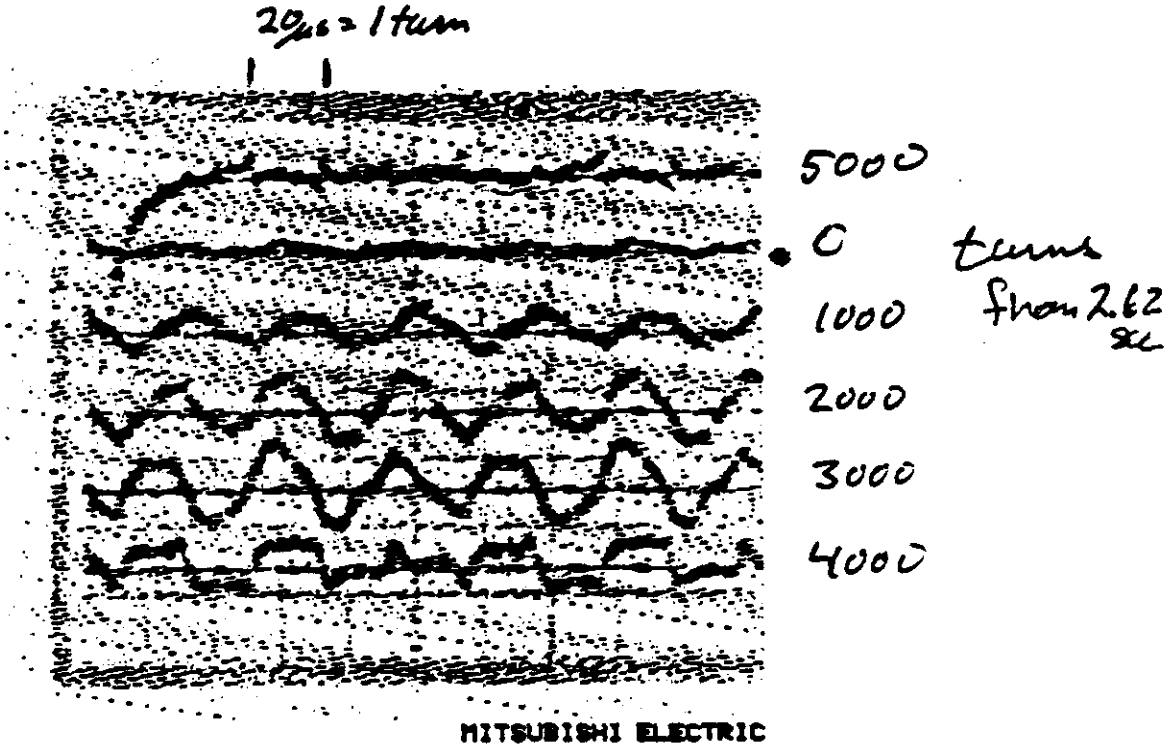

MR Instability -
150 Gev

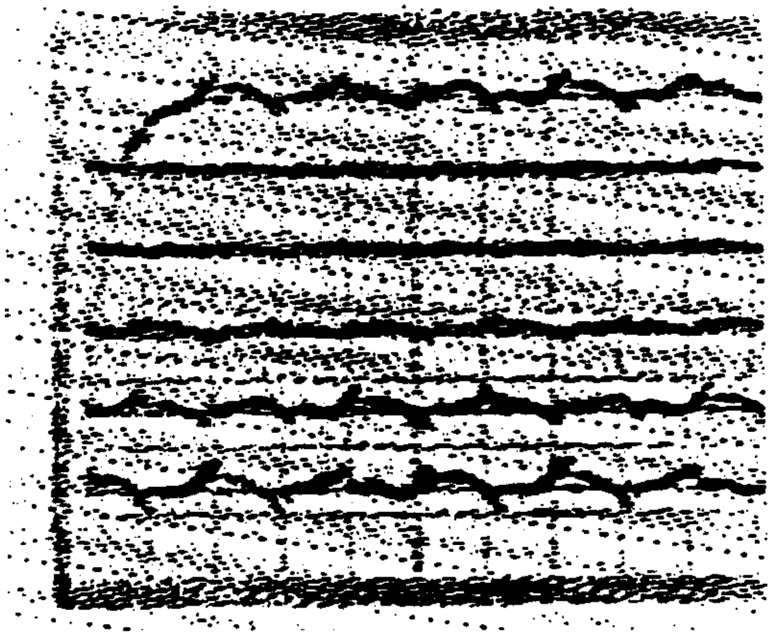

Fig. 1. Observed horizontal displacement of bunches at E44 at 150 GeV. Each line comprises 200 μs or $\sim$ 10 turns. Each line is separated from the next one by $\sim$ 1000 turns.

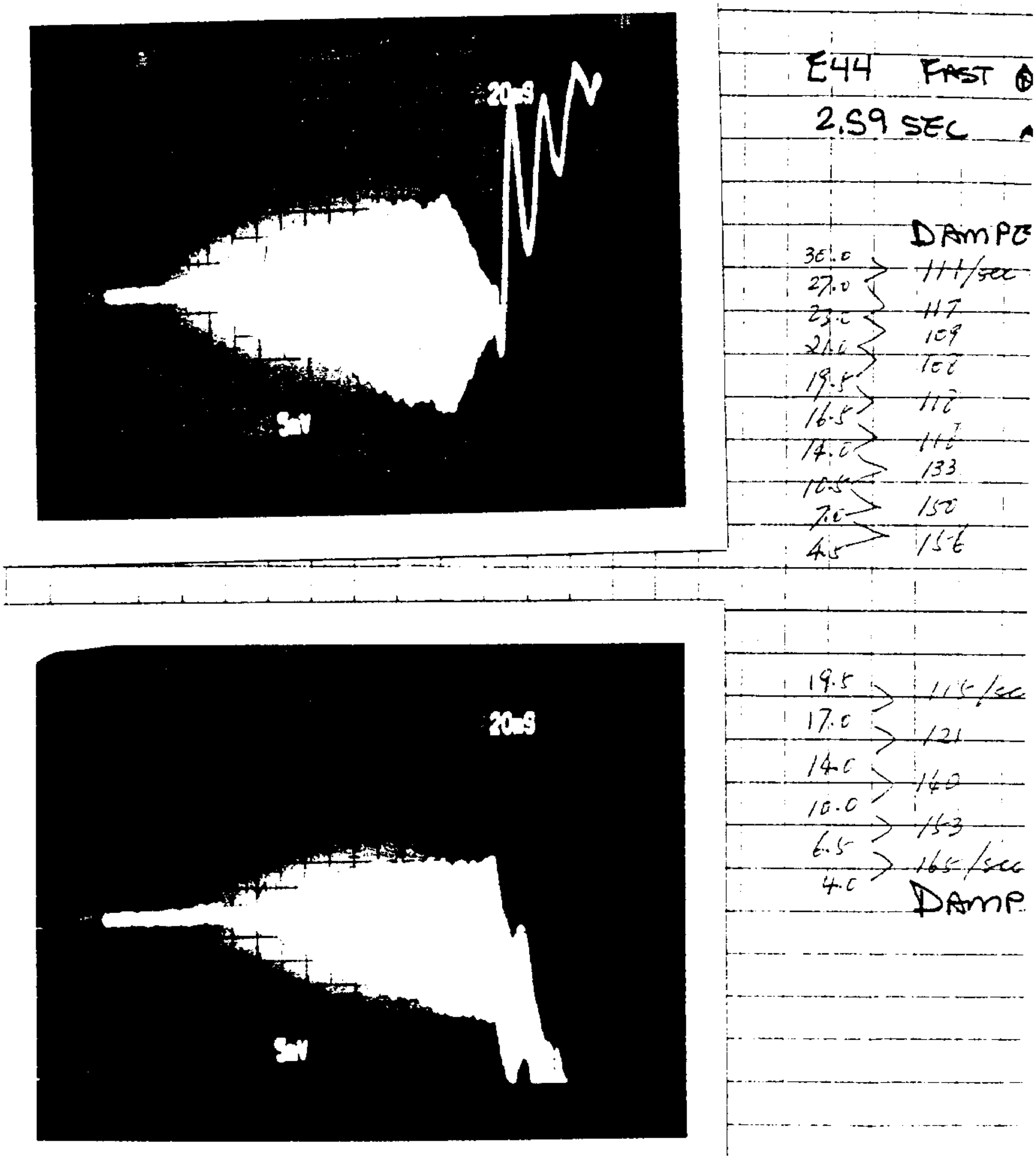

Fig. 2. Observed growth in horizontal emittance at E44 at 150 GeV with horizontal damper off (top) and on (bottom).

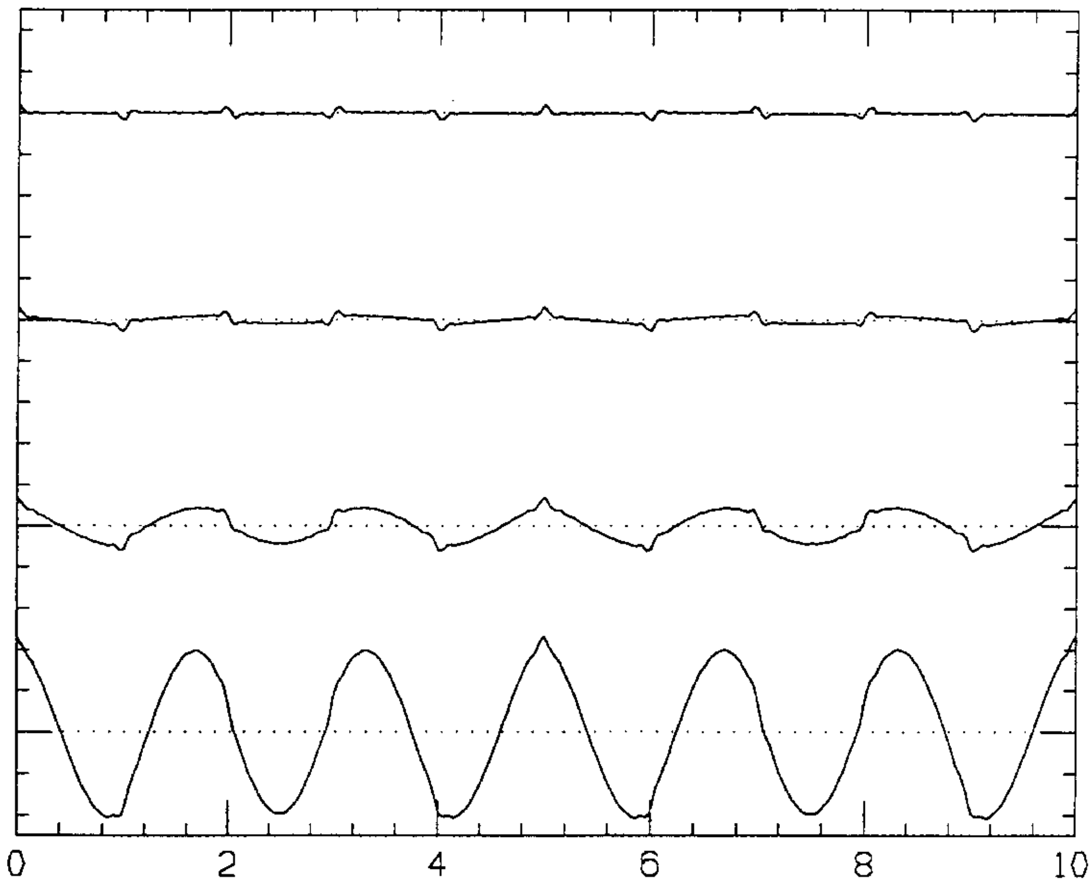

Fig. 3. Simulation of Fig. 1 assuming that the growth is driven by the wall resistivity of the beam pipe and the Lambertsons. Transverse coupled-bunch modes $\mu = 20$ to 27 are included. The coupled amplitudes for each mode are assumed equal initially. The first line is for turn 1 to 10, the second line for turn 1001 to 1010, the third line for turn 2001 to 2010, and the fourth line for turn 3001 to 3010.