national accelerator laboratory

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ONE-TURN EXTRACTION USING PINGERS

L. C. Teng

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In the NAL terminology there are two types of devices for fast transverse deflection of the beam--the kicker and the pinger. A kicker is a superfast delay line type of device which can produce a square-wave deflection with a rise time in the range of tens of nanoseconds. A pinger is a not-so-fast resonant device which can only produce a sine-wave deflection with a period in the range of tens of microseconds and is, therefore, a much simpler and cheaper device. For one-turn extraction the most straightforward way is to use a kicker. We will show, here, that one-turn extraction can be accomplished quite well using a few (2 or 3) pingers with amplitudes and timing properly adjusted.

We shall assume that all the pingers are placed at locations with identical $\beta = \beta_{max}$ (hence $\alpha = 0$) so that the betatron oscillation advances from pinger to pinger as a sinusoidal oscillation with the betatron phase advance as the argument. We also assume that the septum (negligible thickness) is properly located about a quarter wavelength downstream of the first pinger (P_1) with no other pinger in between P_1 and the septum such that the desired combined action of the pingers is a pure angle deflection at P_1 with zero position displacement. All pingers are assumed to have the same period and are crowbarred after the first half-oscillation.

The angle deflection caused by P_1 at P_1 may, then, be written as

$$\sin\phi$$

where ϕ is the pinger phase and where the amplitude is normalized to unity. One revolution around the accelerator corresponds to a pinger phase θ (Fig. 1). The pinger amplitude and period is so adjusted

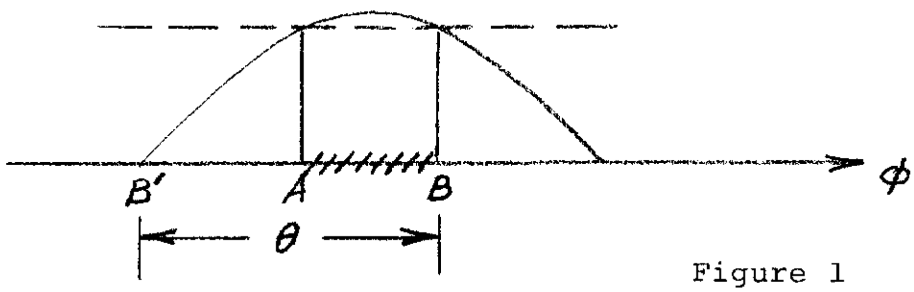

Figure 1

that the shaded part $\overline{AB}$ of the beam receiving the largest angle-deflection is kicked across the septum and extracted out of the machine during the first passage through P_1. The remaining part $\overline{B'A}$ of the beam not having received a sufficiently large kick from P_1 will go around the ring and receive a second kick

$$\sin(\phi+\theta)$$

from P_1. In the meantime the first kick it received ($\sin\phi$) from P_1 will have propagated in betatron oscillation to give an angle-displacement vector at P_1 of

$$e^{i2\pi\nu}\sin\phi$$

where, so written, the real part is the angle and the imaginary part is the displacement. The total of the two kicks received is, therefore, at P_1

$$e^{i2\pi\nu}\sin\phi + \sin(\phi+\theta). \tag{1}$$

Generally, this does not give this part $\overline{B'A}$ of the beam the desired pure angle-deflection of adequate magnitude. This is fixed by additional pingers placed at proper betatron-phase from P_1, turned on in succession at the proper time, and having the proper amplitude.

A. <u>Two-Pinger Case</u>

In this case the optimal arrangement is clearly for $\overline{B'A} = \overline{AB}$ so that

$$\theta = \frac{2\pi}{3} .$$

The second pinger P_2 is turned on with amplitude A_2 when the point B' (the point on the beam which is at P_1 when P_1 is turned on) arrives at P_2. The betatron-phase advance from P_2 to P_1 is denoted by δ_2. The additional angle-displacement vector at P_1 due to P_2 is, hence

$$A_2 e^{i\delta_2} \sin\phi \tag{2}$$

The total angle-displacement vector of beam $\overline{B'A}$ at P_1 on the second turn is the sum of (1) and (2), namely

$$\left(A_2 e^{i\delta_2} + e^{i2\pi\nu}\right) \sin\phi + \sin(\phi+\theta)$$

$$= \left[(A_2 \cos\delta_2 + \cos 2\pi\nu + \cos\theta) \sin\phi + \sin\theta \cos\phi\right]$$

$$+ i\ (A_2 \sin\delta_2 + \sin 2\pi\nu) \sin\phi .$$

In order that this be a pure angle kick with unit amplitude centered in the middle of $\overline{B'A}$ we must have

$$A_2 \sin\delta_2 + \sin 2\pi\nu = 0 \tag{3}$$

and

$$(A_2 \cos\delta_2 + \cos 2\pi\nu + \cos\theta)\sin\phi + \sin\theta\cos\phi = \sin\left(\phi+\frac{\theta}{2}\right)$$

or

$$\begin{cases} 1 = (A_2 \cos\delta_2 + \cos 2\pi\nu + \cos\theta)^2 + \sin^2\theta \\ \quad = 1 + (A_2\cos\delta_2 + \cos 2\pi\nu)(A_2\cos\delta_2 + \cos 2\pi\nu + 2\cos\theta) \\ \tan\frac{\theta}{2} = \dfrac{\sin\theta}{A_2\cos\delta_2 + \cos 2\pi\nu + \cos\theta} \end{cases} \tag{4}$$

Eq. (4) gives either

$$\begin{cases} A_2\cos\delta_2 + \cos 2\pi\nu = 0 \\ \tan\frac{\theta}{2} = \tan\theta \end{cases}$$

which is not a useful solution, or

$$\begin{cases} A_2\cos\delta_2 + \cos 2\pi\nu = -2\cos\theta \\ \tan\frac{\theta}{2} = -\tan\theta \qquad \text{or} \qquad \frac{\theta}{2} = \pi-\theta \qquad \text{or} \qquad \theta = \frac{2\pi}{3} \end{cases} \tag{5}$$

which is the solution we want. With $\theta = \frac{2\pi}{3}$, Eq. (3) and the first of Eq. (5) give

$$\begin{cases} A_2\sin\delta_2 + \sin 2\pi\nu = 0 \\ A_2\cos\delta_2 + \cos 2\pi\nu = 1 \end{cases}$$

or

$$A_2 = 2\sin\pi\nu \qquad \delta_2 = \pi\left(\nu-\frac{1}{2}\right) + 2n\pi \qquad (n = \text{integer})$$

$\left[\delta_2 = \pi\left(\nu-\frac{1}{2}\right) + n\pi\right.$ if the kick by P_2 can have opposite sign.$\left.\right]$

With $\nu = 20\frac{1}{4}$ we have

$$A_2 = \sqrt{2} \qquad \delta_2 = 2n\pi - \frac{\pi}{4}$$

and with $\nu = 20\frac{1}{2}$ we have

$$A_2 = 2 \qquad \delta_2 = 2n\pi$$

The recipe is, now, as follows:

Pinger half-period = $\frac{3}{2}$ (beam revolution time).
Betatron phase from P_2 to P_1 = $\pi(\nu-\frac{1}{2}) + n\pi$.
Amplitude of P_1 adjusted so that a kick of $\cos\frac{\pi}{6}$ (amplitude) displaces the beam by its full width at the septum.
Amplitude of P_2 = 2 $\sin\pi\nu$ (amplitude of P_1).
P_2 turn-on is delayed from P_1 turn-on by the beam transit time from P_1 to P_2.

The angle-deflection of the entire beam at P_1 now looks like (Fig. 2)

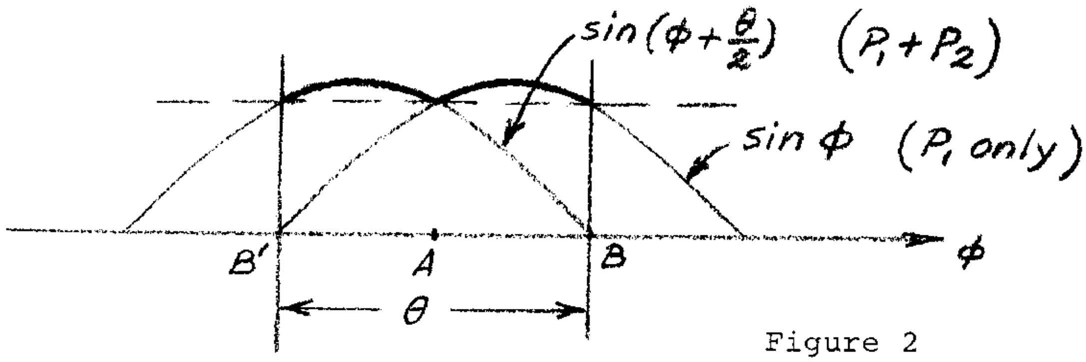

Figure 2

The relative deflection ripple over the entire beam is

$$\frac{1}{\cos\frac{\pi}{6}} - 1 = 15\%.$$

Of course, the beam comes out in the order

AB(B')A.

B. Three Pinger Case

In this case the optimum is for $\overline{B'A} = 2\overline{AB}$ so that

$$\theta = \frac{3\pi}{5} .$$

The second pinger should be so adjusted that

$$A_2 \sin\delta_2 + \sin 2\pi\nu = 0 \tag{6}$$

and

$$(A_2 \cos\delta_2 + \cos 2\pi\nu + \cos\theta) \sin\phi + \sin\theta \cos\phi = \sin\left(\phi + \frac{2\theta}{3}\right)$$

or

$$\begin{cases} 1 = 1 + (A_2 \cos\delta_2 + \cos 2\pi\nu)(A_2 \cos\delta_2 + \cos 2\pi\nu + 2 \cos\theta) \\ \tan\frac{2\theta}{3} = \dfrac{\sin\theta}{A_2\cos\delta_2 + \cos 2\pi\nu + \cos\theta} \end{cases} \tag{7}$$

The solution of Eq. (7) which we want is

$$\begin{cases} A_2 \cos\delta_2 + \cos 2\pi\nu = -2\cos\theta \\ \tan\frac{2\theta}{3} = -\tan\theta \qquad \text{or } \frac{2\theta}{3} = \pi - \theta \qquad \text{or } \theta = \frac{3\pi}{5} \end{cases} \tag{8}$$

Eq. (6) and the first of Eq. (8) give

$$\begin{cases} A_2 \sin\delta_2 + \sin 2\pi\nu = 0 \\ A_2 \cos\delta_2 + \cos 2\pi\nu = -2\cos\frac{3\pi}{5} = \frac{\sqrt{5}-1}{2} \end{cases}$$

or

$$\begin{cases} A_2 = \sqrt{(\sqrt{5}-1)\left(\frac{\sqrt{5}}{2} - \cos 2\pi\nu\right)} \\ \tan\delta_2 = \dfrac{\sin 2\pi\nu}{\cos 2\pi\nu - \frac{\sqrt{5}-1}{2}} \end{cases}$$

With $\nu = 20\frac{1}{4}$ we have

$$\begin{cases} A_2 = \sqrt{(\sqrt{5}-1)\sqrt{5}/2} = 1.1756 \\ \tan\delta_2 = -\dfrac{2}{\sqrt{5}-1} \qquad \text{or} \qquad \delta_2 = 2n\pi - 1.0172 \end{cases}$$

The third pinger P_3 should be turned on with amplitude A_3 when the midpoint C between B' and A (Fig. 3) arrives at P_3. Since the

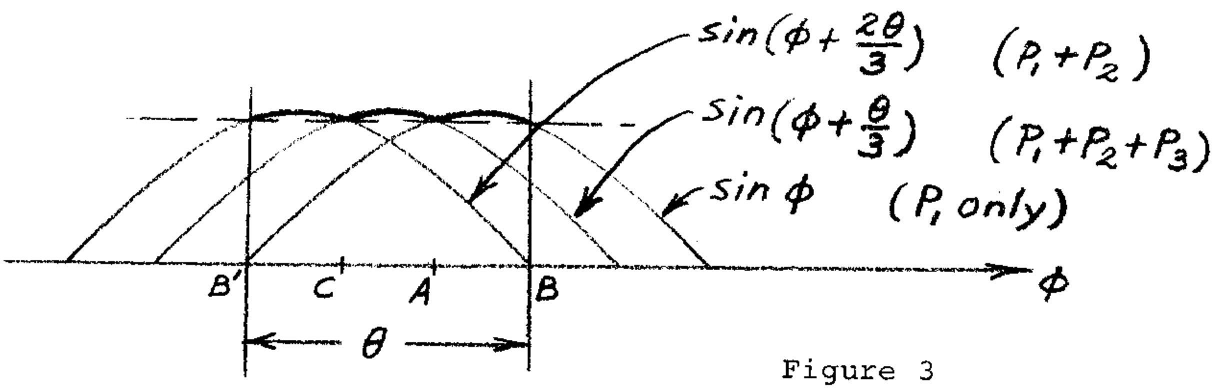

Figure 3

combination of the 2 kicks by P_1 and the kick by P_2 already produces a pure angle deflection at P_1 the betatron-phase from P_3 to P_1 should be an integral multiple of 2π (or π if the kick by P_3 can have opposite sign). The kick by P_3 will, then, produce a pure angle-deflection at P_1 of

$$A_3 \sin(\phi-\phi_3)$$

which when combined with that produced by P_1 and P_2, namely $\sin\left(\phi+\frac{2\theta}{3}\right)$, should give $\sin\left(\phi+\frac{\theta}{3}\right)$. This condition gives

$$\begin{aligned}
A_3 \sin(\phi-\phi_3) &= \sin\left(\phi+\frac{\theta}{3}\right) - \sin\left(\phi+\frac{2\theta}{3}\right) \\
&= \sin\left(\phi+\frac{\theta}{2}-\frac{\theta}{6}\right) - \sin\left(\phi+\frac{\theta}{2}+\frac{\theta}{6}\right) \\
&= -2 \sin\frac{\theta}{6} \cos\left(\phi+\frac{\theta}{2}\right) \\
&= 2 \sin\frac{\theta}{6} \sin\left(\phi+\frac{\theta-\pi}{2}\right).
\end{aligned}$$

For $\theta = \frac{3\pi}{5}$ we have

$$\begin{cases} A_3 = 2 \sin\frac{\pi}{10} = 0.6180 \\ \phi_3 = -\frac{\pi}{5} \text{ (showing that } P_3 \text{ should be turned on when point C arrives at } P_3\text{)} \end{cases}$$

The recipe is, now, as follows:

Pinger half-period = $\frac{5}{3}$ (beam revolution time).

Betatron phase from P_2 to P_1 = $\tan^{-1}\dfrac{\sin 2\pi\nu}{\cos 2\pi\nu-\frac{\sqrt{5}-1}{2}}$.

Betatron phase from P_3 to P_1 = $n\pi$.

Amplitude of P_1 adjusted so that a kick of $\cos\frac{\pi}{10}$ (amplitude) displaces the beam by its full width at the septum.

Amplitude of P_2 = $\sqrt{(\sqrt{5}-1)\left(\frac{\sqrt{5}}{2} - \cos 2\pi\nu\right)}$ (amplitude of P_1).

Amplitude of P_3 = $2 \sin\frac{\pi}{10}$ (amplitude of P_1).

P_2 delayed from P_1 by beam transit time from P_1 to P_2.

P_3 delayed from P_1 by beam transit time from P_1 to P_3 plus $\frac{1}{3}$ (beam revolution time).

The relative deflection ripple over the beam is

$$\frac{1}{\cos\frac{\pi}{10}} - 1 = 5\%.$$

The beam comes out in the order

AB(B')CA

The generalization to 4 or more pinger cases is obvious. But further improvement in reducing the relative deflection ripple over the beam is rather minor and unnecessary.