



IMPROVED NUCLEON-MESON CASCADE CALCULATIONS

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November 21, 1969

I. INTRODUCTION

This is a report of some improvements in the nucleon-meson cascade calculation in shielding materials used by the computer program TRANSK. The most significant changes are,

- (i) Introduction of energy-dependent nuclear nonelastic cross sections;
- (ii) A new formula for the production of neutrons and protons obtained by fitting data from H, Be, Al, Cu and Pb targets. The Trilling formula is used for pion production. It was modified to give better agreement with experimental data for large A materials;
- (iii) Ionization energy losses including electron binding energies and density effect corrections;
- (iv) Separate calculations of p, n, and  $\pi$  flux densities since the nonelastic cross section is energy dependent;

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- (v) Improved treatment of energy conservation using new values for inelasticities and nuclear excitation energies;
- (vi) Introduction of kinematic limits for pion production;
- (vii) Simulation of concrete and heavy concrete shielding materials by the inclusion of elements weighted by their atomic abundance as determined by a chemical analysis of actual NAL concrete samples.

## 2. Elastic and Nonelastic Cross Sections

The total elastic cross sections measured by Bellettini, et al.<sup>1</sup> can be represented by the relation

$$\sigma_{\text{tot. el.}}(A) = 10^{-2.195} A^{1.04} \quad [10^{-24} \text{ cm}^2] \quad (1)$$

where  $A$  is the atomic weight of the target nucleus. The scattering length is determined by

$$\lambda_{\text{el.}}(A) = \frac{A}{\sigma_{\text{tot. el.}}(A) \cdot N_0} \quad [\text{gm/cm}^2] \quad (2)$$

where  $N_0$  is Avogadro's number.

In the cascade calculation only the primary protons are elastically scattered and the cross sections are assumed to be independent of the proton energy since all the incident protons are essentially removed before their energy is significantly reduced.

Nonelastic cross sections of protons, neutrons and pions are needed for energies between tens of MeV and hundreds of GeV. The energy dependence of proton nonelastic cross sections is

introduced into the calculation using the compilation of data by Barashenkov, et al.<sup>2</sup> for energies between 50 MeV and 30 GeV.

Nonelastic cross sections for nine target materials between 50 MeV and 10 GeV are put into a table and stored in a two dimensional array. The cross sections for other target materials are determined by logarithmic interpolation at each energy. A linear interpolation is used to determine the cross sections for other primary momenta. The cross sections are assumed constant above  $P_0 = 10$  GeV/c. The formulae used are

$$\sigma(A) = \sigma_1 \left( \frac{A}{A_1} \right)^B \quad (3)$$

$$B = \log \left( \frac{\sigma_2}{\sigma_1} \right) / \log \left( \frac{A_2}{A_1} \right) \quad (4)$$

Neutron and pion cross sections have been shown to be smaller than proton cross sections<sup>3</sup>

$$\sigma_{nN} = \sigma_{pN} / 1.07 \quad (5)$$

$$\sigma_{\pi N} = \sigma_{pN} / 1.20 \quad (6)$$

The same energy dependence of  $\sigma_{p,N}$  is assumed for  $\sigma_{\pi,N}$  except for pions below 0.35 GeV, where actual experimental values are used<sup>3</sup>.

The production cross sections above 10 GeV as given by Barashenkov, et al.<sup>2</sup> are smaller than those reported by Bellettini<sup>1</sup>. We use Barashenkov's values to get a more

conservative estimate of shielding requirements.

3. Proton and Pion Production Formulas

A fit of the thermodynamical formulae for particle production<sup>11</sup> has recently been completed for the latest experimental data<sup>4,5,6,7</sup>. However, these formulae are impractical in a Monte Carlo calculation of random events. Hence, the modified Trilling formula for proton production<sup>8</sup> and pion production<sup>9</sup> was used since it gives fair agreement with experimental results. A new fit to particle production data in p-p collisions<sup>4, 5</sup> and p-Be, p-Al, p-Cu, p-Pb collisions<sup>7</sup> has also been made.

The measured data are given as

$$\frac{d^2\sigma}{dp d\Omega}$$

where the nonelastic cross sections are used to calculate

$$\frac{d^2N}{dp d\Omega} \left[ \frac{\text{Number of particles}}{\text{GeV/c} \cdot \text{Steradian} \cdot \text{Interacting proton}} \right]$$

At 19.2 GeV, the nonelastic cross sections are<sup>1</sup>

TABLE I.

Target	$\sigma_{ne}$ (barns)
H	0.0315
Be	0.227
Al	0.472
Cu	0.850
Pb	1.750

Nucleon Production Spectra:

The following modified Trilling formula is used to fit the experimental data

$$\frac{d^2N}{dp d\Omega} = \left[ \frac{A_1}{p_0} + \frac{A_2 p}{p_0^2} \left| 1 + \sqrt{1 + \left(\frac{p_0}{m_p}\right)^2} - \frac{p_0}{p} \sqrt{1 + \left(\frac{p}{m_p}\right)^2} \right| \right] \cdot p^2 \cdot \left[ 1 + \sqrt{1 + \left(\frac{p_0}{m_p}\right)^2} - \frac{p_0 p}{m_p^2 \sqrt{1 + \left(\frac{p}{m_p}\right)^2}} \right] \exp(-A_3 p^2 \theta^2) \quad (7)$$

Table II lists the parameters at 19.2 GeV/c for an H target. This energy is chosen since data is also available for other target materials.

TABLE II.

Parameters  $A_i$  of the improved Trilling formula obtained by a fit with data at 19.2 BeV/c

Target Material	$A_1$	$A_2$	$A_3$
H	0.885	0.101	4.256
Be	0.756	-0.192	4.058
Al	0.856	-0.555	3.902
Cu	0.855	-0.712	3.759
Pb	0.744	-0.730	3.580

Table III lists the following integral parameters derived from the spectra: proton multiplicities ( $n_p$ ) inelasticities ( $K_p$ ) and mean transverse momentum ( $\bar{p}_t$ ).

TABLE III.

Multiplicities  $n_p$  inelasticities  $K_p$  and mean transverse momenta of protons resulting from the fitted formula (1) at  $p_0 = 19.2$  GeV/c

Target Material	$n_p$	$K_p$	$\bar{p}_t$ (GeV/c)
H	1.16	0.40	0.40
Be	0.91	0.29	0.41
Al	0.85	0.25	0.42
Cu	0.78	0.21	0.42
Pb	0.63	0.16	0.44

The values obtained for particle production on H targets are in fair agreement with experimental results. The baryon multiplicity should be about  $n_B = 2$ . Since the number of secondary protons in pp collisions is larger than the number of neutrons, the value  $n_p = 1.16$  means that total neutron plus hyperon multiplicity is

$$n_n + n_y = 0.84$$

The inelasticities and mean transverse momentum agree with empirical data. Therefore, one can conclude that proton production on H targets is well represented by formula 7.

About equal numbers of secondary protons and neutrons are expected in p-nucleus collisions. One also expects  $n_p$  to increase with an increase in atomic weight of the target<sup>5</sup> material because of the intranuclear cascade. However, the inelasticity  $K_p \sim 0.3$  should be independent of the target material.

This is in contradiction with the data in Table III. The reason for this disagreement can be explained by the fact that all measurements at  $p_0 = 19.2$  GeV/c were made for secondary momenta  $p > 6$  GeV/c. The parameters in the fit using formula 7 reflect the behavior of the spectra at high momenta. The intranuclear cascade attenuates the production of protons at high secondary momentum. These protons undergo secondary and tertiary collisions inside the nucleus creating particles with lower secondary momentum. Thus the multiplicities should indeed increase if the entire secondary momentum spectrum is included.

Data is not available to perform a fit with a modified formula. The only way to correct the behavior of the formula before such data becomes available is to alter the formula to increase the production at small secondary momentum without affecting the production at high momentum.

This is done in such a way that the inelasticities for all target materials is on the order of 0.28 - 0.30 and the multiplicities  $n_p$  increase to at least  $n_p = 1$  for Be and possibly up to  $n_p = 2$  for heavier materials. The transverse momentum should remain unchanged. This is accomplished by introducing a fourth parameter  $A_4$  in the following manner

$$\frac{d^2N}{dpd\Omega} = \left[ \frac{A_1}{p_0} \left( 1 + \frac{A_4}{(1+p)} + \frac{1.5 A_4}{(1+p^2)} \right) + \frac{A_2 p}{p_0^2} \left| 1 + \sqrt{1 + \left( \frac{p_0}{m_p} \right)^2} - \frac{p_0}{p} \sqrt{1 + \frac{p^2}{m_p^2}} \right| \right] \cdot p^2 \cdot \left[ 1 + \sqrt{1 + \frac{p_0^2}{m_p^2}} \frac{-p_0 p}{m_p^2 \sqrt{1 + \frac{p^2}{m_p^2}}} \right] \exp(-A_3 p^2 \theta^2) \quad (8)$$

By trial and error one finds the following values for  $A_4$

TABLE IV.

Target Material	$A_4$
H	0.0
Be	0.2
Al	0.5
Cu	1.0
Pb	2.0

Table V lists the values of  $n_p$ ,  $K_p$  and  $\bar{p}_t$  obtained using formula 8.

TABLE V.

Multiplicities  $n_p$  inelasticities  $K_p$  and mean transverse momenta  $\bar{p}_t$  of protons resulting from formula (8) at  $p_0=19.2$  GeV/c

Target Material	$n_p$	$K_p$	$\bar{p}_t$
Be	0.99	0.30	0.40
Al	1.17	0.28	0.40
Cu	1.47	0.27	0.40
Pb	1.92	0.28	0.39

The agreement between the measured and calculated spectra at  $p > 6$  GeV/c is unchanged while the spectra at small momenta are strongly enhanced.

The results of formula 8 can be compared with the experimental data of Dekkers, et al<sup>10</sup>. (It should be noted that this data is not included in the fit since the absolute normalization of this data appears doubtful). In this experiment the proton production on Be and Pb targets at  $p_0 = 18.8$  GeV/c was measured for  $\theta = 0^\circ$  down to  $p=1.6$  GeV/c. There is good agreement with the modified formula and these measurements.

The parameters  $A_1$  to  $A_4$  can be plotted as a function of atomic weight of the target. A smooth curve through these points allows an interpolation to parameter values for other target materials. The chances of obtaining measured neutron data is slim. Charge independence of nuclear forces implies no difference between neutron and proton spectra and thus they are assumed equal in the cascade calculations.

Pion Spectra.

The following Trilling formula for pion production is used in the cascade calculations<sup>9</sup>.

$$\frac{d^2N}{dpd\Omega} = A_1 p^2 \exp \left( -A_2 \frac{p}{\sqrt{p_0}} - A_3 p \sqrt{p_0} \theta^2 \right) + A_4 \frac{p^2}{p_0} \exp \left( -A_5 \left( \frac{p}{p_0} \right)^2 - A_6 p \theta \right) \quad (9)$$

This formula does not reproduce experimental results as well as the thermodynamical model but it is more compatible with Monte Carlo methods. In any case, the pions are not as important in shielding calculations as protons and neutrons.

The available data at 19.2 GeV/c is not sufficient to uniquely determine all 6 parameters for  $\pi^+$  and  $\pi^-$  production. However, more data is available on Hydrogen targets and it is possible to obtain parameters for pion production on H. Table VI lists the results using data at 19.2, 30 and 70 GeV/c (allowing for different absolute normalizations).

TABLE VI.

Parameters of the Trilling formula (9) for  $\pi^\pm$  production from H targets normalized to the data at 19.2 GeV/c

Particle	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$
$\pi^+$	3.386	4.146	4.556	7.141	9.600	4.823
$\pi^-$	3.386	4.146	4.556	1.853	9.600	4.823

Only  $A_4$  is allowed to be different for  $\pi^+$  and  $\pi^-$ . The absolute normalization at 30 and 70 GeV/c must be changed by factors of 0.9 and 0.16 to obtain agreement with the respective data. These factors probably only reflect errors in the normalization of the independent experiments. At 70 GeV/c a large disagreement was also observed when fitting the thermodynamical formula<sup>11</sup>.

In fitting  $\pi^+$  production from heavier targets, the parameters  $A_2$  and  $A_3$  of the first term are kept constant. This term contributes mainly at small secondary momentum where experimental data is not available. This procedure yields good fit for the remaining 4 parameters which are listed in Table VII.

TABLE VII.

Parameters of the Trilling formula (9) for  $\pi^\pm$  production from Be, Al, Cu and Pb targets.

Particle	Target	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$
$\pi^+$	Be	3.52	4.15	4.56	3.49	9.87	4.04
$\pi^-$	Be	3.52	4.15	4.56	1.01	9.87	4.04
$\pi^+$	Al	3.88	4.15	4.56	3.04	10.0	3.91
$\pi^-$	Al	3.88	4.15	4.56	0.82	10.0	3.91
$\pi^+$	Cu	4.13	4.15	4.56	2.47	9.68	4.01
$\pi^-$	Cu	4.13	4.15	4.56	0.67	9.68	4.01
$\pi^+$	Pb	3.43	4.15	4.56	1.88	9.94	3.85
$\pi^-$	Pb	3.43	4.15	4.56	0.56	9.94	3.85

The parameters behave as expected.  $A_1$  increases with atomic weight from H to Cu, namely increasing the number of pions with small secondary momentum. The second term in 9 represents the high momentum pions coming from nucleon-pion resonance decay, and is suppressed by the intranuclear cascade. Thus  $A_4$  decreases with atomic number as expected. The calculated multiplicities ( $n_\pi$ ), inelasticities ( $K_\pi$ ) and

mean transverse momenta  $\bar{p}_t$  are given in Table VIII.

TABLE VIII.

Multiplicities  $n_\pi$ , inelasticities  $K_\pi$  and mean transverse momenta  $\bar{p}_t$  for pions calculated from the fitted formula (9) at 19.2 GeV/c

Material	$n_{\pi^+}$	$K_{\pi^+}$	$\bar{p}_{t\pi^+}$	$n_{\pi^-}$	$K_{\pi^-}$	$\bar{p}_{t\pi^-}$
H2	1.11	0.16	0.32	0.73	0.09	0.29
Be	0.97	0.13	0.34	0.71	0.09	0.30
Al	1.00	0.14	0.33	0.76	0.09	0.29
Cu	0.97	0.13	0.32	0.79	0.09	0.29
Pb	0.80	0.11	0.32	0.65	0.08	0.31

These quantities roughly agree with the experiment and the thermodynamical model. The multiplicities are perhaps 20% too low. This decrease with atomic number results from the assumptions made during the fitting. The low momentum part of the spectrum must be measured before the first term of the formula can be fitted properly to each material separately. The values of the parameters for other target material are obtained by interpolation from a smooth curve passing through the fitted points.

The  $\pi^0$  spectrum is expected to be somewhere between the  $\pi^+$  and  $\pi^-$  spectra. Simple Giesch-Gordon algebra arguments<sup>11</sup> indicate that the  $\pi^0$  spectra are expected to agree more favorably with  $\pi^+$  spectra than with the  $\pi^-$  spectra. This is basically the difference in numbers of  $\pi^+$   $\pi^-$   $\pi^0$  in each charge state of isobar decay. Typical ratios for this contribution are  $\pi^+ : \pi^0 : \pi^- = 7 : 10 : 1$ ,

and is in good agreement with fits obtained for  $\pi^+$  and  $\pi^-$  with the thermodynamical model. Thus it is assumed that the  $\pi^0$  spectrum is equal to the  $\pi^+$  spectrum.

#### 4. Ionization Energy Losses

Stopping power is calculated using the following formula<sup>12</sup>

$$-\frac{dE}{dx} = z^2 \frac{Z}{A} K(\beta) \left[ \rho(\beta) - \ln(I) - \frac{C}{Z} - \frac{\delta}{2} \right] \frac{\text{MeV}}{\text{gm/cm}^2} \quad (10)$$

where  $ze$  is the charge of the incident particle.  $Z$  and  $A$  are the atomic number and weight of the target material.  $\beta$  is the relativistic velocity which can be calculated from the momentum  $p$  and energy  $p^2 + m^2$  of the incident particle.

$$\beta = \frac{p}{\sqrt{p^2 + m^2}} \quad (11)$$

The functions  $K(\beta)$  and  $\rho(\beta)$  are

$$K(\beta) = \frac{0.307}{\beta^2} \quad (12)$$

$$\rho(\beta) - \ln \left[ 1.022 \cdot 10^6 \beta^2 / (1 - \beta^2) \right] - \beta^2 \quad (13)$$

The parameter  $I$  is the mean excitation energy of the material measured in eV. Values for the parameter are given by Fano<sup>12</sup> and the interpolation formulae below are given by Barkas, et al.<sup>13</sup>

$$I = Z \cdot (12.0 + 7.0/Z) \text{ eV for } I < 163 \text{ eV} \quad (14)$$

$$I = Z ( 9.76 + 58.8 Z^{-1.19} ) \text{ eV for } I > 163 \text{ eV} \quad (15)$$

The term  $C/Z$  is the tight electron binding energy or inner shell correction. The expression given below by Barkas<sup>13</sup> is only valid for

$$n = \beta\gamma = \frac{\beta}{\sqrt{1 - \beta^2}} \geq 0.13 \quad (16)$$

$$\begin{aligned} C(I, n) = & (0.422377n^{-2} + 0.0304043n^{-4} \\ & - 0.00038106n^{-6}) 10^{-6} I^2 \\ & + (3.85019n^{-2} - 0.1667989n^{-4} \\ & + 0.00157955n^{-6}) 10^{-9} I^3 \end{aligned} \quad (17)$$

Curves for  $C/Z$  can be found in reference 12. The parameter  $\delta$  is the density effect correction which is used if the material is in solid or liquid form. The values given by Sternheimer<sup>14</sup> are

$$\delta = 0 \quad \text{For } X < X_0 \quad (18)$$

$$\delta = 4.606 + C_1 + a (X_1 - X)^m \quad \text{For } X_0 \leq X \leq X_1 \quad (19)$$

$$\delta = 4.606X + C_1 \quad \text{For } X > X_1 \quad (20)$$

$$X = 0.5 \log_{10} \left[ \beta^2 / (1 - \beta^2) \right] \quad (21)$$

The parameters  $a$ ,  $m$ ,  $C_1$ ,  $X_0$  and  $X_1$  are different for each material and depend on its dielectric constant.

Density effect corrections are very important for high energy incident particles where they reduce the stopping power in many cases by 20%.

The relation

$$dp = dE/\beta \quad (22)$$

can be used to calculate the momentum change caused by ionization energy loss.

### 5. Inelasticities, Excitation Energies & Energy Conservation

In the cascade calculation, energy conservation is fulfilled in an average sense, but not in each scattering event. The inelasticities

$$K_{ij} = \frac{1}{E_i} \int E_j \frac{d^2N}{dpd\Omega} dpd\Omega \quad (23)$$

determine the fraction of primary energy carried away by each kind of secondary particle. Table IX lists the inelasticities  $K_{ij}$  used in the calculation.

TABLE IX.

Inelasticities  $K_{ij}$  used in the cascade calculation

Outgoing Particle / Incoming Particle	p	n	$\pi^+$	$\pi^0$
p	0.3	0.3	0.25	0.15
n	0.3	0.3	0.25	0.15
$\pi^+$	0.2	0.2	0.4	0.2

The values for incident protons and neutrons are essentially correct and agree with the measured particle spectra at primary energies between 10 and 30 GeV, and cosmic ray experiments. No energy dependence is assumed since sufficient data is not available. Hyperons and kaons are not treated separately but are included in the nucleon and pion components respectively.

The values given in Table IX are normalized to one. However, a certain part of the energy is used for nuclear excitation. Normally nuclear excitations depend on the incident particle energy and atomic weight of the targets.

In this calculation, one assumes that particles with momentum below 500 MeV/c ( $E \sim 125$  MeV) do not produce secondaries and the energy is deposited as nuclear excitation. At high energies the nuclear excitation energy is assumed proportional to atomic weight and independent of energy,

$$E_{\text{ex}} = \frac{A}{100} \left[ \text{GeV} \right] \text{ for } E > 3 \text{ GeV} \quad (24)$$

for lower energies

$$E_{\text{ex}} = 0.125 + \frac{(E - 0.125)(B - 0.125)}{2.875} \quad (25)$$

for  $0.125 < E < 3$  GeV

with  $B = A/100$ , but not less than 0.125. When  $E < 0.125$ , the value of  $E_x$  is always equal to  $E$ .

All inelasticities for particle production are corrected in the following way

$$K_{ij} = K_{ij} (1 - E_{\text{ex}}/E_0) \quad (26)$$

#### 6. Kinematic Limits

In all cases the Fermi momentum is neglected and it is assumed that the kinematic limit for proton production is given by elastic scattered protons.

$$p + p \rightarrow p + p \quad (27)$$

For pion production, the kinematic limit is calculated from the process



If  $p_0$  is the lab momentum of the incident proton and  $p$  is the lab momentum of the produced particle, then the center of mass energy is

$$E_{cm} = \sqrt{2m_p^2 + 2m_p \sqrt{p_0^2 + m_p^2}} \quad (29)$$

The maximum center of mass energy of a produced proton is

$$E_p^* = E_{cm} / 2 \quad (30)$$

The maximum center of mass energy of a produced pion is

$$E_\pi^* = \frac{(E_{cm}^2 + m_\pi^2 - 4m_p^2)}{2E_{cm}} \quad (31)$$

In the laboratory system the maximum and minimum momentum for secondary protons are

$$P_p \text{ max} = P_0 \quad \theta = 0^\circ \quad (32)$$

$$P_p \text{ min} = 0 \quad (33)$$

The maximum and minimum momentum for pion production in the laboratory are

$$P_\pi \text{ max} = \gamma (P_\pi^* + \beta E_\pi^*) \quad \theta = 0^\circ \quad (34)$$

$$P_\pi \text{ min} = \gamma (-P_\pi^* + \beta E_\pi^*) \quad \theta = 180^\circ \quad (35)$$

For a given incident momentum, the maximum scattering angle of a secondary pion with laboratory momentum  $p$  is

$$\cos \theta_{\max} = \frac{E^* + \gamma \sqrt{p^2 + m^2}}{\beta \gamma p} \quad (36)$$

where  $E^*$  is its maximum center of mass energy.

### 7. Particle Densities

In introducing energy dependent cross sections in the cascade calculation, the number of particle tracks and star densities are no longer proportional to each other. Thus, proton, neutron and pion track densities are calculated independently. The muon track density calculation is unchanged.

### 8. Concrete Targets

In order to simulate targets of ordinary or dense concrete, a chemical analysis of these materials was made at N.A.L. The radiation length, nonelastic mean-free path, and elastic mean-free path are then determined by summing these quantities for all constituents and weighting each contribution by the atomic abundance of that element.

### 9. New or Changed Subroutines in Program TRANSK

DPDX (Fortran function) calculates the ionization energy loss  $dP/dx$  in units of GeV/cm. The tight binding and density effect corrections are employed.

DEDX (Fortran function) first calculates a table of  $dP/dx$  using function DPDX and then uses the table for interpolation purposes.

SIZL (Subroutine) calculates the nonelastic cross sections and mean-free paths of protons, neutrons and pions as functions of atomic weight A of the target, and momentum p of the particle.

PPIM (Fortran function) calculates the maximum pion momentum allowed for pions created in pp collisions.

EEXI (Fortran function) calculates nuclear excitation energies (including the energy of all low momentum particles which are not separately considered).

EKINE (Fortran function) calculates the kinetic energy of a particle as a function of momentum.

AINEL (Fortran function) calculates the inelasticities for the creation of p, n,  $\pi^+$  and  $\pi^0$  as a function of the energy E and atomic weight A.

Other routines which were modified are DISEK, ENION, RANGE, VFE, SEKDIS, KASDEN, ZEICH, EDIST, and CONST.

The new array EKE(LO) stores the kinetic energy of the particle being considered in DISEK.

PTHR defined at the beginning of DISEK is the lower cut-off momentum.

ZINT, which is defined at the beginning of DISEK, determines the depth Z (gm/cm<sup>2</sup>) where the primary protons start interacting. Particle densities in the backward direction are obtained when ZINT > 0.

ACKNOWLEDGEMENTS

We would like to thank Drs. M. Awschalom and T. White for many helpful suggestions and discussions concerning the work reported here. One of the authors, J. Ranft, is grateful for the opportunity of performing this work during a recent visit to the Radiation Physics Section at N.A.L.

REFERENCES

1. G. Bellettini, G. Cocconi, A. N. Diddens, E. Lillethun, G. Matthiae, J. P. Scanlon, A. M. Wetherell, Nucl Phys 79, 609 (1966).
2. V. S. Barashenkov, K. K. Gudima, V. D. Toneev, "Energy Dependence of the Nuclear Cross Sections for Nucleons at Energy Above 50 MeV", Dubna Preprint P2-4183.
3. J. Ranft, CERN-MPS/INT. MU/EP 66-8.
4. E. W. Anderson, E. J. Bleser, G. B. Collins, T. Fujii, J. Menes, F. Turkot, R. A. Carrigan, Jr., R. M. Edelstein, N. C. Hien, T. J. McMahon, I. Nadelhaft, Phys Rev Let 19, 198 (1967).
5. J. V. Allaby, F. Binon, A. N. Diddens, P. Duteil, G. Giacomelli, R. Meunier, J. P. Peigneux, K. Schlupmann, M. Spighel, C. A. Stahlbrandt, J. P. Stroot, A. M. Wetherell, paper submitted to Vienna Conf (1968), "Particle Production in Proton-Proton Collisions at 19.2 GeV".

6. Yu. B. Bushnin, S. P. Denisov, S. V. Danskov,  
A. F. Dunaitsev, Yu. P. Gorin, V. A. Kachanov,  
Yu. S. Khodirev, V. I. Kotov, V. M. Kutyin,  
A. I. Petrukhin, Yu. D. Prokoshkin, E. A. Razuvaev,  
R. S. Skuvalov, D. A. Stoyanova, J. V. Allaby,  
F. Binon, A. N. Diddens, P. Duteil, G. Giacomelli,  
R. Meunier, J. P. Peigneux, K. Schlupmann, M. Spighel,  
C. A. Stahlbrandt, J. P. Stroot, A. M. Wetherell,  
Phys Let 29b, 48 (1969).
7. Unpublished results of the group from Ref. 5, made  
available prior to publication by A. N. Diddens.
8. J. Ranft, CERN-MPS/INT. MU/EP 66-3 (1966)
9. G. Trilling, UCID 10, 148 (1966).
10. D. Dekkers, J. A. Geibel, R. Mermod, G. Weber,  
T. R. Willitts, K. Winter, B. Jordan, M. Vivargent,  
N. M. King, E. J. N. Wilson, Phys Rev 137, 962 (1965)
11. R. Hagedorn, J. Ranft, Supl Nuovo Cimento 6, 169, (1968),  
and unpublished results.
12. U. Fano, Ann Rev Nucl Sci 13, 1 (1963).
13. W. A. Barkas, M. J. Berger, NAS-NRC Publication 1133,  
"Tables of Energy Losses and Ranges of Heavy Charged  
Particles".
14. R. M. Sternheimer, Phys Rev 88, 851 (1952).