national accelerator laboratory

FN-187
2040

# LONGITUDINAL SPACE-CHARGE EFFECTS AT TRANSITION IN NAL BOOSTER AND MAIN RING

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May 20, 1969

## INTRODUCTION

The longitudinal space-charge blowup studied in a series of papers by Sørenssen and Hereward[1,2,3,4] affects both the booster and the main ring of the NAL synchrotron.

The effect is the following (for more detail, see (1) and (4)): Space charge repulsion between the particles in a bunch in a synchrotron counteracts the rf-focusing force responsible for phase stability below the transition energy, and reinforces it above transition energy. The strength of this effect is proportional to the cube of the longitudinal charge density; since, because of the adiabatic variation of parameters, the bunch is shortest at transition, the effect is strongest there.

A bunch that is initially matched in phase space to the shape of the rf bucket remains matched until shortly before transition. Near the transition energy the bucket changes shape very rapidly (becoming infinite in the $\Delta p/p$ dimension); the bunch cannot keep up but attains a certain shape depending on the rate of passage through transition. In the absence of space-charge forces, the dynamics after transition are just a time reflection of those before, and it can easily be seen that the phase space matching re-establishes itself. Therefore, if the transition phase shift is performed perfectly, no oscillations of bunch shape are set up.

When space-charge forces are present, the effective focusing force is weakened before transition and reinforced after (because of the "negative mass" characteristic of the beam above transition). Therefore the "ideal" shape for a bunch of given phase space area is different before and after transition; the ideal matched bunch before transition is longer (with smaller momentum spread) than without space charge, while after transition the ideal bunch is shorter and has a larger momentum spread.

Thus a bunch that was matched to the bucket before transition finds itself mismatched. As a result, oscillations of bunch shape are set up; the bunch oscillates - at twice synchrotron-oscillation frequency - between a configuration with shorter length and higher momentum spread than the ideal one, and one with longer azimuthal extent and smaller momentum spread. This is disadvantageous for the operation of a synchrotron for several reasons:

(a) The maximum length of the bunch may be close to the limits of phase stability, i.e. the width of the bucket. This can lead to a loss of particles.

(b) The maximum momentum spread is increased, and the corresponding spread in radial position may exceed the portion of the radial aperture not required for other purposes (betatron oscillations, magnet errors, sagitta).

(c) Because the phase oscillations are nonlinear, the particles executing large oscillations have a lower frequency than those with small oscillations. This produces "filamentation": The initial phase space ellipse is wound up into a spiral, and effectively the phase space occupied by the bunch grows, in spite of Liouville's theorem. This means that the beam from the booster has a larger momentum spread, width, and angular extent than it otherwise would, leading to more severe requirements on injection into the main ring. Similarly the beam from the main ring is degraded, leading to more severe requirements on the extraction system (in some respects this may also be beneficial - the beam is less sharply bunched, so that the microscopic duty cycle of the slow beam is increased).

In the following, we summarize the theory, examine several methods of compensation, and estimate the effects for the NAL booster and main ring.

SUMMARY OF THEORY

In the absence of space charge, a matched bunch at transition has a length $\theta_o$ and a momentum spread $\Delta p/p$, where

$$\theta_o = \frac{2^{1/3}3^{1/6}}{\pi^{5/6}} \Gamma(\tfrac{2}{3}) \left(\frac{h}{\gamma_t}\right)^{2/3} \left(\frac{\sin \phi_s}{\cos^2 \phi_s (Mc^2)^2 eV}\right)^{1/6} \left(\frac{Sc}{R}\right)^{1/2} \qquad (1)$$

and

$h$ = harmonic order

$\gamma_t$ = transition energy in $Mc^2$ units

$eV$ = peak rf voltage per turn at transition

$\phi_s$ = stable phase angle at transition

$S$ = canonical phase space area of one bunch (in energy-time units);

$$\Delta_o = \frac{2}{\pi\sqrt{3}} \frac{h}{\beta\gamma Mc^2} \frac{Sc}{R\theta_o} \quad . \tag{2}$$

Space charge effect: Ratio of rf focusing to space charge force is (eq. (29) of ref. 1) 1)

$$\eta = - \frac{3\pi r_p}{R} \frac{h}{\gamma^2} \frac{Mc^2}{eV\cos\phi_s} \frac{g_o}{\theta^3} \; N \tag{3}$$

where $N$ is the total number of particles in the machine, $g_o$ is a geometrical factor, numerically about equal to 4.5, and $r_p = e^2/Mc^2$ is the classical electrostatic particle radius. The parameter $\eta_o$, which plays a critical role in the space charge theory, is simply eq. (3) evaluated at $\gamma = \gamma_t$ with $\theta$, the bunch half length, equated to $\theta_o$, the bunch half-length at transition calculated in the absence of space charge forces.

Note that this ratio $\eta$ is negative before transition ($\cos\phi_s > 0$) and positive after ($\cos\phi_s < 0$).

If nonlinearity of phase oscillations is neglected, the equation for the bunch half-length $\theta$ may be written in the form

$$\frac{d}{dt}\left(\frac{1}{\Omega^2}\frac{d\theta}{dt}\right) + \left(1 \pm \eta_o \frac{\theta_o^3}{\theta^3}\right)\theta - \left(\frac{2\ h\ S}{eV\cos\phi_s}\right)^2 \frac{\Omega^2}{\theta^3} = 0, \tag{4}$$

where $\Omega^2$ is the square of the instantaneous phase oscillation frequency. The last term in (4) is the term converting the equation of motion of the individual particle into the envelope equation[5]. The sign in the $\eta_o$ (space charge) term switches from negative to positive at transition.

Equation (4) can be converted to a dimensionless form involving only the one parameter $\eta_o$. Near transition, $\Omega^2$ is proportional to the time from transition. We define a characteristic time T by

$$\Omega^2 = |t|/T^3 \tag{5}$$

with

$$T = \frac{1}{\omega}\left[\frac{2\pi^2\gamma^4}{h\ \sin\ \phi_s\ \cos\phi_s}\left(\frac{Mc^2}{eV}\right)^2\right]^{1/3} \tag{6}$$

and write

$$x = t/T = \Omega^2 T^2. \tag{7}$$

We normalize $\theta$ by defining

$$\Theta = \frac{3^{1/6}}{\pi^{1/2}}\Gamma\left(\frac{2}{3}\right)\frac{\theta}{\theta_o} = 0.9175\ \frac{\theta}{\theta_o}\ ; \tag{8}$$

the constant 0.917.5 is introduced for convenience in some of the formulas. Then (4) transforms to

$$\frac{d}{dx}\left(\frac{1}{x}\frac{d\Theta}{dx}\right) + \left[1 \pm \eta_o\left(\frac{.9175}{\Theta}\right)^3\right]\Theta - \frac{x}{\Theta^3} = 0 \tag{9}$$

which is the equation treated by Sørenssen[4].

The actual bunch half length in a particular synchrotron is obtained from the solution of (9) by using (1) and (8). The momentum spread $\Delta p/p$ is given by

$$\frac{\Delta p}{p} = \frac{3^{2/3}\Gamma\left(\frac{2}{3}\right)}{2\pi^{1/2}} \left[\left(\frac{1}{x}\,\frac{d\Theta}{dx}\right)^2 + \frac{1}{\Theta^2}\right]^{1/2} \Delta_o \ . \tag{10}$$

A computer program has been written to solve equation (9) with initial conditions selected so that the bunch is matched, i.e. executes negligible shape oscillations, before transition. The results of the computation, in agreement with Sørenssen[4], show that the bunch shape oscillates after transition, with a peak bunch length and peak relative momentum spread increasing with $\eta_o$ as shown in Fig. 1.

COMPENSATION METHODS

Several ways of ameliorating or even - optimistically - eliminating this blowup effect may be considered. These include:

a) Adjustment of transition timing. The time at which the phase of the accelerating voltage is switched from $\cos\phi_o<0$, corresponding to the switch from the negative to positive sign in the third term of Eq. (9), can be adjusted to occur at a later (or earlier) time than the exact passage through transition, i.e. at some value of x other than x=0.

In fact, in the actual operation of a synchrotron the operator is likely to tune transition timing empirically to obtain optimum performance rather than to coincide with the instant when x=0. Thus this method is likely to be employed automatically.

The computer program has been modified to permit the change of sign of the space charge term at any time $x_2$. Approximate values of the optimum $x_2$ and corresponding improvements in maximum $\theta$ and $\Delta$ are as follows:

| $\eta_o$ | $x_2$ | $\theta_{max}/\theta_{max}(\eta_o=o)$ | $\Delta_{max}/\Delta_{max}(\eta_o=o)$ |
|---|---|---|---|
| 1.0 | 0.5 | 2.00 | 1.12 |
| 3.0 | 1.0 | 2.71 | 2.04 |
| 6.0 | 1.5 | 3.32 | 3.25 |

(b) The triple switch. The phase may be switched back and forth repeatedly; if this is done at the appropriate moments the bunch distortion can be completely compensated when $\eta_o < 3$, and partially compensated for larger values of $\eta_o$, as described by Sørenssen in (3) and (4).

The triple switch has been tried experimentally at CERN, but with only limited success. Therefore, even though its theoretical effectiveness is high, it seems worthwhile to explore other compensation methods.

(c) Feedback damping. The bunch shape oscillations induced by the **mismatch** can be detected by pickup electrodes, they manifest themselves as modulations (at twice synchrotron-oscillation frequency) of the height of the beam envelope with the instantaneous height of the envelope proportional to $1/\theta$.

Thus a signal proportional to $d\theta/dt$ can be obtained from the pickup electrode, assuming the electronic differentiation can be performed in a reasonably short time. In this connection we note that the characteristic time T is of the order of a millisecond; therefore it should be sufficient to perform the electronic differentiation in a few microseconds.

This method has been employed with some success by Raka[6] at the Brookhaven AGS.

If the amplitude of the applied voltage is modulated by this signal proportional to $d\theta/dt$, equation (9) is modified by the addition of a term

$$a\ \theta \frac{d\theta}{dx} \tag{11}$$

on the left-hand side. With positive a this may be expected to produce damping of the oscillation.

Computations bear this out. As an example, Fig. 2 shows the oscillations after transition for $\eta_o = 3.0$ with $a = 0$ and $a = 0.02$ (a level at which the resultant modulation of the RF is limited to about 10%). It is seen that for $x \approx 20$ the oscillations have damped to a small amplitude, nearly the same as in the absence of space charge.

However, another feature of this method, also seen from Fig. 2, is that immediately after transition the amplitude of the first few oscillations is nearly as large with damping as without.

Therefore damping does not materially reduce the maximum expansion of the beam length $\theta$ or of the momentum spread $\Delta$ right after transition. If either of these threaten to exceed the allowable limits imposed by the rf voltage or the aperture, this damping process is therefore not sufficient. On the other hand, the damping eventually restores the oscillations to the behavior they would show without space charge; therefore with damping the longitudinal phase space in the booster at injection into the main ring would be back to its ideal configuration.

This latter conclusion, however, depends on the linear approximation for phase oscillation (which is implicit in the theory used here). Since the oscillations are in fact nonlinear their frequency depends on amplitude. This leads to the process of "filamentation" and an effective growth of phase-space area in approximately the time it takes for large-amplitude oscillations to lag in phase behind the small-amplitude oscillations by an amount of the order of $\pi/2$. Therefore, even with damping, an increase in the effective longitudinal phase-space area can be expected unless the damping essentially eliminates the bunch-shape oscillations by that time. The value of our dimensionless variable x corresponding to this phase lag depends on the initial maximum excursion $\theta_o$ $\Theta_{max}$ and on the stable phase angle.

(d) Sudden change of transition energy. If the value of $\nu_x$, and with it the transition energy, is suddenly decreased just before transition energy is reached, the transition region may simply be skipped. Or, if $\nu_x$ is decreased at a rapid but finite rate the rate of passage through transition is increased. In either case the blowup of oscillations can be reduced.

Hardt and Möhl[7] show that such a jump can be obtained for the proposed CERN booster (similar to the NAL booster) with fairly modest pulsed quadrupole lenses; recently this scheme has been successfully tried at CERN[8].

A modification of our computer program simulates a jump by skipping, in the integration of equation (9), from $x=-x_3$ to $x=+x_3$. This corresponds to a sudden jump of $\gamma_t$ from $\gamma_o$ to $\gamma_o - \Delta\gamma$ at the moment when the beam has reached the energy $\gamma_o - \Delta\gamma/2$. The relation between $x_3$ and $\Delta\gamma$ is

$$\Delta\gamma = 2\dot{\gamma}\, x_3 T$$

where T is defined by (6).

Computations show that with $x_3 = 1$ (corresponding to $\Delta\gamma = 0.125$ for the NAL booster) and for $\eta_o = 3$ the maximum amplitude in $\theta$ and in $\Delta$ are reduced by factors of 1.4 and 1.67, respectively. With $x_3 = 2$ the reduction factors became 1.57 and 2.40.

(e) Reactive Loading of Chamber Walls.

The geometric factor $g_o$ in the expression (3) for $\eta_o$ depends on a balance between inductive and capacitive impedence of the system consisting of the beam and the vacuum chamber walls. Briggs and Neil[9] have proposed changing this balance by loading the inside of the chamber wall with a dielectric layer or with closely spaced fins; this can in principle reduce the factor $g_o$, and therefore the space-charge effect, to zero at transition. An alternate method, proposed by Sessler and Vaccaro[10], is to use helical inserts so as to increase the inductance. It should be possible to use one of these methods in the otherwise unusual straight sections of the booster; Sørenssen and Courant[11] have shown, in a rough preliminary calculation, that this aim can be accomplished using one or two booster straight sections. However, the loading characteristics must have a band width reading to wave lengths of the order of the bunch length, and the details have not been worked out. This possibility will be discussed in a future paper; for the present we base our estimates on the assumption that $g_o$ is given and has a value around 4.5.

Numerical Values for NAL Accelerators

We assume the following parameters:

| | Booster | Main Ring |
|---|---|---|
| $R$ | 75 m | 1000 m |
| $h$ | 84 | 1113 |
| $\gamma_t$ | 5.5 | 19.61 |
| $eV$ | .80 MeV | 3.47 MeV |
| $\sin\phi_s$ | 0.9 | 0.767 |
| $\gamma_{injection}$ | 1.213 | 11.66 |
| $\zeta_{inj}$ | .648 | .00475 |
| $N$ | $3.46\text{x}10^{12}$ | $4.5\text{x}10^{13}$ |
| $S$ | .0208 eV-sec | .0208 (per bucket) |

Here it is assumed that there is no blowup of longitudinal phase space at transition or at transfer from the booster into the main ring.

These parameters lead to the following values of the bunch half-length $\theta_o$, the momentum spread $\Delta p/p$, the space charge parameter $\eta_o$, and the "characteristic time" T:

| | Booster | Main Ring |
|---|---|---|
| $\theta_o$ | .193 radians | .0846 radians |
| | = 11.1 degrees | = 4.84 degrees |
| $\Delta p/p$ | $\pm 2.59\text{x}10^{-3}$ | $\pm 1.64\text{x}10^{-3}$ |
| $\eta_o$ | 3.33 | 6.3 |
| $T$ | 227 μsec | 2.43 msec |

These parameters enable us to convert the results of the computations (solutions of Eq. (9) and its modifications for various compensation schemes) into estimates of the lengths, widths, and oscillation amplitudes of the bunches in these two accelerators.

Taking $\eta_o$ in the booster to be 3.0, we find the following maximum excursion $\theta_{max}$ and maximum momentum spreads; with and without delay of the phase jump, and with various jumps $\Delta\gamma_t$ effected by quadrupoles. Damping is not included; computations show, as mentioned before, that damping hardly affects the initial amplitudes of oscillation but only reduces the amplitudes later in the acceleration cycle.

| Phase jump delay (in units of T = .227 ms) | Jump in $\gamma_t$ | $\theta_{max}$ | $\left(\frac{\Delta p}{p}\right)_{max}$ |
|---|---|---|---|
| 0 | 0 | $38^o$ | $\pm 9.2 \times 10^{-3}$ |
| 0.75 | 0 | $29^o$ | $\pm 5.8 \times 10^{-3}$ |
| 1.0 | 0 | $30^o$ | $\pm 5.3 \times 10^{-3}$ |
| 0 | 0.125 | $25^o$ | $\pm 5.3 \times 10^{-3}$ |
| 1.0 | 0.125 | $21^o$ | $\pm 3.5 \times 10^{-3}$ |
| 0 | 0.25 | $21^o$ | $\pm 3.7 \times 10^{-3}$ |
| 0 | 0.25 | $20^o$ | $\pm 2.75 \times 10^{-3}$ |

We see that even in the best cases, the maximum excursion in $\theta$ is about ±20 to 30 degrees - computed on the linearized theory of synchrotron oscillations. But the nominal booster parameters call for a stable phase angle of 65$^{\circ}$ and for this value the limits of phase stability are only from 40$^{\circ}$ to 115$^{\circ}$; i.e. only 25$^{\circ}$ on one side, 50$^{\circ}$ on the other side of $\phi_s$. Clearly this means that we are perilously close to the limits of phase stability; it would appear very advisable to raise the RF voltage by something like 10%, obtaining $\phi_s$ 55$^{\circ}$, limits of phase stability 19$^{\circ}$ to 125$^{\circ}$.

The peak excursion in $\frac{\Delta p}{p}$ causes trouble if it consumes excessive aperture. The aperture allowance in the booster, according to the design book, is 43 mm for betatron oscillations and 5.5 mm for synchrotron oscillations at injection. At transition betatron oscillations have damped down by the usual $\sqrt{p}$ factor to 15 millimeters, leaving 33 millimeters available for synchrotron oscillations. With a peak momentum compaction factor $x_p$ = 3.16 cm for $\Delta p/p$ = 1%, this means that $\Delta p/p$ may be allowed to be at most 1.04%. Thus, with phase jump delay, we are comfortably within the aperture as far as momentum spread is concerned.

It may be expected that, even with damping and other compensation, there will be a residual blowup of effective longitudinal phase space. Therefore the phase space at transfer from booster to main ring will be larger than the ideal value. This will, of course, reduce $\eta_o$ in the main ring by a factor equal to the 3/2 power of the phase space dilution. Assuming the phase space dilution to be a factor of 2 or 4, we find, for the main ring, $\eta_o$

$\eta_o$ = 2.2 for booster dilution factor of 2

$\eta_o$ = 0.8 for booster dilution factor of 4.

The values of $\theta_o$ and $\Delta_o$ are increased over the ideal values by the square root of the dilution factor. In addition, the same blowup effect in the main ring increases the maximum excursion further.

With booster dilution of a factor of 2, we have $\theta_o$ = 6.84 degrees, $\Delta_o = 2.32\text{x}10^{-3}$, $\eta_o$ = 2.2.

At this level of $\eta_o$ we may expect, even with compensation, a further blowup of $\theta$ by a factor of 2 and of $\Delta_o$ by a factor between 1.5 and 2, just as in the booster. The lengthening of the bunch is still trivial, well within the bucket; but the increase in $\Delta p/p$ brings it to $\pm\ 4\text{x}10^{-3}$. With a momentum compaction function $x_p$ = 5.2 in the main ring, this implies an aperture requirement of ±2cm in the main ring.

If we are more pessimistic about compensation in the booster and take a blowup factor of 4, $\eta_o$ in the main ring is reduced to 0.8, but $\theta_o$ and $\Delta_o$ are doubled compared to their values without blowup, to $\theta_o$ = 9.7 degrees, $\Delta p/p$ = $3.28\text{x}10^{-3}$. Now the further blowup in the main ring will be small, but may again be enough to bring the maximum $\Delta p/p$ to $4\text{x}10^{-3}$, leading to the same aperture requirements.

In either case, since the excursion in $\theta$ is small, the phase oscillations will be very nearly linear in the main ring. Therefore filamentation should not occur, and feedback damping should be capable of tuning the oscillations so that the final phase space at ejection is about equal to its value at injection into the main ring.

## CONCLUSION

Longitudinal space charge effects, at the intensities of the NAL accelerator, cause increases in phase oscillation amplitudes of the booster. Even with the compensation methods studied here, the amplitudes will be large enough to make it most advisable to raise the rf voltage level from the value contemplated in the design report - a 10% increase should suffice.

In the main ring, the most serious effect is an increase of the momentum spread in the vicinity of transition. This leads to appreciable aperture requirements and makes it inadvisable to reduce the radial aperture below the value adopted in the design report.

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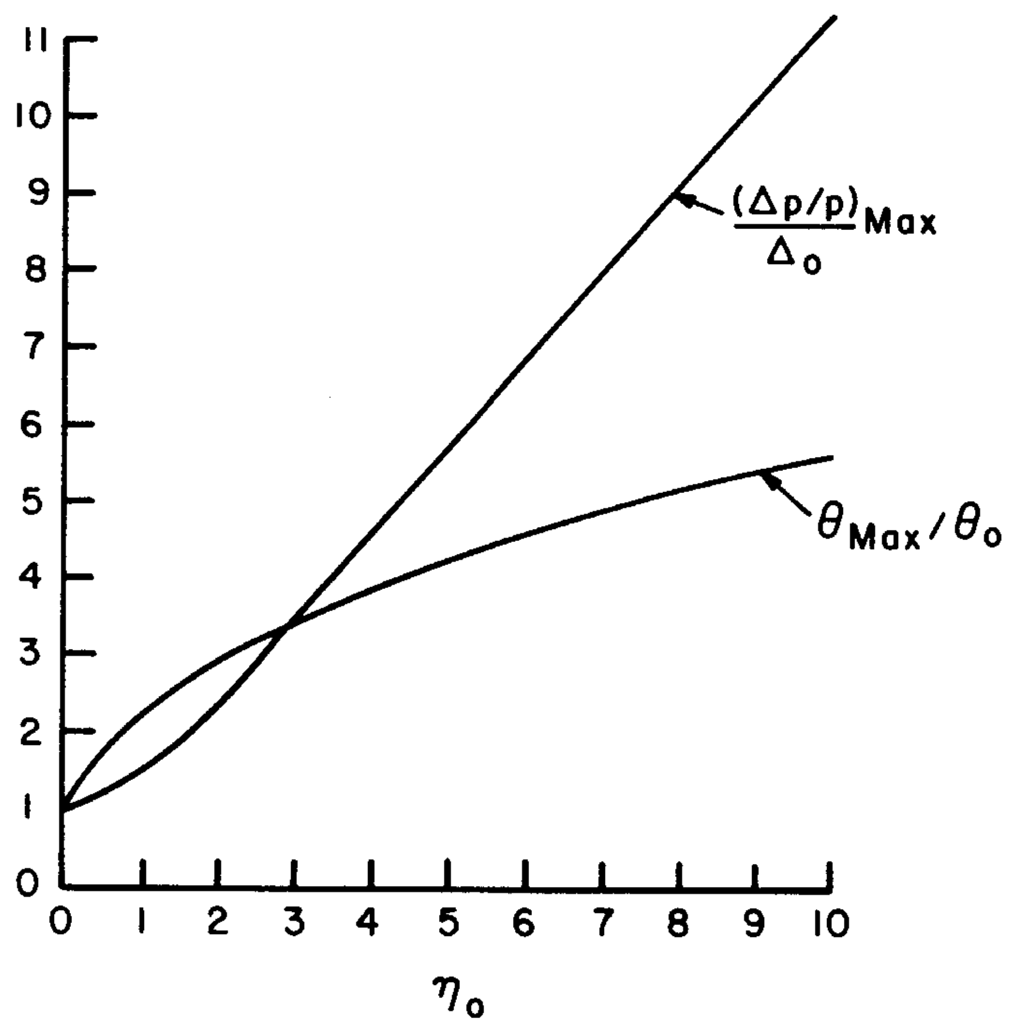


Fig. 1 - Ratio of maximum amplitude in $\theta$ and $\Delta p/p$ to amplitudes at transition without space charge, for solution of Eq. (9) as function of $\eta_o$ .

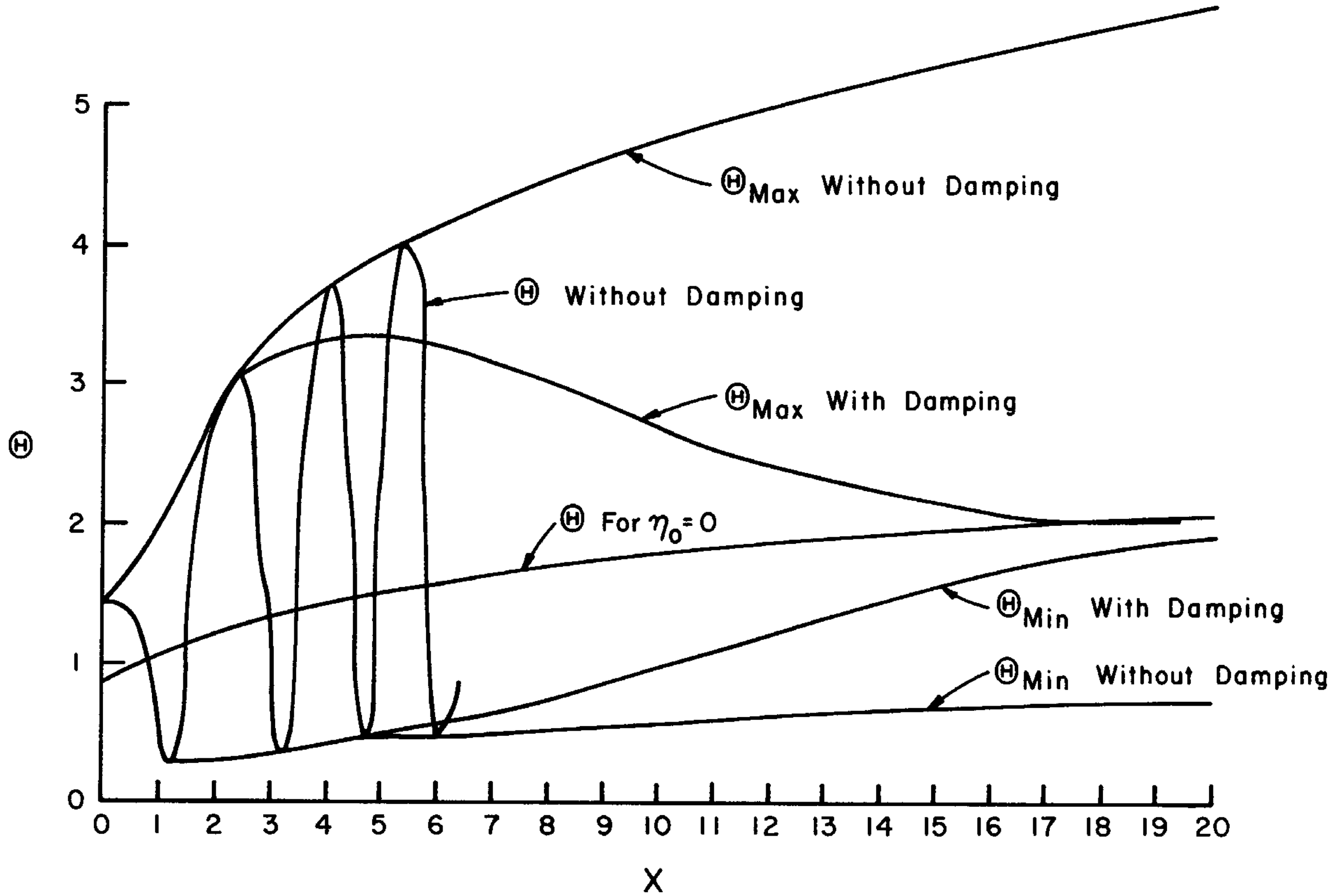


Fig. 2 - Amplitudes of $\Theta$ (eq. 9) for $\eta_o = 0$ and $\eta_o = 3$. $\Theta_{max}$ and $\Theta_{min}$ are envelopes of oscillation, calculated for solution of eq. (9) (no damping) and eq. (9) with damping term (11) added.