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Error and Tolerance Analysis for the Main Accelerator

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Various imperfections (here defined as systematic deviations from ideal values recognized in the design such as field distortions due to saturation of magnet iron) and errors (here defined as uncontrollable random deviations) in field and alignment of magnets in the main accelerator are analyzed. These imperfections and errors are classified as follows:

A. Bending magnet (BM) field imperfections and errors.

	Type	Effect on Orbit
1.	Random B error	Radial distortion of closed orbit
2.	Systematic B' imperfection (caused by saturation of Fe)	ν-shift
3.	Random B' error	Negligible
4.	B'' imperfection and error (to be compensated by sextupole magnets)	

B. Quadrupole magnet (QM) field imperfections and errors.

	Type	Effect on Orbit
1.	Systematic B' imperfection (caused by saturation of Fe)	ν-shift
2.	Random B' error	Excitation of 1/2 integral resonance
3.	B'' imperfection and error (to be compensated by sextupole magnets)	

C. Random misalignments of BM

	Type	Effect on Orbit
1.	Longitudinal displacement	Negligible
2.	Transverse displacements	Negligible
3.	Rotation about longitudinal axis	Vertical distortion of closed-orbit
4.	Rotation about transverse axis	Negligible

D. Random misalignments of QM

	Type	Effect on Orbit
1.	Longitudinal displacement	Negligible
2.	Transverse displacements	Distortions of closed-orbit
3.	Rotation about longitudinal axis	Coupling of vertical and radial oscillations
4.	Rotation about transverse axis	Negligible

In the following we will treat these effects one by one.

(A-1) Random B error in BM

The RMS radial excursion amplitude of the closed-orbit A_x^{RMS} is given by

$$A_x^{RMS} = \frac{\pi C_B}{\nu_x \left|\sin \pi\nu_x\right|} \tag{1}$$

where

$$C_B = \frac{R}{\sqrt{M_B}} \left(\frac{\Delta B}{B}\right)^{RMS}$$

and

$$\begin{cases} M_B = \text{total number of BM} = 744 \\ R = \text{machine radius} = 1000 \text{ m} \\ \nu_x = \text{radial oscillation wave number} = 20\text{-}1/4 \end{cases}$$

Numerically (1) gives

$$A_x^{RMS} \text{ (cm)} = 804 \left(\frac{\Delta B}{B}\right)^{RMS} \tag{2}$$

For a tolerance of $\left(\frac{\Delta B}{B}\right)^{RMS} = 5 \times 10^{-4}$ we have $A_x^{RMS} = 0.402$ cm

Another interesting relation is that for a 20th harmonic field error with amplitude $\left(\frac{\Delta B}{B}\right)^{(20)}$ the amplitude of the 20th harmonic radial excursion of the closed-orbit is given by

$$A_x^{(20)} = \frac{R^2/\rho}{\nu_x^2 - 20^2} \left(\frac{\Delta B}{B}\right)^{(20)} \tag{3}$$

where ρ = orbit radius in BM = 743 m. Numerically this gives

$$A_x^{(20)} \text{ (cm)} = 1.34 \times 10^4 \left(\frac{\Delta B}{B}\right)^{(20)} \tag{4}$$

For $\left(\frac{\Delta B}{B}\right)^{(20)} = 5 \times 10^{-4}$ we get $A_x^{(20)} = 6.7$ cm showing that a 20th harmonic error is much more harmful than an RMS error.

(A-2) Systematic B' imperfection in BM

The saturation of the BM iron will produce a non-zero $k \equiv \frac{1}{B}\frac{dB}{dx} = \frac{B'}{B}$ at radial positions away from the center of the vacuum chamber. At a non-central radial position then, the ν-shift is given by

$$\Delta\nu = \frac{R}{2\nu} k \tag{5}$$

For $R = 10^3$ m, $\nu \cong 20$ and allowable $\Delta\nu = \pm 1/4$ we get as tolerable limits on k

$$-0.1 < k < +.01$$

(A-3) Random B' error in BM

This type of error generally excites the half-integral resonance. But for the BM since non-vanishing values of B' are already small quantities of the first order resulted from saturation of the magnet iron, random B' errors will be second order small quantities whose effects are negligible.

(A-4) B'' imperfection and error in BM

To first order the sextupole component of the field in the BM caused by saturation of the magnet iron can be compensated by sextupole magnets placed in the short (mini-) straights and producing an equal and opposite $B''L$. With one sextupole in each short straight (total number $N_S \cong 192$) at a length of $L_S \approx 1$ ft. and $B''_s = 2000$ kG/m^2 (2.5 kG on the pole tip 5 cm from the axis) we can compensate for a sextupole field component in the BM of

$$B'' = \frac{N_S L_S}{2\pi\rho} B''_s = 25 \text{ kG/m}^2$$

which is more than the sextupole component of the BM field expected up to 20 kG.

(B-1) Systematic B' imperfection in QM

When parts of the QM iron are saturated, B' will vary across the aperture. In addition, a difference in saturation of the iron between the BM and the QM will also cause a systematic $\frac{\Delta B'}{B'}$ in the QM. For the FODO lattice and using the thin lens approximation, the ν-shift is given by

$$\Delta\nu = 2\,\frac{\nu}{\mu}\,\frac{1-\cos\mu}{\sin\mu}\left(\frac{\Delta B'}{B'}\right) \tag{6}$$

where μ = betatron oscillation phase advance per cell $\approx$ 1.235. Numerically this gives

$$\Delta\nu = 23.3\left(\frac{\Delta B'}{B'}\right) \tag{7}$$

which shows that the tolerance in the B' imperfection of the QM should be

$$-1\% < \frac{B'}{B'} < +1\%$$

(B-2) Random B' error in QM

Uncorrelated random error in B' of the QM will excite the half-integral resonance (Here, the $\nu = \frac{41}{2}$ resonance). The full width W of the resonance band and the growth rate of the amplitude A in the middle of the resonance are given by

$$\left\{\begin{aligned} W &= \frac{2}{\nu}\,C_Q \\ \frac{d\ln A}{dn} &= \frac{\pi}{\nu}\,C_Q \end{aligned}\right. \tag{8}$$

where $\frac{d}{dn}$ means increment per turn, and

$$\left\{\begin{aligned} C_Q &= \frac{F_Q}{\sqrt{M_Q}}\left(\frac{R^2B'}{B\rho}\right)\left(\frac{\Delta B'}{B'}\right)^{RMS} \\ F_Q &= \text{fraction of the circumference of ring occupied by QM} \\ M_Q &= \text{total number of QM} \end{aligned}\right.$$

Numerically this gives

$$W = 8.24 \left(\frac{\Delta B'}{B'}\right)^{RMS}$$

$$\frac{d\ln A}{dn} = 13.0 \left(\frac{\Delta B'}{B'}\right)^{RMS} \qquad (9)$$

For a tolerance of $\left(\frac{\Delta B'}{B'}\right)^{RMS} = 2 \times 10^{-4}$ we have $W = 0.0016$ and $\frac{d\ln A}{dn} = \frac{1}{385}$.

(B-3) B'' imperfection and error in QM

As in (A-4) these imperfections and errors can be compensated by the sextupole magnets. Since the fraction of the circumference of the main accelerator occupied by QM is much smaller than that by BM, the sextupoles discussed in (A-4) are more than adequate to compensate for B'' imperfection and error in the QM field.

(C-1) Random longitudinal displacement error of BM

As stated before, this type of misalignment has negligible effect on the particle orbit.

(C-2) Random transverse displacement error of BM

Because the BM field is uniform this type of misalignment has negligible effect on the particle orbit.

(C-3) Random rotational error about longitudinal axis of BM

The RMS vertical excursion amplitude of the closed-orbit A_y^{RMS} is given by an expression similar to (1), namely

$$A_y^{RMS} = \frac{\pi K_B}{\nu_y \left|\sin \pi\nu_y\right|} \qquad (10)$$

where

$$K_B = \frac{R}{\sqrt{M_B}} \quad (\Delta\theta)^{RMS}$$

and $\Delta\theta$ is the angle of rotation. Numerically, this gives

$$A_y^{RMS}\,(\text{cm}) = 804\,(\Delta\theta)^{RMS} \tag{11}$$

For a tolerance of $(\Delta\theta)^{RMS} = 0.25$ mrad we have $A_y^{RMS} = 0.201$ cm.

(C-4) Random rotational errors about transverse axes of BM

Because of the uniform field in the BM, rotational error about a vertical axis produces negligible effect on the particle beam and because a rotation about a radial axis produces only a field component along the direction of motion of the particles, the effect of such a rotational error is negligible.

(D-1) Random longitudinal displacement error of QM

This type of error produces negligible effects on the focusing properties of the QM chain.

(D-2) Random transverse displacement errors of QM

The RMS transverse excursion amplitudes A_x^{RMS} and A_y^{RMS} of the closed-orbit are given by relationships similar to (1), namely

$$A_x^{RMS} = \frac{\pi D_Q}{\nu_x \,|\sin \pi\nu_x|} \tag{12}$$

with

$$D_Q = \frac{F_Q}{\sqrt{M_Q}} \left(\frac{RB'}{B\rho}\right) (\Delta x)^{RMS}$$

and analogous formulas for A_y^{RMS} in terms of $(\Delta y)^{RMS}$. Numerically, this gives

$$A_x^{RMS} = 18.8\ (\Delta x)^{RMS} \qquad \text{(same for } A_y^{RMS}) \tag{13}$$

For a tolerance of $(\Delta x)^{RMS} < 0.01$ cm, we get $A_x^{RMS} < 0.19$ cm.

(D-3) Random rotational error about longitudinal axis of QM

This type of error excites the coupling resonances between the radial and the vertical oscillations. The widths and the growth rates are given by

$$\left\{ \begin{array}{ll} W = \dfrac{4A_xA_y}{\nu_x A_x^2 \pm \nu_y A_y^2}\ K_Q & \\ \dfrac{dA_x}{dn} = \dfrac{\pi A_y}{\nu_x}\ K_Q & \dfrac{dA_y}{dn} = \pm \dfrac{\pi A_x}{\nu_y}\ K_Q \end{array} \right. \tag{14}$$

where the + sign is for the sum-resonance $\nu_x + \nu_y = 41$ and the - sign is for the difference-resonance $\nu_x - \nu_y = 0$, and where

$$K_Q = \frac{F_Q}{\sqrt{M_Q}} \left(\frac{R^2B'}{B\rho}\right) 2\ (\Delta\theta)^{RMS}$$

$\Delta\theta$ being the angle of rotation. Equation (14) shows that the sum-resonance producing growth in both the radial and the vertical oscillations is more damaging than the difference-resonance which causes only a turn-over between the radial and the vertical oscillations. On the other hand, the width of the difference-resonance tends to be bigger than that of the sum-resonance. Numerically, we have

$$K_Q = 170\ (\Delta\theta)^{RMS}$$

For $A_x \sim A_y \sim 1$ cm, we have $\nu_x A_x^2 \pm \nu_y A_y^2 \sim 20\ \text{cm}^2$ and

$$W \sim 34\,(\Delta\theta)^{RMS}$$

$$\begin{cases} \dfrac{dA_x}{dn}\,(\text{cm/turn}) = 26\,(\Delta\theta)^{RMS} \\[2ex] \dfrac{dA_y}{dn}\,(\text{cm/turn}) = \pm\,26\,(\Delta\theta)^{RMS} \end{cases} \qquad (15)$$

For a tolerance of $(\Delta\theta)^{RMS} = 0.25$ mrad, we get

$$\begin{cases} W \sim 0.0085 \\[1ex] \dfrac{dA_x}{dn} = 0.0065 \text{ cm/turn} \qquad \dfrac{dA_y}{dn} = \pm\,0.0065 \text{ cm/turn} \end{cases}$$

(D-4) Random rotational errors about transverse axes of QM

These types of error produce negligible effects on the particle orbits.