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NOTES ON RIPPLE REQUIREMENTS FOR SLOW EXTRACTION AND SOME PARAMETERS RELATIVE TO CORRECTING THE SPILL

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Now lets look at the ripple requirement in the presence of sextupole and energy spread.

$$\frac{\Delta p}{p} = \nu^2 \frac{\Delta r}{R} \qquad \therefore \ \Delta r = \frac{R}{\nu^2} \frac{\Delta p}{p}$$

the ν spread in the beam is given by:

$$\delta \nu = \frac{\partial \nu}{\partial r} \delta r = \frac{\partial \nu}{\partial r} \frac{R}{\nu^2} \frac{\delta p}{p}$$

Now this determines $\dot{\nu}$ on the flat-top.

$$\dot{\nu} = \frac{\partial \nu}{\partial r} \frac{R}{\nu^2} \frac{\dot{p}}{p} + \Delta\nu\, \omega \cos \omega t$$

the 100% modulation condition is therefore given by:

$$\Delta \nu = \frac{\partial \nu}{\partial r} \frac{R}{\nu^2} \frac{\dot{p}}{p} \cdot \frac{1}{\omega}$$

Now we can express $\Delta \nu$ in terms of $\frac{\Delta I}{I}$.

$$\frac{\Delta I}{I} = \frac{\nu^2}{R} \Delta r \qquad \Delta r = \frac{1}{\frac{\partial \nu}{\partial r}} \Delta \nu$$

$$\therefore \quad \Delta \nu = \frac{\partial \nu}{\partial r} \frac{R}{\nu^2} \frac{\Delta I}{I}$$

∴ for 100% modulation we have the condition:

$$\frac{2v}{2r}\frac{R}{v^2}\frac{\Delta I}{I} \leq \frac{2v}{2r}\frac{R}{v^2}\frac{\Delta p}{p}\cdot\frac{1}{\omega}$$

$$\therefore \quad \frac{\Delta I}{I} \leq \frac{\Delta p}{p}\cdot\frac{1}{\omega}$$

Now we can learn something from this relation.

1.) the ripple does not depend upon $\frac{2v}{2r}$.

2.) For a given ripple current, a large $\Delta p/p$ is a help

Also $\Delta V = L\omega \Delta I$

$$\frac{\Delta V}{L\omega} \leq \frac{\Delta p}{p}\cdot\frac{I}{\omega}$$

$$\therefore \quad \Delta V \leq \frac{\Delta p}{p} I L$$

This tells us that things are better at higher currents. Also, that high inductance is a good thing.

A change in ν of .0025 units is enough to make the beam go from stable to fully unstable.

The normal procedure for producing a spill would be to change ν by this amount over the period of the flat top (1 sec, say).

Now if ν is modulated by ripple in addition, the condition for 100% modulation is:

$$\dot{\nu} = 0 . \qquad \dot{\nu} = K + \Delta\nu\omega \cos\omega t$$
$$\text{when } \nu = Kt + \Delta\nu \sin\omega t$$

$$\therefore \quad \Delta\nu = \frac{K}{\omega}$$

The lowest frequency is 60 cycles, and $\Delta\nu = \dfrac{.00250}{377}$

$$\therefore \quad \Delta\nu \approx 10^{-5} .$$

Now consider that $\frac{\partial v}{\partial r} \approx .1$, say.

1 amp will cause a modulation of $\Delta v = .06 = 6 \times 10^{-2}$.

$\therefore$ 1 ma $= 6 \times 10^{-5}$, which is severe.

$$\Delta V = L\omega \Delta I$$
$$= 1 \times .377 \times 5 \cong 2 \text{ Volts}$$

And this is 6 times too large.

The only way to achieve this stability is to feed the magnet ripple voltage to the quadrupoles, with the proper strength, and make use of the fact that $\frac{\partial v}{\partial r} \propto$ const., independent of r.

Now a ripple in the current will cause a spill modulation mainly because of a change of ν with radius.

$$\frac{\Delta\nu}{\nu} = \frac{\Delta I}{I} + \frac{K}{\nu^2}\frac{\Delta I}{I}$$

$\therefore$ 1 amp of ripple will cause $\Delta\nu = \frac{20}{3500} = \frac{1}{175} = .0057$

if K, the sextupole component is zero.

Now $\Delta\nu_s = \frac{\partial\nu}{\partial r}\Delta r = \frac{\partial\nu}{\partial r}\frac{R}{\nu^2}\frac{\Delta I}{I}$

i.e., $K = \frac{\partial\nu}{\partial r} \times R$

for these to cancel we must have:

$$\frac{\partial\nu}{\partial r}\frac{R}{\nu^2} = 1 \qquad \frac{\partial\nu}{\partial r} = \frac{\nu^2}{R} = \frac{400}{40{,}000} = .01$$

Now we can convert this result to a sextupole moment uniformly distributed around the machine.

$$S = a x^2 \qquad a \equiv \text{sextupole moment}$$

$$\delta\nu = \frac{1}{4\pi} \times \frac{G l \beta}{p} \qquad G = 2 a x$$

$$\delta\nu = \frac{1}{2\pi} \, \frac{a x \beta l}{p} \qquad B\rho = p$$

$$\therefore \frac{\delta\nu}{2\pi} = \frac{a\beta l}{2\pi p} \cong \frac{a\beta \times 2\pi\rho}{2\pi B\rho} = \frac{a\beta}{B_0} = .01$$

$$\therefore a = \frac{B_0 \times .01}{2000}$$

$$\therefore \frac{a}{B_0} = \frac{1}{2} \times 10^{-5}$$

In 10,000 Gauss, .05 gauss deviation at 1" will put us out of business. This is a thoroughly unreasonable tolerance to hold on the field.

The same stunt on the CEA requires about 15 times larger $\frac{a}{B_0}$.

For a 1 gauss error, at 1", $a = 1$

$$\frac{\delta\nu}{2\pi} = .2$$

Most likely the sextupole term will dominate the problem.

Now lets look at what kind of quadrupole strengths are required.

Assume $\frac{\Delta p}{p} = 2 \times 10^{-3}$ ~~say~~

$$\Delta \nu_{\text{bend}} = \frac{\partial \nu}{\partial R} \Delta R = \frac{\partial \nu}{\partial R} \frac{R}{\nu^2} \frac{\Delta p}{p}$$

$$\frac{10^5}{400} \times 2 \times 10^{-3} \Rightarrow \Delta \nu = \frac{1}{2} \frac{\partial \nu}{\partial R}$$

Now suppose we have $\frac{\partial \nu}{\partial R} = .1/\text{cm}$ say. $\Rightarrow .25/\text{inch}$

then $\Delta \nu = .05$ This is a rather large value. $\Delta \nu$ could easily be as small as $.01$.

Now $\delta\mu = \frac{G\ell}{2p} \times \beta = 2\pi \delta\nu$

At 200 Gev, $G\ell = \frac{4\pi p}{\beta} \delta\nu$

$\beta = 100$ meters $\Rightarrow 4 \times 10^3$ inch $\quad p = 2.6 \times 10^8$ gauss-in

$\therefore \; G\ell = 2.6 \times \pi \times 10^5 \times \delta\nu = 8.2 \times 10^5 \, \delta\nu$ gauss.

If we assume that there are 96 quads, then $G\ell$/quad

$= 8.5 \times 10^3 \, \delta\nu$ gauss/quad

$= 85$ gauss/quad

∴, if we have a 12" long guard, this means ≈ 10 gauss/inch ⇒ 40 gauss/inch.
With a $r = 2.5"$ guard, this is 25 – 100 gauss at the pole tip.

With an 8 turn coil, 300 amp/turn ⇒ 40 amps.
The load will look mainly resistive, probably.

One of the advantages of this system is that it leaves a substantial ω in the beam, which can help to damp incoherent oscillation which might otherwise arise. Also, the spill servo in the special guard circuit, which is completely orthogonal to the power supply, which merely attempts to remain as "flat" and ripple free as possible.
A back-leg winding sort of pickup is satisfactory for this kind of closed loop.

Now lets look at the analysis for the quadrupole current supply which regulates the spill:

$$\dot{\nu} = \left[\frac{\partial \nu}{\partial r}\,\frac{R}{\nu^2}\,\frac{\Delta p}{p}\right]^{\cdot} + \Delta\nu\,\omega \sin\omega t$$

($\Delta\nu$ in the beam)

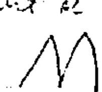
Now $\Delta\nu$ is the ripple current in the quad circuit,

$$\Delta\nu = K\,\Delta I$$

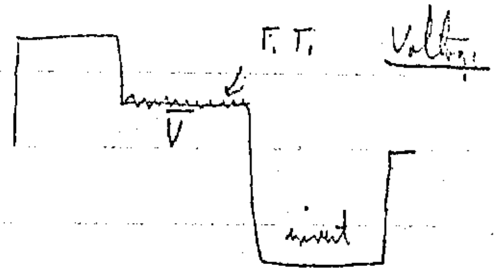
$$\therefore \quad \Delta I \leq \frac{1}{K\omega}\cdot\frac{\partial \nu}{\partial r}\,\frac{R}{\nu^2}\cdot\left(\frac{\Delta p}{p}\right)^{\cdot}$$

Now we see that a large $\frac{\partial \nu}{\partial r}$ is an asset.
We can use sextupoles to correct this term.
Now the absolute current required is just given by

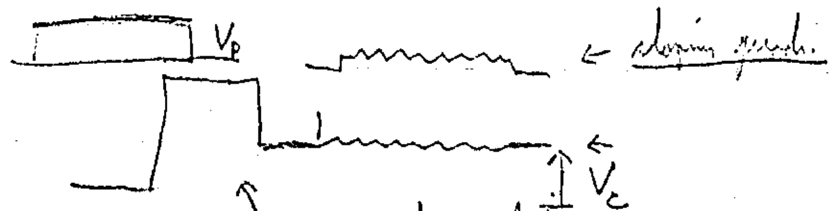
$$I_{max} = \left[\frac{1}{K\omega}\,\frac{\partial \nu}{\partial r}\,\frac{R}{\nu^2}\,\frac{\Delta p}{p}\right]^{\cdot}$$

The only requirement then is that $\dot{I}_q \neq 0$.
This means that the voltage form must be better than 1 phase, full wave:  $\Rightarrow$  in the spill.

If we take the direct ripple voltage and feed it to the quads, then we must be sure that the current-voltage relations are matched. A passive low level matching pad could take care of this.



We will run this into an adder:



pre program for constant current
then switch a bipolar signal.

$V_p - \overline{V} = V_c$, ~~...~~ when $V_c \geq x\Delta V$

$V_c = I_o R$

Lets consider the ripple requirements:

$\Delta r = 100 \frac{\Delta p}{p}$ inches.

If $\frac{\Delta p}{p} = 2 \times 10^{-4}$, $\Delta r = .02$ inches

Now we can have $\frac{\Delta p}{p} = 2 \times 10^{-3}$, $\Delta r = .2$ inches $= \frac{1}{2}$ cm.

Now since $\frac{\Delta p}{p} = \frac{\Delta I}{I}$, we require regulation of 1 part in 1000 over the flat top. Current regulation a factor of 2 better than this would be considered excellent.

The current ripple requirement is given by the relation:

$$\frac{\Delta I_\omega}{I} = \frac{1}{\omega} \frac{\Delta p}{p} \quad \text{for a 1 sec. flat-top}$$

To convert this to volts, we have:

$$\Delta I . L\omega = \Delta V_\omega$$

$$\Delta V_\omega = L I \frac{\Delta p}{p}$$

$I \approx 2000$ amps, $L \approx 5$ henries

$$\Delta V_\omega \cong 10^4 \frac{\Delta p}{p}$$

Now if we feed the voltage back to the quads we expect another factor of at least 10 in decreased sensitivity to ripple.

$$\therefore \Delta V_\omega = 10^5 \frac{\Delta p}{p} \text{ is tolerable.}$$

for 2×10^{-4} this is 20 Volts.

For the case when the energy spread is enlarged obtain $V_\omega = 200$ Volts. The goal therefore should be to shoot for a max. ripple of ~ 20 volts, although one expects good performance even with 100 volt ripple.

In order to keep beam motion down in the 1/4 cm region, we require a limit on ΔI.

$$\frac{\Delta I_\omega}{I} \leq 5 \times 10^{-4}$$

$$\Delta I_\omega \leq 1 \text{ amp}$$

$$\Delta I_\omega = \frac{\Delta V_\omega}{L\omega} \leq 1$$

$$\therefore \Delta V_\omega \leq L\omega$$

for the lowest frequency, 60 cps, say, this requirement is less stringent than the ripple requirement, and therefore the ripple modulation will not move the beam in the 200 volt case by more than .5 mm.