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NOTE ON MAGNETS FOR USE IN HIGH ENERGY BEAMS AT THE PROPOSED 200 GEV ACCELERATOR

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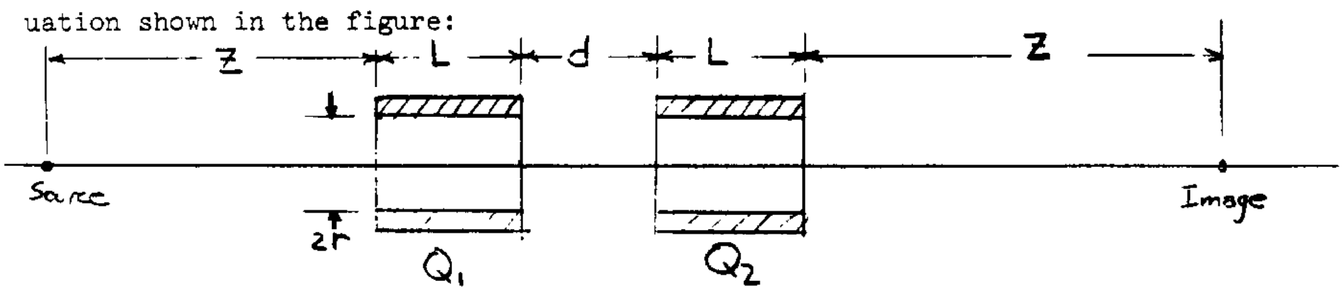
In considering possible experiments for the proposed accelerator, one is soon led to a consideration of beam design. In order to design a beam, he must first make a guess as to what kinds of magnets will be available. This note presents the author's "guesses" based on a study of requirements for typical experimental situations. It is not meant to be a comprehensive study of magnet design, but rather to suggest what kind of magnets would be desirable for typical experiments involving beams with energies from 50 to 200 GeV.

It is clear from the outset that a major limitation on general-purpose magnets will be cost — both for fabrication and power. It is, therefore, not reasonable to just scale up the length of magnets currently in use at existing accelerators. It is also likely that large superconducting magnets will be either unavailable or economically unattractive during the first few years of accelerator operation so that we consider here only magnets of the conventional type. This means that the maximum field in bending magnets is approximately 20 kg and that the maximum field gradient in quadrupoles is approximately $12r^{-1}$ (kg/in) where r is the radius in inches.

Quadrupoles

At low beam energies, quadrupoles with a radius of 6 inches or more are often used. However, it is not clear that such quadrupoles would be practical at high energies due to the limitation on the maximum field gradient mentioned above.

In order to get a feeling for the problem, let us consider a typical situation shown in the figure:



The image and object distances are taken to be equal for simplicity and "by symmetry" the field gradients in the two quadrupoles will be equal and opposite. We define a parameter

$$\theta = \left[\frac{L^2 G}{1312\ P} \right]^{\frac{1}{2}},$$

where L is the length of each quadrupole section (in inches), G is the gradient (kg/inch), and P is the beam momentum (GeV/c). If we replace the quadrupole doublet by an equivalent thin lens of focal length

$$f = \tfrac{1}{2}\left(Z + L + \frac{d}{2}\right),$$

then we find that for $\theta^2 \ll 1$,

$$\frac{1}{f} \approx \frac{\theta^4}{L} \left(\frac{d}{L} + \frac{2}{3}\right) = \frac{L^3 G^2}{(1312P)^2} \left(\frac{d}{L} + \frac{2}{3}\right) \qquad (1)$$

This formula turns out to be an excellent approximation for $\theta \lesssim 0.5$. The solid angle accepted by the quadrupole is approximately

$$\Delta\Omega \cong \frac{\pi r^2}{\left(Z + L + \frac{d}{2}\right)^2} = \frac{\pi r^2}{4f^2} = \frac{\pi r^2}{4} \cdot \frac{\theta^8}{L^2} \left(\frac{d}{L} + \frac{2}{3}\right)^2$$

If we take the maximum allowable gradient to be

$$G_{max} = \frac{12}{r} \left(\frac{kg}{in.}\right),$$

then

$$(\Delta\Omega)_{max} = \frac{\pi L^6}{4(1312P)^4} \cdot \frac{(12)^4}{r^2} \left(\frac{d}{L} + \frac{2}{3}\right)^2 \qquad (2)$$

This leads to the surprising result that the solid angle accepted varies <u>inversely</u> with the square of the radius. This result, however, is only correct for θ small, so eq. (2) is not valid for small <u>r</u>. We have also assumed that the quadrupole can be brought as close to the source as desired, an assumption that may not always be true in practice. It is also true that we can always get a larger solid angle by increasing r <u>provided</u> that we increase L appropriately at the same time. Eventually, however, as L is increased and the entrance of the quadrupole moves closer to the source the gain in useful solid angle with increasing L becomes very small. (See table IV and discussion below).

In order to investigate the problem more quantitatively the configuration shown below has been studied in detail, using the computer program "OPTIK".[1]

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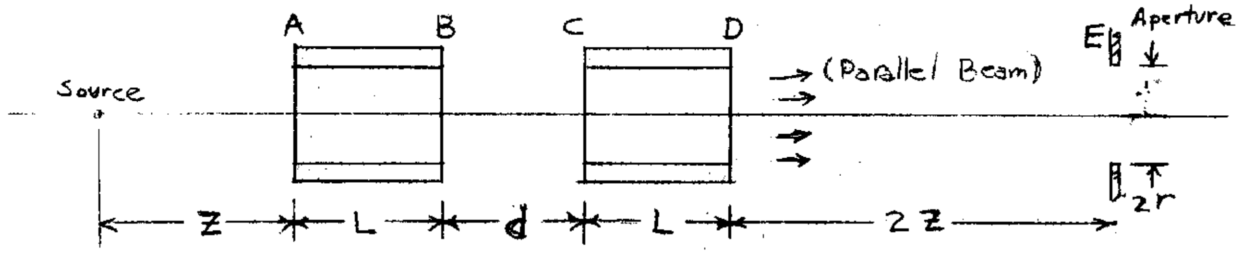
[1]Thomas J. Devlin, "Optik, a Computer Program for the Optics of High-Energy Beams." UCRL-9727, Sept. 15, 1961 (unpublished).



The gradients in the two quadrupoles were adjusted to give a parallel beam out of the second quadrupole. This represents a very common experimental situation. The object distance Z was chosen to give a gradient in the first quadrupole of $12r^{-1}$ (kg/inch). The effect of the various apertures in the system at positions A, B, C, D, and E was then determined using OPTIK. The radius of each aperture was taken as _r_, the radius of the quadrupole. The aperture at E represents, for example, the entrance to another quadrupole. The limitation imposed by the various apertures on the phase space accepted from the source for various values of _r_ is shown in Fig. 1. The case illustrated is for L = 64", d = 64" and P = 100 GeV/c with quadrupoles of radii 0.5", 1.0", 2.0", and 4.0". In these plots each aperture projects into a line in phase space. Only those rays emanating from the source for which the corresponding points in phase space are closer to the origin than this line will be passed by the aperture. The results are summarized in Table 1.

TABLE 1

L = 64", d = 64", P = 100 GeV/c

					Maximum Source Dimensions			
					without Aperture E		with Aperture E	
r (inches)	G_1 (kg/in)	G_2 (kg/in)	Z (in.)	$\Delta\Omega$ (ster.) x 10^6	CD	DC	CD	DC
0.5	23.9	12.4	114.	12.0	±0.5"	±0.19"	±0.14"	±0.19"
1.0	12.0	8.92	350.	11.0	1.0	0.57	0.37	0.57
2.0	6.00	5.42	1195.	5.7	2.0	1.46	0.99	1.46
4.0	3.00	2.92	4490.	2.1	4.0	3.50	2.59	3.50

For comparison, about 70% of the pion flux at 100 GeV/c is contained within a solid angle of 6.0×10^{-6} ster. according to the CKP formula. In this case the 0.5" quadrupole accepts the largest solid angle, but it would only be suitable for use with rather small sources (~ 0.25" diameter). It is expected that the external beam of the 200 GeV accelerator can be focussed to spot ~ 0.1" diameter or less[2] so in general this is not a severe limitation. It is interesting to note that if the beam is made convergent rather than parallel on leaving the second quadrupole, then the smaller-bore quadrupoles are favored even more over the larger ones.

In Table II we show the same quantities for L = 64", d = 128".

TABLE II

L = 64", d = 128", P = 100 GeV/c

r (inches)	G_1 (kg/in)	G_2 (kg/in)	Z (in.)	$\Delta\Omega$ (ster.) $\times 10^6$	Maximum Source Dimensions: without Aperture E, CD	without Aperture E, DC	with Aperture E, CD	with Aperture E, DC
0.5	24.0	9.05	93.	4.4	±0.34"	±0.14"	±0.12	±0.16
1.0	12.0	7.21	264.	10.1	1.0	0.46	0.31	0.46
2.0	6.00	4.39	821.	13.5	2.0	1.28	0.82	1.28
4.0	3.00	2.31	2902.	10.7	4.0	3.13	2.23	3.13

In this case the 1" quadrupole subtends a slightly larger solid angle than the others. Table III shows the effect of increasing L to 128" while keeping d = 128".

TABLE III

L = 128", d = 128", P = 100 GeV/c

r (inches)	G_1 (kg/in)	G_2 (kg/in)	Z (in.)	$\Delta\Omega$ (ster.) 10^6	Maximum Source Dimensions: without Aperture E, CD	without Aperture E, DC	with Aperture E, CD	with Aperture E, DC
0.5	24.0	4.24	1.5"	13.8	±0.12"	±0.05"	--	--
1.0	12.0	3.76	68.5	28.2	0.44	0.21	0.24	0.22
2.0	6.00	3.10	227.	47.0	2.0	0.77	0.57	0.77
4.0	3.00	2.23	700.	44.0	4.0	2.28	1.47	2.28

[2] Glen Lambertson, private communication.

This causes a significant increase in solid angle with the 2" radius quadrupole subtending the largest solid angle. Even with 128" sections the object distance of the 4" radius quadrupoles would be 700 inches at 100 GeV/c and over twice as long at 150 GeV/c.

Table IV shows the effect of increasing r while maintaining Z constant at 250" and at the same time increasing L so that the gradients do not exceed $12\ r^{-1}$.

TABLE IV - Variation of Flux accepted with Radius of Quadrupole (Z = 250", $\frac{d}{L} = 1$)

r	L	$G(\frac{KG}{in})$	G_2	$\theta_x(x10^3)$	$\theta_y(x10^3)$	Fraction of total flux accepted (100 GeV/c	"Weight"
1"	74"	12.0	7.8	1.35	3.7	.32	1
2"	123"	5.96	3.25	1.9	6.9	.50	~6
4"	200"	3.05	1.36	2.7	12.6	.65	~40
6"	270"	2.00	0.79	3.17	17.3	.75	~120

In all cases d = L and P = 100 GeV/c. θ_x and θ_y are the half-angles of the acceptance cone in the DC and CD planes respectively. For comparison 70% of the available pion flux is contained within an angle $\theta \approx 4.4 \times 10^{-3}$. In order to maintain the same fraction of the total flux at lower momenta, say 50 GeV/c, we must double r, thereby halving the maximum gradient and increasing θ_x and θ_y by 2. Thus a 4" radius quadrupole will accept 50% of the flux at 50 GeV/c. In table IV, we also give the "weight" of the quadrupoles taking the smallest as the unit. Since the cost tends to increase linearly with weight, this is relevant to the economics of the choice.

The rather small gain in flux with increasing r is due mainly to the fact that most of the increase in solid angle goes into θ_y which is already large compared to the "Cocconi angle" The choice of Z = 250" is arbitrary of course, but if it is increased, r must be even larger to get the same fraction collected (though L can be shorter). If d/L is decreased to make θ_x and θ_y more nearly equal, then L must be made larger to compensate for the weakening of the lens.

The available pion fluxes range from 5×10^7 sec^{-1} at 150 GeV/c to 1.3×10^{10} sec^{-1} at 25 GeV/c with a ± 1% momentum bite.* Even with only 10% of the total collected, these fluxes would be enough to swamp any experiment with counters or spark chambers in the beam. At present, the most practical means of separating beams at these energies seems to be Cerenkov counters so it is quite possible that most of the "nonpermanent" beams may be limited to total fluxes ~ 10^7 particles/sec. For "permanent" beams such as those for neutrino experiments, and perhaps muon experiments; and those with r.f. separators it is reasonable to assume that special quadrupoles will be used. Unless a new technique for mass separation at high energies is developed, it appears there is in general little reason for trying to capture most of the available flux. Thus, for most beams 1" or 2" radius quadrupoles seem to be quite adequate. Quadrupoles with 0.5" -radius bores would be too small for general-purpose use though they may be useful for special applications such as in forming a beam of short-lived particles. If they could be constructed economically and if the assumed gradients of ≈ 24kg/in. could actually be achieved in practice, they could be useful for fairly low-intensity beams. They could not be used economically for transporting beams over large distances because of the short focal lengths involved.

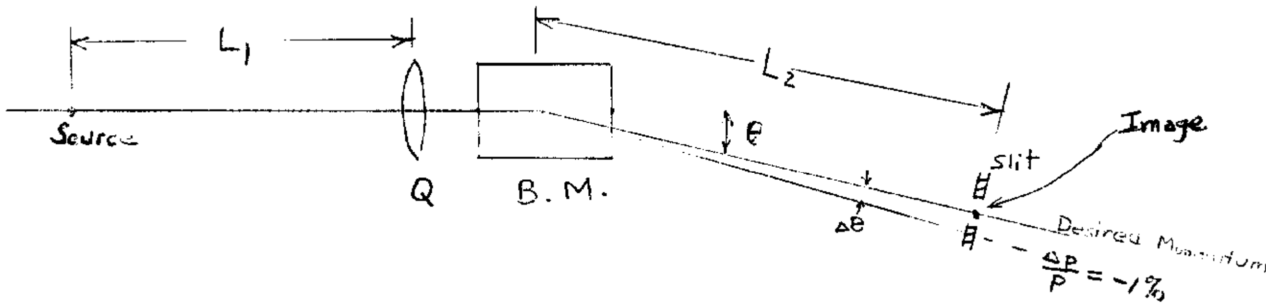
The smaller quadrupoles also have the advantage of smaller overall demensions which makes them easier to bury in a shielding wall and transport. Because of the massive shielding required, it is quite probable that many quadrupoles will remain buried for long periods even though not in use. This is a strong argument for having many inexpensive quadrupoles available.

It would appear that quadrupole "modules" of length approximately 64" and bores of both 1" and 2" radius would be most economical and would satisfy most experimental requirements. For beams of momenta $\gtrsim$ 100 GeV/c two or more of these modules could be put end-to-end to obtain larger solid angles. In certain applications such as muon beams larger bores may be desirable.

* This assumes the CKP formula with a proton intensity of 5×10^{12} pps and a target efficiency of 1/3.

Bending Magnets

Once the radii of the quadrupoles is set, it is reasonable to choose bending magnets with the same gap as the quadrupoles (4"). In order to decide on a reasonable length we must set up some criterion for determining the bending angle required. The figure below illustrates schematically the usual situation.



We require that the spatial separation between the image of particles of the desired momentum and that for particles with momentum differing by say 1%, be large compared to the diameter of the images. The most favorable situation is when the quadrupole and the bending magnet are close together. If we neglect their separation, then we require that

$$L_2 \cdot \Delta\theta > \frac{L_2}{L_1} \cdot \text{(diameter of source)}$$

where

$$\Delta\theta \cong \theta \cdot \frac{\Delta p}{p}$$

A reasonable requirement is

$$\Delta\theta = .01 \cdot \theta = 2 \cdot \left(\frac{\text{diameter of source}}{L_1}\right)$$

Referring to Tables I, II, and III, we find $L_1 \sim 400''$ (measured from source to center of quadrupole) for 100 GeV beams. If we assume a source diameter $\approx 0.1''$ as discussed previously, then we need $\theta = .05$ at 100 GeV/c. This requires a magnet approximately 25 feet long with a field of 20 kg. It is probably more practical to build modules 10 or 12 feet long. The width of the pole tip should be at least 10 to 12 inches to allow fairly large bends at lower momenta.

Conclusions

An attempt has been made to determine suitable dimensions for quadrupole and bending magnets to be used for transporting beams of momentum 50 to 200 GeV/c. Quadrupole modules approximately 64" long with 2" and 4" -diameter bores appear to be optimal for most applications. Bending magnets approximately 10 feet long with 4" gaps and 10" wide pole tips would satisfy most experimental requirements. No real attempt has been made to determine the most economic choice of parameters, but those suggested seem very reasonable from an economic point of view. Also a complete study should be made of the magnets required near the production target. In situations where the first quadrupole must be a long distance from the source, it might be desirable to use quadrupoles with an $\approx$ 8" diameter bore; but the length of each section would have to be $\approx$ 12 ft. long for a 150 GeV/c beam.

A 150 GeV/c beam has been designed using magnets of the sizes suggested and is discussed in another report.[3] On the basis of current estimates of available pion fluxes, a beam of 10^7 pions/sec at 150 GeV/c with a momentum spread of $\pm$ 1% is easily obtainable with a proton beam of 5×10^{12}/sec.

The author wishes to thank Dr. Glen Lambertson for many helpful discussions and suggestions.

[3] Michael J. Longo, "A 150 GeV/c Beam for Spark Chamber Experiments".

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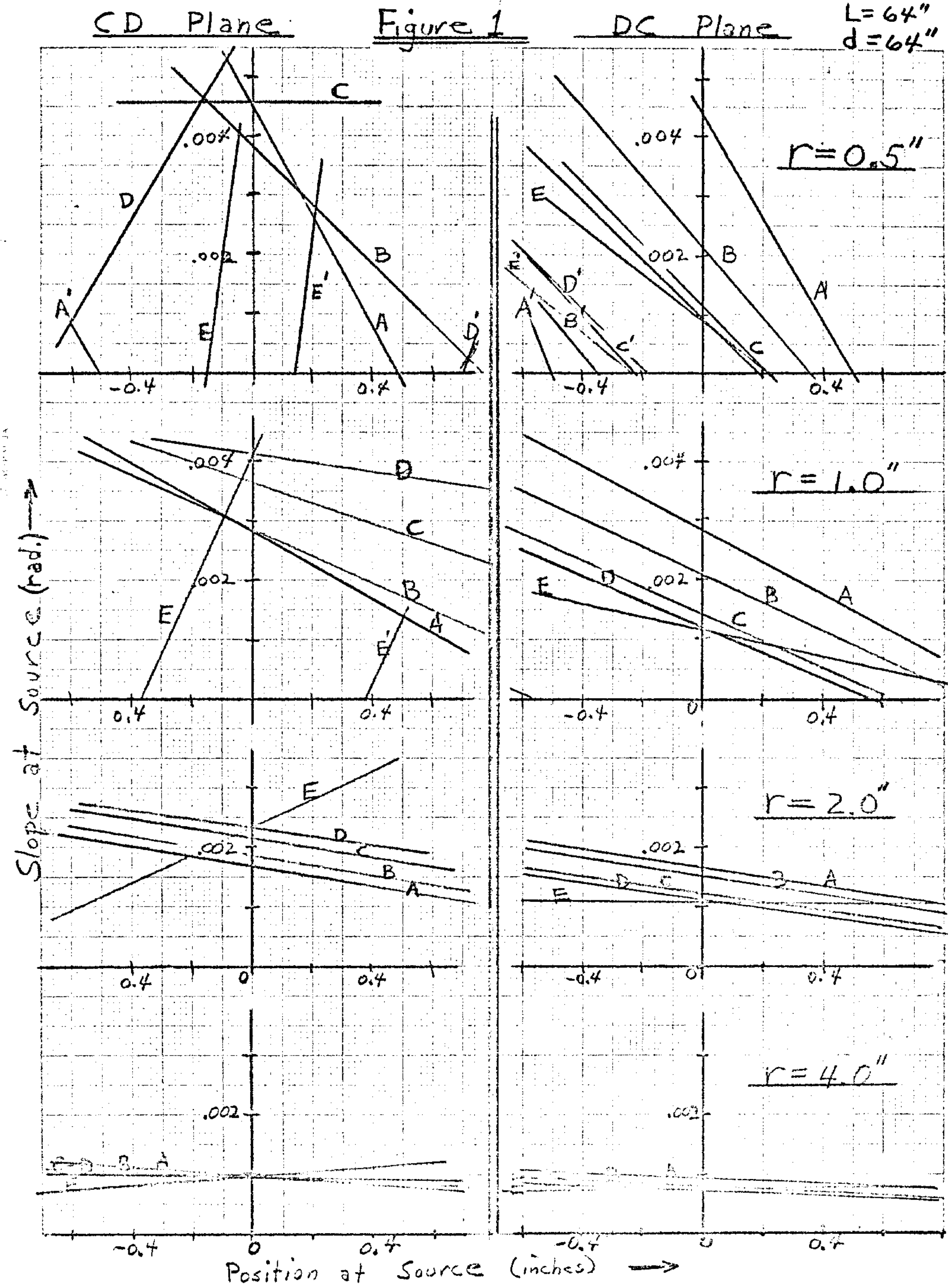

CD Plane
Figure 1
DC Plane
L=64"
d=64"
r=0.5"
r=1.0"
r=2.0"
r=4.0"
.004
.002
-0.4
0
0.4
A
B
C
D
E
A'
D'
E'
B'
C'
Slope at Source (rad.)
Position at Source (inches)