

SOME NOTES RELATED TO

- A. DIRECT INJECTION INTO THE MAIN RING
- B. VERTICAL VERSUS HORIZONTAL INJECTION
- C. MAIN RING INJECTION

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August 10, 1967

Direct Injection into the Main Ring, Linac 200 MeV

For a reduced scope program direct injection from the linac into the main ring has been considered. Assume the following parameters:

$$(v_{rf})_i \cong 28 \text{ Mc/s} \quad (v_{rf})_f \cong 50 \text{ Mc/s} \quad h \cong 1000$$

First minimum rf related to required bucket area:

$$(\hat{eV})_{rf} = \frac{\pi h |\eta| E_s}{8 N F^2 (\varphi_s) \beta^2} \left( \frac{\Delta E}{E} \right)^2 \quad (\text{Bruck; Cole--Morton Paper})$$

( $\Delta E$  refers here to full energy width.)

Assume  $\varphi_s = 30^\circ$  (Bruck definition  $\varphi_s = 60^\circ$ ). Bucket width  $\cong 50\%$ , i.e.,  $F(\varphi_s) = 0.585$  (take  $N = 1$ ).  $\left( \frac{\Delta P}{P} \right)_{linac} \cong \pm 10^{-3}$ , this gives  $\left( \frac{\Delta E}{E} \right)_{full \text{ width}} \cong 10^{-3}$ , assuming some allowance for jitter, etc. This results in

$(\hat{eV})_{rf} \cong 2.4 \text{ MeV/turn}$ . Also, with  $v_s^2 = \frac{h |\eta| (\hat{eV})_{rf} \cos \varphi_s}{2 \beta^2 \pi E_s}$ , it follows that at injection  $v_s \cong 1.0$ .

For synchronous acceleration only, with  $(\Delta E_s) = \frac{(\hat{eV})_{rf}}{\sin \varphi_s} = \frac{2\pi \rho_o R_o \dot{B} (e)}{\sin \varphi_s}$

and  $\rho \cong 0.75 \cdot 10^3$ ,  $R \cong 10^3$ ,  $\sin \varphi_s = 0.5$ , it follows that, with a reasonable initial  $\dot{B}$  value of  $0.1 \text{ T/sec}$ ,  $(\hat{eV})_{rf} \cong 1 \text{ MeV/turn}$ .

These parameters, in addition to the greater complexity of the rf system, essentially rule out further consideration of direct injection from the linac followed by acceleration.

[For completeness sake consider a second rf system in the main ring (with all its complications of transfer during the acceleration cycle),  $(v_{rf})_i \cong 2.8 \text{ Mc/s}$ ,  $(v_{rf})_f \cong 5.0 \text{ Mc/s}$ ,  $h = 100$ ,  $(\hat{eV})_{rf} \cong 300 \text{ KeV/turn}$ ,  $v_s \cong 0.1$ ].

The space charge limit and stacking capability has been calculated for 200 MeV injection. By reducing the main ring (Co + Ins) aperture allowance by a factor of two and adding this to betatron acceptance it seemed possible

to obtain  $1.5 \cdot 10^{12}$  protons/pulse. This would not be unreasonable for an initial reduced scope program.

Another objection to this mode of operation is the significantly lower injection field ( $\cong 30$  gauss) by a factor of approximately ten, compared with 10 BeV injection.

In general direct injection looked sufficiently unfavorable that no further attention has been devoted to it.

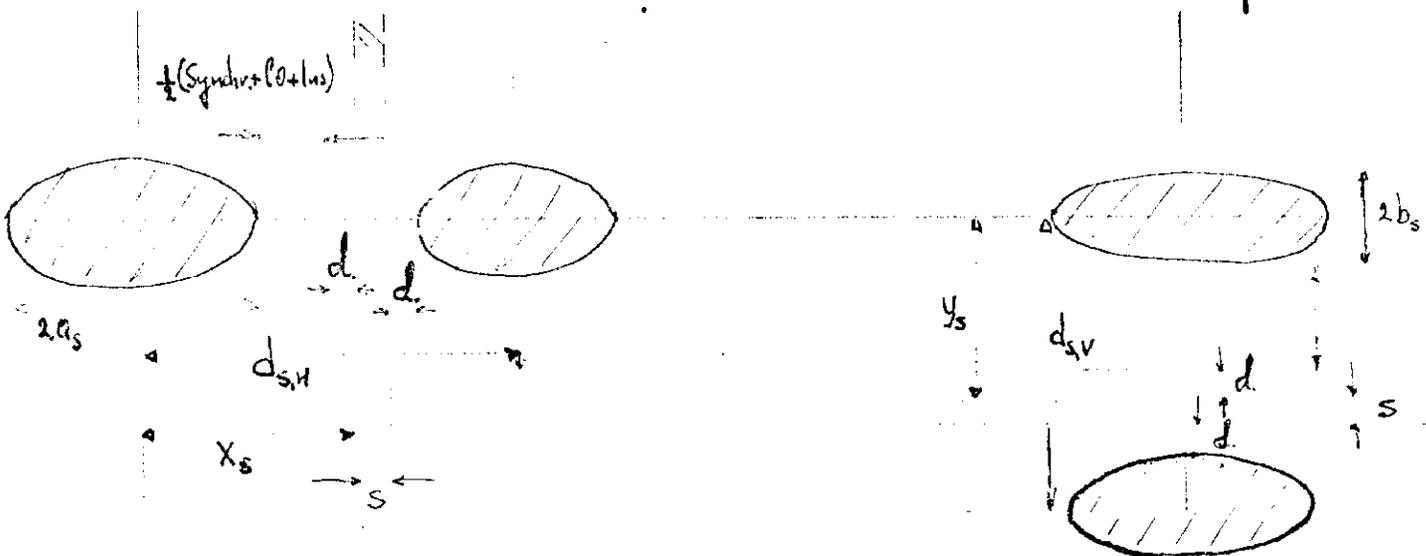
Vertical Versus Horizontal Injection

(The following was stimulated by and benefitted from discussions with J. Peterson on the relative merits of vertical versus horizontal injection.)

Because of the relative magnitudes of booster beam emittance and main ring admittance single turn injection is assumed. Further, the injection component sequence magnetic inflector (septum magnet) fast kicker combination is used to bring the beam onto the (essentially unperturbed) equilibrium orbit. The kicker magnet must have the full main ring aperture. Consider the following, related to beam deflection at the location of the septum.

Horizontal Inj.

Vertical Inj.



$$d_{s,h} = 2a_s + \frac{1}{2} (\text{Ins.} + \text{Co} + \text{Synchr}) + 2(d) + S.$$

$$d_{s,v} = 2b_s + \frac{1}{2} (\text{Ins.} + \text{Co}) + 2(d) + S$$

After commissioning of the accelerator orbit deformations will be used near injection to eliminate the  $\frac{1}{2}$  Co allowance, or  $\frac{1}{2}$  (Co + Ins.) aperture allowance. Here Ins. is preserved for beam blow-up phenomena, etc. and taken to be proportional to the beam size, i.e.,  $\frac{1}{2}$  (Ins. + Co)  $\rightarrow$   $\frac{1}{2}$  (Ins.)  $\rightarrow \cong 0.4 a_s, 0.4 b_s$  respectively. Further, it is unreasonable to assume that clearance for beam "tails" and steering errors should be taken proportional to beam width  $\rightarrow (d) \cong 0.2 a_s$  or  $0.2 b_s$ , respectively.

$$d_{s,h} = 2.6 \cdot a \sqrt{\frac{\beta_{h,s}}{\beta_{h,max}}} + A_{ss} \quad \text{with } A_{ss} = (\frac{1}{2} \text{ Synchr} + S)$$

and

$$d_{s,v} = 2.6 \cdot b \sqrt{\frac{\beta_{v,s}}{\beta_{v,max}}} + S.$$

Using now the well known relationships for a fast kicker:

$$V_k = \frac{(B1)_k w}{\tau_r} \quad \left[ \text{recall also that } I_k = \frac{(B1)_k h_k}{l_k} \right]$$

one finds

$$\frac{V_{k,h} \tau_r}{(B1)_{k,h}} = w_{k,h} = 2a \sqrt{\frac{\beta_{k,h}}{\beta_{h,max}}} + (\text{Ins.} + \text{Co} + \text{Synchr})_h = 2a \sqrt{\frac{\beta_{k,h}}{\beta_{h,max}}} + A_h$$

$$\frac{V_{k,v} \tau_r}{(B1)_{k,v}} = 2b \sqrt{\frac{\beta_{k,v}}{\beta_{v,max}}} + B_v \quad \text{where } B_v = (\text{Ins.} + \text{Co})_v$$

Typically, for NAL main ring  $A_h \cong 6$  cm,  $B_v \cong 3$  cm.

Further, with kicker and septum magnet located in the same medium straight,

$$\frac{d_{s,h}}{l_{k,s}} = \theta_{k,h} = \frac{(B1)_{k,h}}{B\rho} \quad \text{and} \quad \frac{V_{k,h} \tau_r}{(B\rho)} = \theta_{k,h} \left[ 2a \sqrt{\frac{\beta_{k,h}}{\beta_{h,max}}} + A_h \right]$$

yields, with a similar approach for the vertical coordinate,

$$\frac{V_{k,h}}{V_{k,v}} = \frac{\left[ 2a \sqrt{\frac{\beta_{k,h}}{\beta_{h,max}}} + A_h \right] \left[ 2.6 a \sqrt{\frac{\beta_{h,s}}{\beta_{h,max}}} + A_{ss} \right]}{\left[ 2b \sqrt{\frac{\beta_{k,v}}{\beta_{v,max}}} + B_v \right] \left[ 2.6 b \sqrt{\frac{\beta_{v,s}}{\beta_{v,max}}} + S \right]}$$

Keeping some realistic values in mind, this may be simplified with

$$A_{ss} \ll 2.6 \sqrt{\frac{\beta_{h,s}}{\beta_{h,max}}}, \quad S \ll 2.6 \sqrt{\frac{\beta_{v,s}}{\beta_{v,max}}} \quad \text{and} \quad \beta_{v,max} \cong \beta_{h,max} \cong \beta \quad \text{to}$$

$$\frac{v_{k,h}}{v_{k,v}} = \frac{a}{b} \sqrt{\frac{\beta_{h,s}}{\beta_{v,s}}} \left[ \frac{2a \sqrt{\frac{\beta_{k,h}}{\beta_{h,max}}} + A_h}{2b \sqrt{\frac{\beta_{k,v}}{\beta_{v,max}}} + B_v} \right]$$

Maintaining  $\tau_T$  ( $\cong 15$  nsec required here) it is desirable to choose minimum  $V_k$ . Substituting in the last equation appropriate values will already guide

the choice of vertical versus horizontal injection. To illustrate this

further, assume

$$A_h \cong 2a \sqrt{\frac{\beta_{k,h}}{\beta_{h,max}}}, \quad B_v \cong 2b \sqrt{\frac{\beta_{k,v}}{\beta_{v,max}}}, \quad \text{which is quite realistic}$$

in the present case.

This results in

$$\frac{v_{k,h}}{v_{k,v}} \cong \frac{a^2}{b^2} \sqrt{\frac{\beta_{h,s} \beta_{h,k}}{\beta_{v,s} \beta_{v,k}}} \quad \text{or}$$

$$\left[ V_k \cong \text{Const. } E \sqrt{\beta_k \beta_s} \right]_{h \text{ or } v} \quad \text{where } E = \text{beam emittance}$$

Two conclusions may be drawn from this, first, with freedom to locate the kicker-septum magnet combination, minimize  $\sqrt{\beta_k \beta_s}$ ; second, with  $\frac{a^2}{b^2} \cong 10$  (or  $\frac{E_h}{E_v} \cong 10$ ) for the main ring, vertical injection is clearly more favorable.

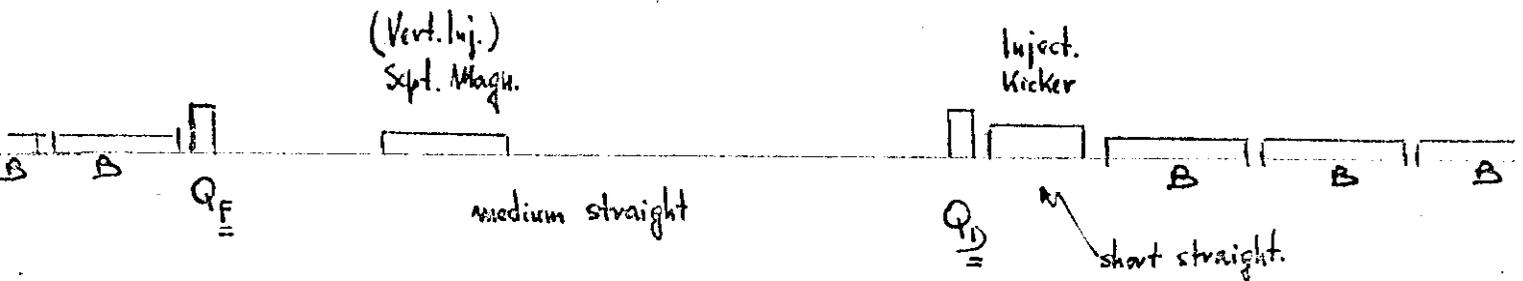
For a regular lattice, i.e., kicker and septum magnet in different straights, it follows, analogously, with  $\theta_k = \frac{d_s}{\sqrt{\beta_k \beta_s} \sin(\mu_s - \mu_k)}$  that

$$\frac{v_{k,h}}{v_{k,v}} = \frac{a^2}{b^2} \frac{\sin_v(\mu_s - \mu_k)}{\sin_h(\mu_s - \mu_k)} \quad \text{which is useful, when considering booster ejection,}$$

typically, i.e., it is clearly desirable to choose the distribution of ejection straights in the booster such that  $\sin(\mu_s - \mu_k) \cong 1$ . (Further examination of the ejection orbits indicate that  $\mu_s - \mu_k \cong 70^\circ$  to  $80^\circ$  seems most favorable, in general.)

Main Ring Injection

During the process of evolving the main ring lattice a variety of lattice arrangements, obtained by A. Garren, were tested for suitability of injection. This was especially critical since initially the medium straight was obtained by eliminating only two bending magnets rather than four ( $\cong 15$  m rather than  $\cong 28$  m straight). Since all the calculated combinations involved the 15 m straight the results are no longer relevant (especially where out of necessity in some cases injection through the horizontally focusing quadrupole had been assumed) and will not be reproduced here. For the present lattice the following injection arrangement is assumed:



Using arguments similar as in Appendix IV, Add. C, i.e., use of closed orbit bumps, beam blow-up insurance, beam blow-up proportional to beam size, etc. a range of required displacements at the location of the injection septum magnet is obtained. These, depending on the various assumptions, range from a conservative value of  $\cong 3.5$  cm to a minimum value of  $\cong 1.7$  cm, taking into account that minimum phase space separation at the location of the septum magnet of injected beam and equilibrium orbit beam is desirable to reduce injection errors due to kicker amplitude variations. One (preferred) case is cited:  $d_s = 2.6$  cm, with the septum magnet located close to the vertical defocusing quadrupole (at  $\beta_v \cong 30$  m) and further, a closed orbit deformation of 0.7 cm, with the "insurance" aperture allowance invested in the possibility of a larger beam size, for either beam blow-up

or the option of (future) two turn injection. With presently a kicker septum magnet separation of  $\cong 25$  m, a kick of  $\cong 1$  mrad would be required in this case. For 10 BeV protons, it seems possible to achieve,  $\cong 2$  mrad per meter total kicker length (modules of 0.3 m length have been assumed) while still "approaching" the required  $\mathcal{L}_T$ . Conservatively a total kicker magnet of 2 m length has been assumed here.