The Use of the Endcoupled Cavity System for the Linac Upgrade

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Introduction.

Bosi Wang[1] has suggested a new form of coupled cavity system as a possible alternative to the sidecoupled (SCS) or disc and washer (DAW) structures. The cavities are $\beta\lambda/4$ long, all on axis, and the phase shift per cavity is $\pi/2$ as in the SCS and DAW structures, where alternate coupling cells or modes are effectively off axis and do not accelerate the beam. The potential advantages are; (1) lower ratio of peak field to field on axis, allowing higher gradient operation ("nose cones" are not needed), (2) higher transit time factor, since the cells are $\beta\lambda/4$ long instead of $\beta\lambda/2$, (3) reasonable sensitivity to errors in cavity resonant frequency, (4) similar sensitivity to errors in coupling constants[2], and (5) "alternating phase focussing" (APF) avoiding the need for quadrupole focusing while maintaining phase focusing. There has been considerable discussion about whether this mode (+--++--++-.....) can be excited in a cavity chain, since it violates most people's intuition. This report, based on talks given at linac upgrade meetings 1/22/88 and 1/29/88, addresses this question and also the question of whether such a system would be useful in the linac upgrade.

Coupled Cavity Systems

The following equation, found extensively in the literature[3,4], and derived for lumped circuits, accurately describes the electric or magnetic fields in a chain of coupled, high Q resonators in the neighborhood of the resonant frequency ω_m;

$$D_m X_m = K^+ X_{m+1} + K^- X_{m-1}; \quad D_m = 1 - (\omega_m/\omega)^2 + (1-j)/Q \qquad 1.1$$

If we restrict ourselves to aperture coupling, avoiding coupling which can introduce phase shifts such as transmission lines or separate cavities, then the K's are real, very nearly the same whether X is an electric or magnetic field, and slowly varying with frequency. Since Q is large, we can generally ignore the Q term in D, its main role being to assure that an uncoupled cavity oscillates freely at a frequency lower than ω_m and has a decrement.

Standing waves (modes) in a system of identical coupled cells can always be viewed as being composed of a sum of positive going and negative going waves leading to a pattern of excitation of the cells, all cells having a common time dependence, say $\cos\omega t$ or $\sin\omega t$. The actual pattern of excitation in a chain of finite number depends on the phase velocity, or phase shift per cell, and on the termination of the system at the two ends. Wave solutions are of the form $X_n = A \exp j(\omega t - n\phi)$ where A is a complex constant. Substituting this into 1.1, neglecting the Q term, and assuming all cells to be the same, we find the dispersion relation, relating the frequency ω to the phase shift ϕ per cell.

$$D(\omega) = 2\,K \cos\phi \qquad 1.2$$

We note that at each frequency we find two equal and opposite values of ϕ, corresponding to the opposite going waves. When $\omega = \omega_n$, $\phi = \pm\pi/2$, independent of the value of K. This is obviously a desirable choice. Among other properties, the slope of the $\omega(\phi)$ curve is greatest, giving the maximum group velocity, and the maximum frequency separation between waves of given phase velocities. Note that if we had taken $K^+ = -K^-$, the solution at $\omega = \omega_n$ would be $\phi = 0, \pm\pi$.

In a chain of a finite number of cavities, the solutions are of the form

$$X_n = A^+ e^{j(\omega t - n\phi)} + A^- e^{j(\omega t + n\phi)} \qquad 1.3$$

ω, (and therefore ϕ), A^+, and A^- must be chosen to fit the termination conditions at the ends of the chain. As an example of this, consider the SCS, where there are an odd number N+1 of cells, the end cells only being coupled to one neighbor. In what follows, we will restrict ourselves to the case where $\phi = \pi/2$. The general mode spectra are derived in the literature[3,4]. Applying 1.1 to n =0 yields $A^+ = A^-$, while applying 1.1 to n = N requires N to be even. Then the excitation pattern is (+0-0+0-0.........0±) depending on whether N/2 is odd or even.

For the ECS, shown in Figure 1, again consider $\phi = \pi/2$. Apply 1.1 to cavities $1,\alpha,\beta,\gamma$ (D = 0)

$$KX_2 + K_4X_\alpha = 0$$

$$K_4X_1 + K_3X_\beta + K_1X_\gamma = 0$$

$$K_3X_\alpha + K_2X_\gamma = 0$$

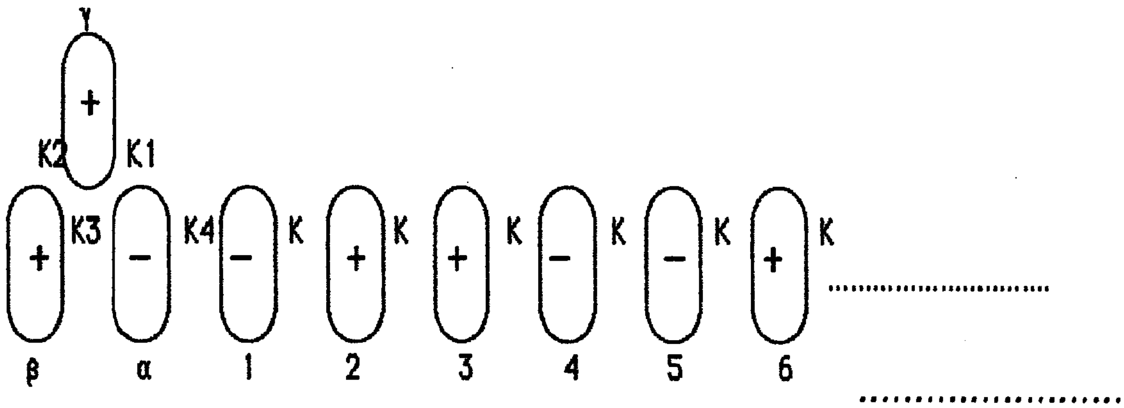

Figure 1. End Coupled Cavity Chain.
The beam passes through cavities βα123456.... in succession.
The K's are the coupling coefficients between cavities.

$$K_2X_\beta + K_1X_\alpha = 0 \qquad 1.4$$

We can solve these for the ratio

$$X_1/X_2 = -B\,,\quad B = 2(KK_1K_3)/(K_2K_4^2) \qquad 1.5$$

Substituting 1.3 for n = 1 and 2 yields

$$A^+/A^- = (j-B)/(j+B) \qquad 1.6$$

Choosing B = 1 yields

$$A^+/A^- = j = e^{j\pi/2} \qquad 1.7$$

That is, the waves are $\pi/2$ out of phase as opposed to in phase for the SCS. Choosing

$$K_{1,2,3} = K_4/2 = K/2 \qquad 1.8$$

gives B = 1 and the pattern shown in Figure 1, with equal magnitude of excitation amplitude in all cells. Renaming β = 0, α = 1 etc, gives

$$X_n = \sqrt{2}\cos(n\pi/2 + \pi/4) \qquad 1.9$$

Similar conditions must pertain at the other end of the chain. A transmission line analogy is given by terminating a line with a ring line of the proper length and impedance. Thus it appears that this mode is physically realizable. Achieving it requires precision in the coupling constants at the end, a property not shared by the SCS and DAW.

Beam Dynamics

It remains to be determined Whether this mode is useful for the linac upgrade. This depends on whether it provides enough phase and radial focusing. Let us consider a chain of cavities each $L_n = \beta\lambda/4$ long with transit time factor T on the axis, average field EX_n. Consider particles which arrive at cavity 0 when the phase is $\Phi + \pi/4 + \delta\phi_0$,That is, the time dependence of the fields is $\sin[\omega t + \Phi + \pi/4]$ and the particle passes the center of cavity 0 at $t = \delta\phi_0/\omega$. Due to velocity differences the $\delta\phi_0$ becomes $\delta\phi_n$ in cell n. For the energy gain in cell n, we find

$$\Delta E_n = eTEL\{ \sin(\Phi+\delta\phi_n) + (-1)^n \cos(\Phi + \delta\phi_n)\}/\sqrt{2} \qquad 2.1$$

The highest average acceleration comes when $\Phi = \pi/2$, and is $eTEL/\sqrt{2}$. We find also the acceleration of the particle with a phase error

$$\delta E_n/\delta\phi_n = eTEL\{\cos\Phi - (-1)^n \sin\Phi\}/\sqrt{2} \qquad 2.2$$

To get AG focussing, choose $\Phi = \pi/2$. Finally the transit time depends on radius as $I_0(2\pi r/\beta\gamma\lambda)$ so the radial momentum kick is given by a deflection theorem

$$\Delta P_n = -\partial^2 \Delta E_n/\omega\partial r\partial\Phi = eTE\beta\pi r(-1)^n/4\sqrt{2}c(\beta\gamma)^2 \qquad 2.3$$

This is equivalent to a series of lenses of alternating strength $\pm f$ separated by $L = \beta\lambda/4$ where

$$1/f = eTE\beta\pi/4\sqrt{2}Mc^2(\beta\gamma)^3 \qquad 2.4$$

The two cell (one period) transfer matrix is

$$M = \begin{bmatrix} 1 & 0 \\ -1/f & 0 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/f & 0 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+L/f & 2L+L^2/f \\ -(1+L/f)/f & 1-L/f-(L/f)^2 \end{bmatrix} \qquad 2.5$$

Then $\cos\mu = 1-(L/f)^2/2$, and for small μ, $\beta = 2f + L$. For the high field case at 116 MeV, the most favorable to the radial focusing, we find $\mu_r = 2.3\ 10^{-3}$ and $\beta = 37$ m. In the SCS or DAW with quadrupoles, β is about 5m, so an auxiliary set of quadrupoles would be needed for the ECS.

For the longitudinal motion we need to know the variation of $\delta\phi$ with energy departure in passing from one cell center to the next. It is

$$\delta\phi = -\pi\beta\delta E/2(\beta\gamma)^3 Mc^2 = -\delta E/E_2 \qquad 2.6$$

From 2.2,

$$\delta E = \pm eTEL\delta\phi/\sqrt{2} = eTE\beta\lambda\delta\phi/4\sqrt{2} = E_1\delta\phi \qquad 2.7$$

The transfer matrix is the same as 2.5 with $L \to -1/E_2$ and $f \to 1/E_1$. Then $\cos\mu = 1 - (E_1/E_2)^2/2$, or $\mu \approx E_1/E_2$ and "β" $\approx 2/E_1 - 1/E_2$. (Since the sense of rotation in $E-\phi$ space is opposite to that in $r-r'$ space, $M_{12} = -\beta\sin\mu$.) The phase focussing is relatively weak as well. A comparison is made in Table 1 below.

RF Power

There is a penalty for operating at high field, namely RF power. If we operate at E = 30 MV/m, T = .9 we need 14.9 m of structure to accelerate by 284 MeV. If Z = 117MΩ/m, the total power needed is about 93 Mw. In the SCS or DAW, if E = 7.5MV/m, T = .88, Φ_s = -32°, we need about 51 m of structure. If ZT^2 = 52 MΩ/m, the total (cavity) power is about 43 MW, or 2 times less. There will be a cost penalty associated with this additional power. Cooling will undoubtably be a more serious problem in the cavities as well. It is interesting to consider a low field version in which the average acceleration gadient is the same as in the SCS or DAW. In table one below, parameters of the SCS/DAW design and low and high field ECS designs are given. Here μ is the longitudinal phase advance in a length $\beta\lambda/2$, "β" is the ratio of phase to energy width in a matched ellipse, and $\Delta\phi$ is the half width of the matched ellipse containing 0.15 MeVnsec, the nominal bunch area. The numbers in brackets are those at 400 MeV, those without brackets are at 116 MeV.

Table 1

		SCS/DAW	ECS (low)	ECS (high)
E	MV/m	7.5	8.8	30
T		.88	.9	.9
Φ_s	°	-32	-	-
ZT^2	MΩ/m	52	117	117
E_{accel}	MV/m	5.6	5.6	19.1
L	m	50.7	50.7	14.9
P	MW	42.5	27.2	92.7
μ	rad	.058 [.032]	.0014[.00042]	.0046[.0014]
"β"	$(MeV)^{-1}$	.195[.368]	8.37 [5.33]	2.45[1.56]
$\Delta\phi$	rad	.216[.297]	1.42[1.13]	.767[.612]
ΔE	MeV	.903 [.807]	.170[.212]	.313[.392]

Conclusions

There are three principal reasons why the ECS structure is problematical for the linac upgrade. (1) There is not enough phase focusing to avoid dilution of the beam from the Alvarez structure except possibly at the highest fields. (2) The gain in transit time factor does not offset the loss in accelerating gradient occasioned by the +--++--..... field pattern, necessitating higher power in the cavities. (3) The radial focusing gained by the APF feature is inadequate in this energy and frequency range, so that quadrupole focusing is still required.

References

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4. E. A. Knapp, et. al., RSI **39**, 979-991, July, 1968