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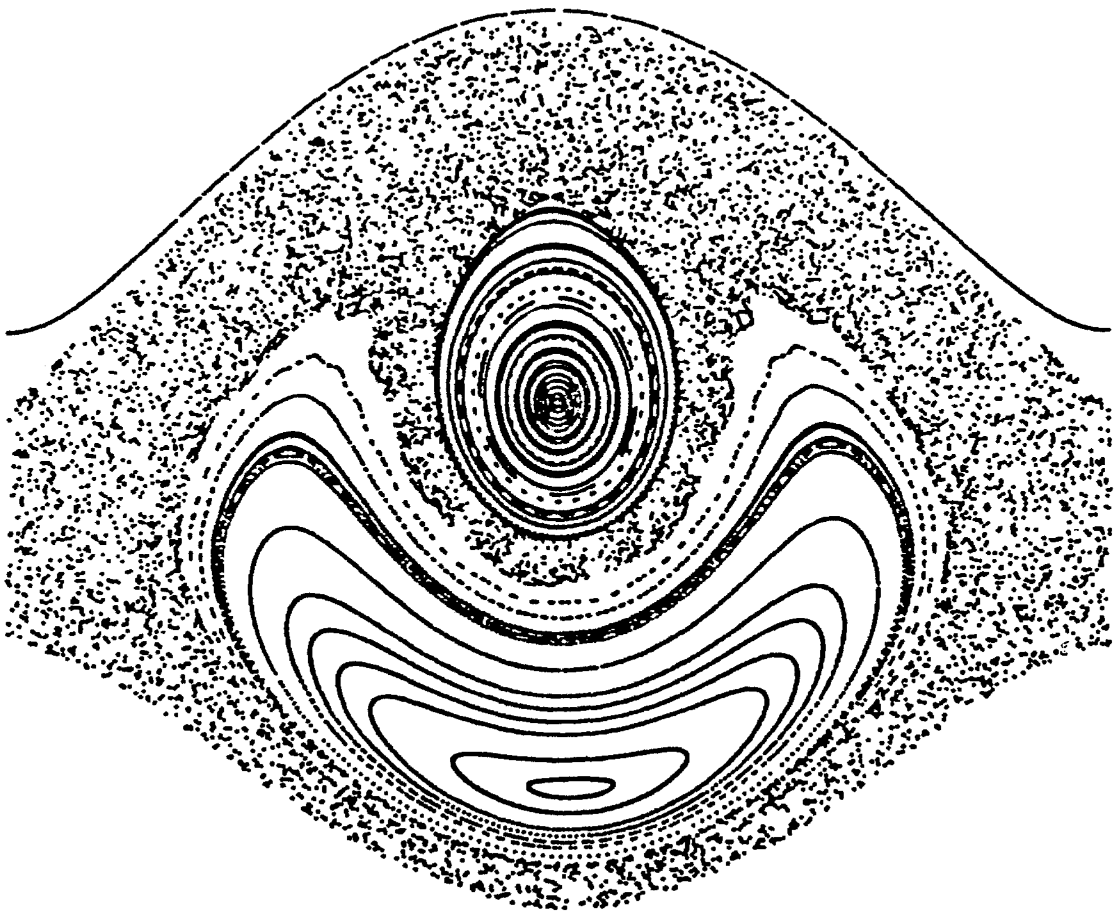

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Comments on Solenoid Coupling Correction in BEPC

S. Peggs

Abstract

First, a mathematical sketch of the solenoid coupling correction problem is drawn, illustrating the main features of a solution using pairs of skew quadrupoles close to the solenoid. Then a comparison is made of the various correction schemes in use, or once used, at a selection of electron colliders worldwide. Finally, a potential BEPC scheme in the current luminosity optics is described, using two pairs of rotated quadrupoles and one pair of skew quadrupoles, all within 20 meters of the collision point.

A mathematical overview of the problem

Consider the general case of the electron collider shown in Figure 1a, with an experimental solenoid surrounded by several skew quadrupoles. It is convenient to take the collision point at the center of the solenoid as a primary reference point, and to refer to it as $*$, as in $\beta_V^* = 0.085$ centimeters. Figure 1 also introduces two other locations, E and W, which are beyond the most "Easterly" and "Westerly" skew quadrupoles. The one turn transfer matrix T_{**} with the coupling magnets all turned ON is related to the one turn matrix with all couplers OFF through the equation

$$T_{**}(\mathrm{ON}) \; = \; P_W \; T_{**}(\mathrm{OFF}) \; P_E \tag{1}$$

Both T and P are 4 by 4 matrices. The "projection" matrix P_E, for example, is given by

$$P_E \; = \; T_{E*}^{-1}(\mathrm{OFF}) \; T_{E*}(\mathrm{ON}) \tag{2}$$

as illustrated in Figure 1b. The matrix T_{E*} represents linear motion from $*$ to E . The west projection matrix P_W is found by substituting W for E in the same expression.

It can be shown[1,2] that, to first order in solenoid strength θ and skew quadrupole strengths q_i, the projection matrix P_E is given by

$$P_E \quad = \quad I_4 + \theta \begin{pmatrix} 0 & I_2 \\ -I_2 & 0 \end{pmatrix} + \sum_i q_i \begin{pmatrix} 0 & Q_i \\ -Q_i^\dagger & 0 \end{pmatrix} \tag{3}$$

Here I_2 and I_4 are identity matrices, the sum is over all skew quads between * and E, and the superscript dagger † denotes matrix transposition. The solenoid angle is given by

$$\theta = \frac{B_s L_s}{2\,(B\rho)} \tag{4}$$

where B_s and L_s are the solenoid magnetic field and (total) length, and $B\rho$ is the magnetic rigidity. It is convenient to define the strength of a thin skew quadrupole as the dimensionless quantity

$$q = \frac{(\beta_H\,\beta_V)^{1/2}}{f} \tag{5}$$

where f is its focal length. With this definition, the skew quadrupole matrix Q becomes

$$Q = \begin{pmatrix} -\,s_H\,c_V \left(\dfrac{\beta_H{}^*}{\beta_V{}^*}\right)^{1/2} & -\,s_H\,s_V\,(\beta_H{}^*\beta_V{}^*)^{1/2} \\ \dfrac{c_H\,c_V}{(\beta_H{}^*\beta_V{}^*)^{1/2}} & c_H\,s_V \left(\dfrac{\beta_V{}^*}{\beta_H{}^*}\right)^{1/2} \end{pmatrix} \tag{6}$$

Shorthand is used to write trigonometric functions of betatron phases as

$$s_H \equiv \sin(\phi_H), \qquad c_H \equiv \cos(\phi_H) \tag{7}$$

et cetera. The phase origin is at the collision point, $\phi_H{}^* = \phi_V{}^* = 0$.

Returning to examine equation (3), note that I_2 is the transpose of itself, so that the matrix elements in the upper right block of P_E are the same as the elements in the lower left block (although they are reorganized). All other elements are either zero or one. This shows that at most four independent parameters are necessary to describe all possible P_E matrices. Consideration of equation (6) confirms that all four are truly independent, since betatron phases ϕ_H and ϕ_V may be chosen so as to set any three out of four Q matrix elements to zero. If both phases are 2π, for example, then $s_H s_V = 0$, et cetera, and only the bottom left term in Q survives.

How many skew quadrupole pairs?

Now that a framework is available in which to solve the problem, how rigorous a solution is required? How many conditions must be met by the skew quadrupole correctors?

Least rigorous - two skew quad pairs. A minimal requirement is that the single turn transfer matrix should be block diagonal and unmodified, when evaluated at a reference point OUTSIDE the local region of solenoid and skew quadrupoles. That is, motion through the interaction region should be the same whether or not the solenoid and its compensating skew quadrupoles are turned on. This "transparency" requirement is mathematically expressed as

$$P_E P_W = I_4 \qquad (8)$$

To first order in coupler strengths, the left hand side of (8) is simply given by (3), where the sum is now extended to include ALL skew quadrupoles (and the solenoid term is doubled in strength). Now, assume from here on that the optics are symmetric about the collision point, including the arrangement of skew quadrupoles as logically related pairs. If one such pair is denoted as $-i$ and i, then $\phi_{Hi} = -\phi_{H-i}$, and inspection of the right hand side of equation (6) shows that $Q_i - Q_{-i}$ is a diagonal 2 by 2 matrix. That is, if skew quadrupole pairs are antisymmetrically powered, $q_i = -q_{-i}$, then only two independent conditions remain to be met in order to satisfy equation (8). Only two skew quadrupole pairs are required to guarantee the transparency of the interaction region.

Most rigorous - four skew quad pairs. The downfall of the two pair scheme is that it allows the collision point optics - β_H^*, β_V^* et cetera - to be disturbed. One way to prevent this is to insist that motion through both the East and the West segments should be transparent. Mathematically,

$$P_E = I_4, \qquad P_W = I_4 \qquad (9)$$

If (9) is satisfied, then equation (8) is also trivially satisfied. As discussed above, four independent conditions must be satisfied (in general) in order to set either P_E or P_W to the identity matrix. However, with antisymmetrically powered skew quadrupole pairs placed in otherwise symmetric optics, it easy to show that the the same solution satisfies both equations (9). Four skew quadrupole pairs are required to guarantee the transparency of both sides of the interaction region.

Minimum acceptable - three skew quad pairs. Only three skew quad pairs are strictly necessary in an electron collider, where the vertical beam size is much less than the horizontal size[1]. This configuration guarantees that the vertical beam size will not increase (lowering the luminosity), and also avoids destructive perturbation of the beam-beam dynamics. TRISTAN currently operates in this configuration, for example, and CESR ran with three skew quad pairs for some years after its original mini-beta optics were installed.

Table 1 shows the various schemes that are, or were, in use at various electron colliders around the world. The parameters of solenoids in proton collider experiments are very similar to those in electron colliders, at energies are typically two orders of magnitude larger, so that proton solenoid angles θ are negligibly small.

Name	Number of skew quad pairs	Comment
BEPC	1	1/4 way round ring, with magic phases
CESR	4	2 partially rotated quad pairs, 2 purely skew pairs
DORIS	0	antisolenoids inside the experiment
LEP	4	complete correction
PETRA	0	approximate global compensation between 3 experimental solenoids
TRISTAN	3	"some difficulties at injection"

Table 1 Comparison of solenoid compensation schemes in electron colliders

The present remote correction scheme in BEPC

The BEPC lattice currently consists of four identical arc quadrants A, combined with the symmetry

$$\mathrm{BEPC} \;=\; A, -A, A, -A \tag{10}$$

Three pairs of skew quadrupoles are powered during injection, including the QR3 pair that is the only pair powered in collision optics[3]. The east QR3 skew quad is located about 4 meters beyond the end of the first quadrant. Betatron phases at this location are constrained to be close to the "magic" values

$$\phi_H = 3\,\pi, \qquad \phi_V = 3.5\,\pi \tag{11}$$

In the mathematical language developed above, this translates into

$$c_H \approx -1, \quad s_H \approx 0, \quad c_V \approx 0, \quad s_V \approx -1 \tag{12}$$

Inserting this relationship into equation (6), it is seen that only the bottom right term in the Q matrix is significantly strong - though it is suppressed relative to the top left term by the large factor

$$\beta_H^* / \beta_V^* \approx 1.30 / 0.085 = 15.3 \tag{13}$$

Note in passing that equations (3) and (6) are readily combined to reproduce the same four conditions that were independently derived as equation (15) in reference [3].

As noted in reference [3], the present BEPC solution with one skew pair cannot solve the four conditions exactly. However, an evaluation of the effects of the residual coupling, using the code PETROS[4], shows that the vertical emittance incurred is not very significant. Collider runs with the solenoid and the QR3 skew sextupoles OFF, and ON, show little reduction in attainable luminosity. Although the decoupling scheme works well with the present optics, it may well be a major constraint for future luminosity upgrades. The installation of new mini-beta optics, and/or the conversion to a lattice with a single low beta section, may make the constraints of equation (11) hard to achieve. Even today, these constraints limit operations to a region of the tune plane just below the tune values $(Q_H, Q_V) = (6.0, 7.0)$. Further, it is probably essential for any future multi-bunch scheme that the outermost skew quadrupoles are inside the first electrostatic separators launching the closed orbit "pretzels". Fortunately, a preliminary investigation suggests that a local compensation scheme that would overcome all these objections is possible. The scheme uses three pairs of skew quadrupoles.

A possible local correction scheme in BEPC

Suppose that 6 bunches are to be stored in each beam of a multi-bunch scheme. All skew quads in a local correction scheme would then have to be located within about ±20 meters of the collision point, corresponding to 1/12 of the BEPC circumference of 240.4 meters. A slightly more restrictive constraint on a generalized local scheme is that all skew quadrupoles should be inside the first horizontal bend in order that no vertical dispersion (and no vertical emittance) should be generated outside the correction region. The first bend begins about 14 meters from the collision point. Note that requiring all skew quads to be in a straight line geometry is sufficient to avoid spurious vertical dispersion, even if the horizontal dispersion η^* at the collision point is non-zero[1].

Quantity	units	value
B(solenoid)	Tesla	0.8
L(solenoid)	meters	3.6 (total)
Energy	GeV	1.55
β_H^*	meters	1.30
β_V^*	meters	0.085
Q_H		5.83
Q_V		6.69

Table 2 Summary of BEPC optics used in the evaluation of a local correction scheme.

Figure 2 illustrates the performance of a local three skew pair scheme as calculated by the code SS[5], in the present BEPC collision optics summarized by Table 2. The first two thick quadrupoles are rotated by θ_1 and θ_2 radians, while a third thin skew quadrupole of integrated field strength $(B'L)_3$ Tesla is placed s_3 meters from the collision point. This is essentially the geometry that was originally used in CESR immediately after the mini-beta upgrade of 1982. What is the optimum location of the third pair? Figure 2 answers this question graphically, by plotting the three skew quadrupole strengths as a function of s_3, and by plotting the amplitude of the vertical dispersion wave generated outside the interaction region straight. The vertical dispersion amplitude is scaled to a vertical beta function of $\beta_V = 25$ meters.

The optimum location for the third skew quad pair appears to be at $s_3 \approx 14.7$ meters, between the first and second horizontal bends. This location minimizes the strength requirements, with modest rotation angles of about 0.1 radians and an integrated strength $(B'L)_3 \approx 0.42$ Tesla, but still leaves the vertical dispersion negligibly small. Note that, since

$$(B'L)_3 \quad = \quad \frac{B_{pole}}{R_{pole}} L_3 \tag{14}$$

and if the field at the pole of the third skew quad is conservatively rated at $B_{pole} = 0.5$ Tesla, then its length L_3 may still be less than the pole radius, R_{pole}. The third skew quad is indeed thin.

This is far from being a complete analysis of a workable scheme. The performance of the scheme in different configurations, such as in present injection optics, in the proposed mini-beta optics, and in multi-bunch optics, also need to be considered. It is crucial to evaluate whether or not it is possible to avoid mechanically rotating the quadrupoles during the energy ramp between injection and collision conditions. Nonetheless, the scheme appears to be promising.

Acknowledgements

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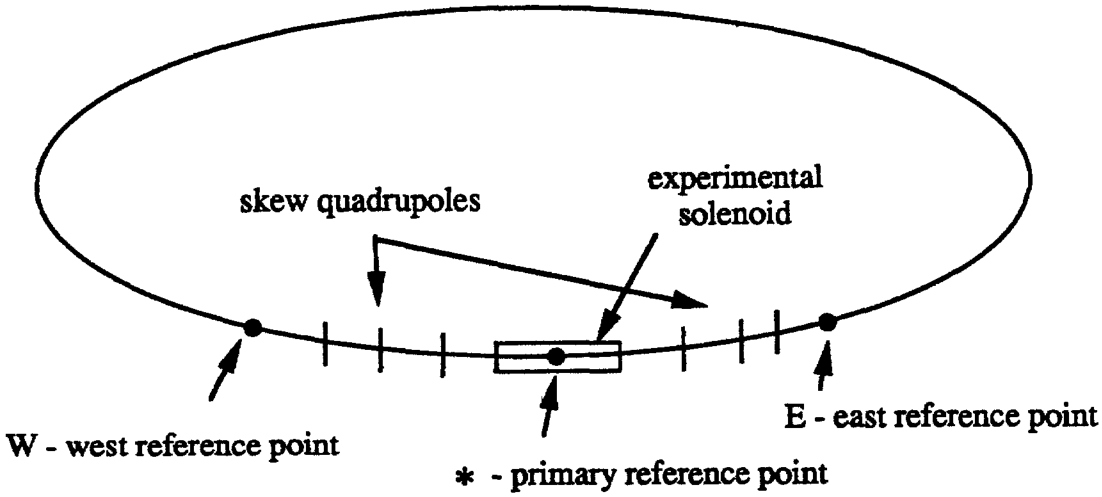

Figure 1a Generalized layout of an interaction region in an electron collider, including an experimental solenoid and several skew quadrupole correctors.

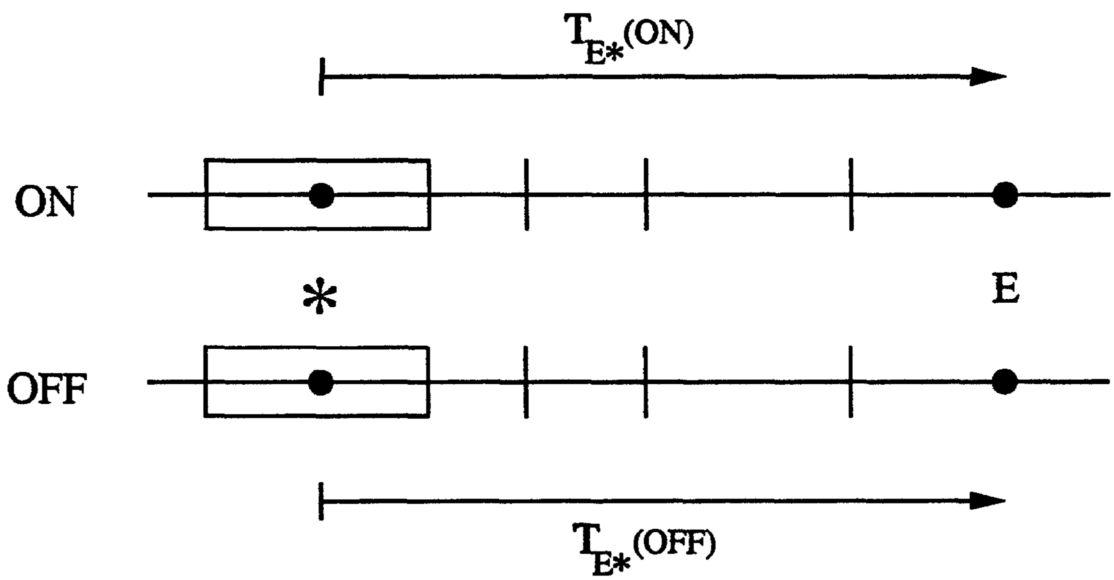

Figure 1b Transfer matrices used in the evaluation of the P_E projection matrix.

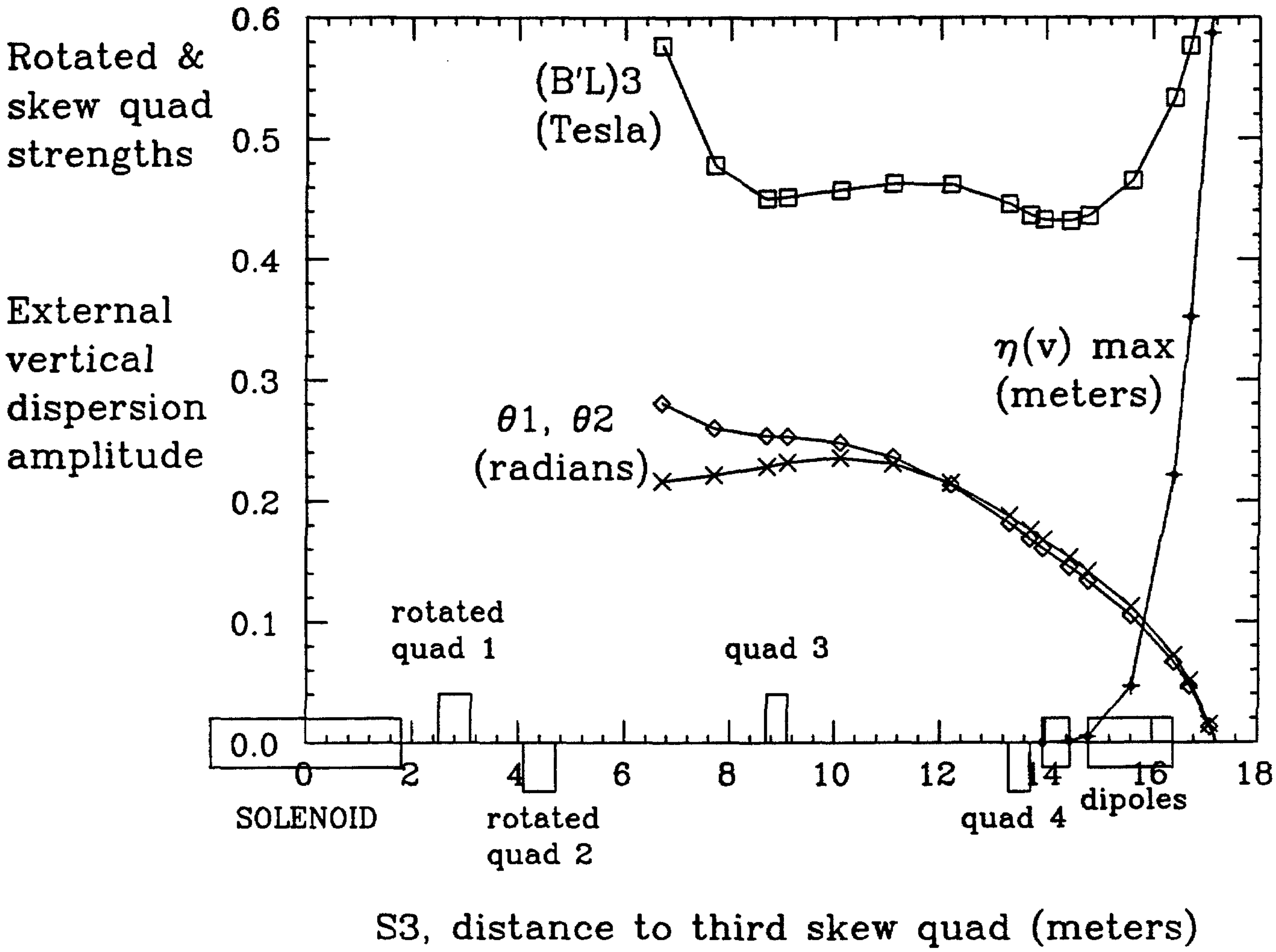

Figure 2 The dependence of rotated quad angles, third skew quadrupole pair strength, and external vertical dispersion amplitude, as a function of the location of the third skew quadrupole pair, in the present BEPC luminosity optics. The optimum location appears to be around 14.7 meters from the center of the solenoid.