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TITLE: Proton Mining - Dual Frequency Amplitude Modulation

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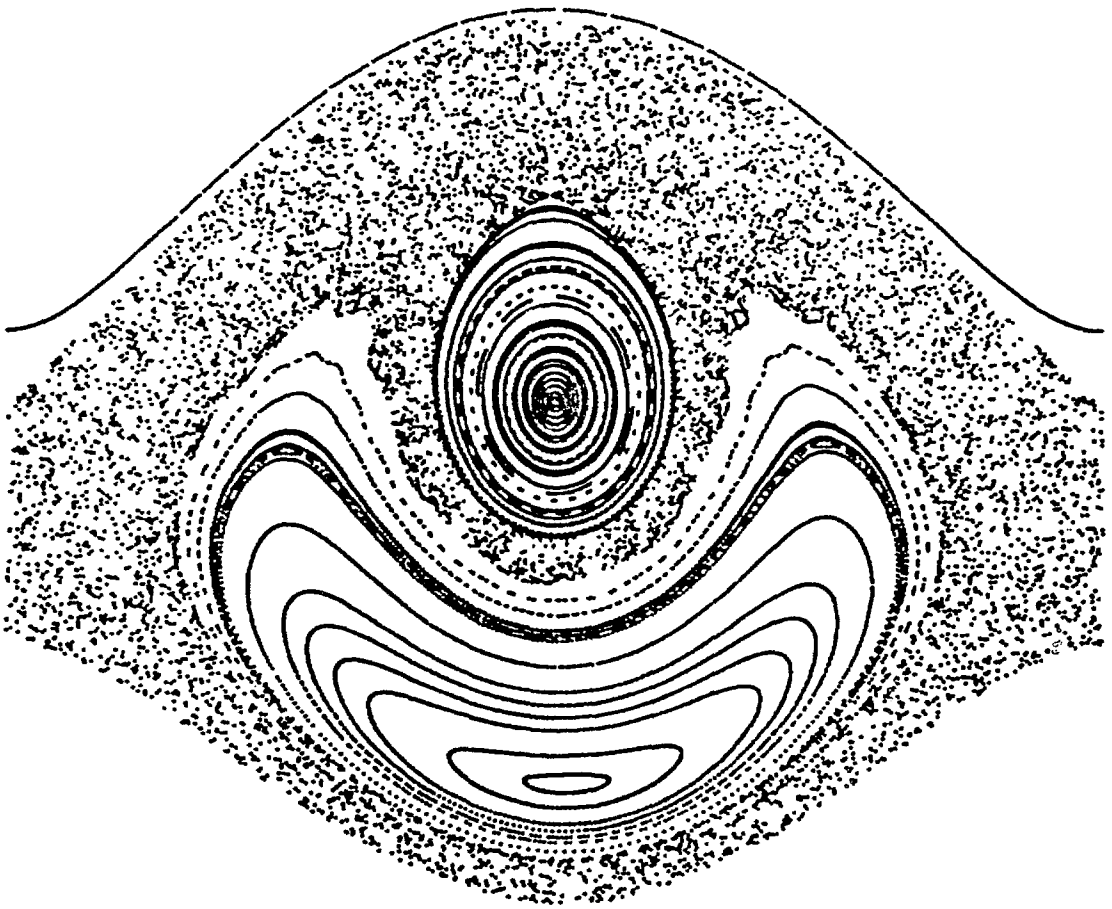

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Proton mining - dual frequency amplitude modulation

S. Peggs

Introduction

The Supercollider Fixed Target experiment (SFT) proposes to extract beam parasitically, while colliding beam experiments are also in operation. Setting the betatron tune close to an advantageous resonance is not allowed, so most attention has turned to Radio Frequency (RF) manipulations. Various types of manipulation have been proposed, such as amplitude or phase modulation, excitation at a single frequency, at dual frequencies, with wide band noise, or with narrow band noise. All of these schemes have the same general arrangement, as follows.

There is a crystal close to the beam at a place where the horizontal dispersion is large. (Insofar as the horizontal beta function is small, we are justified in temporarily ignoring betatron motion.) Protons in the RF bucket may strike the crystal, if the amplitude of synchrotron oscillations is large enough. Suppose that a particular proton just misses the crystal at the extreme displacement of one oscillation, but one synchrotron period later, at its next extremum, has a large enough amplitude to enter the crystal deeply. This proton is successfully extracted if its relative motion from one turn to the next carries it more than about 1 micron - the surface dead zone - into the crystal, and if it is then successfully channeled.

This note sketches the performance of a dual frequency amplitude modulation scheme. One of the modulation frequencies is fixed, and has a relatively deep modulation depth. In the nominal example typical of the Tevatron that is depicted below, the frequency of this "drive" modulation is about 1.5 times the small amplitude 900 GeV synchrotron tune of $Q_{s0} \approx 0.00089$, and its depth is 2% of the nominal RF voltage of 1.4 Megavolts per turn. The drive period is conveniently set, here, to an exact integer $T_{drive} = 750$ accelerator turns. A second, much weaker, "feed" modulation is used to capture protons in the longitudinal tails of the bunch (but closer to the core than the crystal). By adiabatically lowering the feed frequency, the associated resonance islands move outwards, eventually feeding the protons to the strong influence of the drive resonance. A nominal feed modulation depth of 0.2% is used below, one tenth of the drive depth, with a period T_{feed} in the range from 600 turns to 725 turns. $T_{s0} \approx 1127$ turns.

Moving selected protons up the walls of the RF bucket onto the crystal is vaguely analogous to a using a dragline bucket in an open cast mine. Hence, the process *might* be thought of as proton mining.

Estimate of response to the drive modulation

If $\delta = \Delta p/p$ and ϕ is the RF phase, then the linearized solution to the first order equations of motion is

$$\delta = \delta_{max} \cos(2\pi Q_s t) \tag{1}$$

$$\phi_n = \delta_{max} \sin(2\pi Q_s t) \tag{2}$$

where the subscript n means "normalized", and Q_s is the synchrotron tune. The normalized phase is given by

$$\phi_n = (\delta_{max}/\phi_{max})\, \phi \tag{3}$$

Differentiation of (1) with respect to time, and the substitution of (2), gives

$$\frac{d\delta}{dt} = -2\pi Q_s \,.\, \phi_n \tag{4}$$

This representation of a difference system by differential equations is reasonable, since $Q_s << 1$. Recognizing that the right hand side of (4) results from the RF voltage term $-V \sin(\phi)$, after linearization and normalization, then it is seen that a voltage modulation of

$$\frac{\Delta V}{V} = \alpha \cos(2\pi Q_d t + \beta) \tag{5}$$

is introduced to the model by rewriting (4) as

$$\frac{d\delta}{dt} = -2\pi Q_s \left(1 + \frac{\Delta V}{V}\right) . \phi_n \tag{6}$$

or

$$\frac{d\delta}{dt} = -2\pi Q_s \left[1 + \alpha \cos(2\pi Q_d t + \beta)\right] \phi_n \tag{7}$$

where $Q_d = 1/T_{drive}$ is the tune of the modulation, and β is its initial phase.

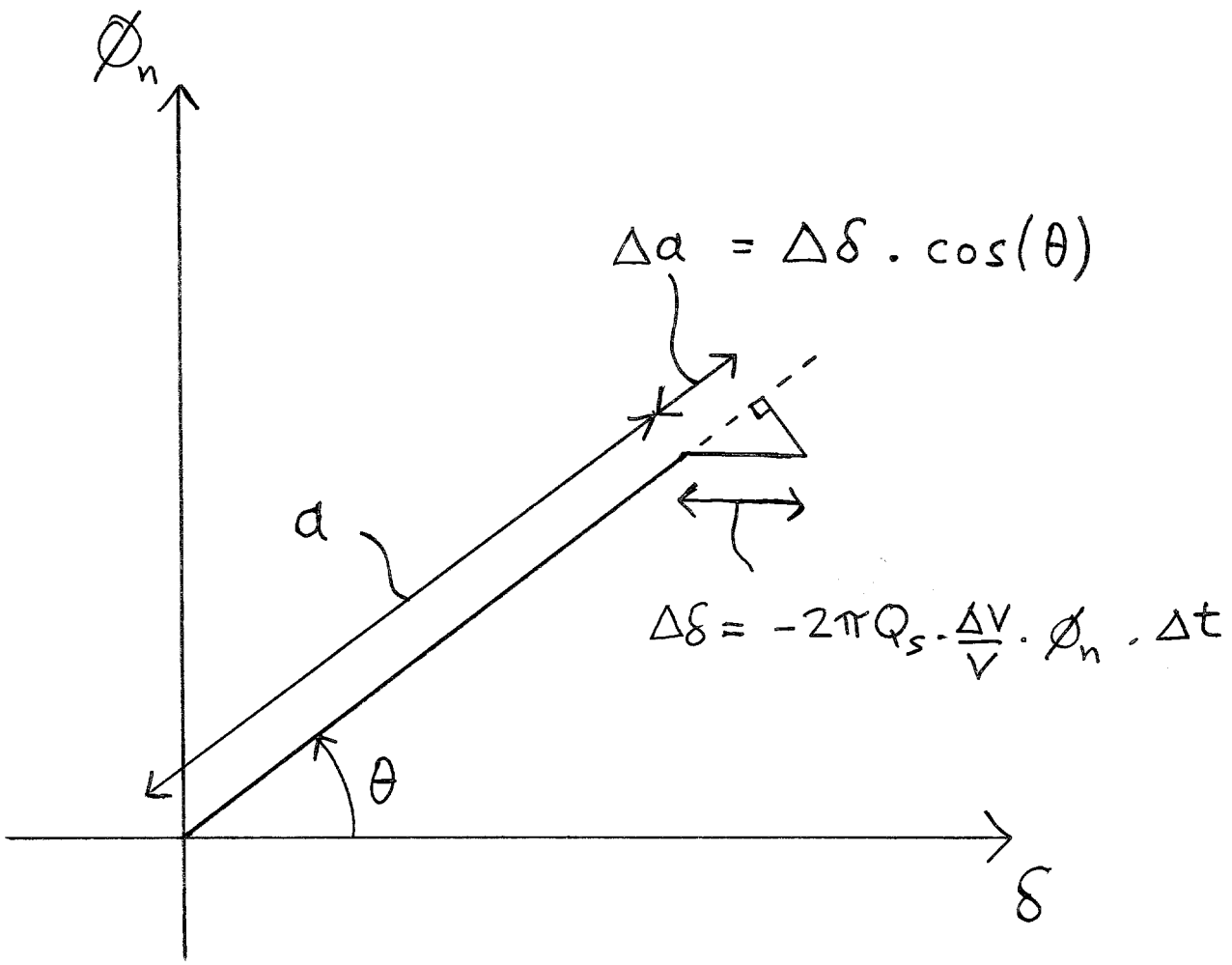

Figure 1 Conversion from δ and ϕ_n to polar coordinates a and θ, and calculation of the increase in a due to a voltage error, ΔV.

The introduction of nonlinearities is easier in polar coordinates a and θ, given by

$$\delta = a \cos(\theta) \tag{8}$$

$$\phi_n = a \sin(\theta) \tag{9}$$

and illustrated crudely in Figure 1. Except for the perturbation term in (7), the amplitude a is constant. When the modulation is included, the rate of change of a is given by

$$\frac{da}{dt} = -2\pi Q_s \; a \; \sin(\theta)\cos(\theta) \frac{\Delta V}{V} \tag{10}$$

Substituting (5) and approximating $\theta = 2\pi Q_s t$, this becomes

$$\frac{da}{dt} = -\frac{\pi Q_s}{2} \; a \; \alpha \left[\sin(2\pi\{2Q_s + Q_d\}t + \beta) + \sin(2\pi\{2Q_s - Q_d\}t - \beta)\right] \tag{11}$$

If the drive tune and the synchrotron tune are approximately equal, $Q_s \approx Q_d$, for example by thinking of $Q_s < Q_{s0}$ as the detuned value of a proton with a moderate synchrotron amplitude, then only the second term in (11) survives integration over a synchrotron period. The relative increase in amplitude over one period is

$$\frac{\Delta a}{a} = \alpha \frac{\pi}{2} \sin(\beta) \tag{12}$$

For example, the nominal drive depth $\alpha = 0.02$ results in a maximum relative growth rate per period of about 3%, independent of synchrotron tune, et cetera. This assumes only that the proton remains locked on resonance - is on a resonance island.

Figure 2 shows two plots of the phase space response to the nominal drive. Figure 2A shows a turn-by-turn plot of two protons launched with different initial conditions. The inner one is hardly affected by the drive resonance at all. The outer particle is caught on resonance, and spirals out at a rate in rough agreement with equation (12). Figure 2B and all the rest of the Figures that follow, below, are Poincare plots. That is, they plot only one point per complete modulation period, 750 turns in this case. Strong resonance islands are seen at a relatively large amplitude in Figure 2B, with smooth stable motion inside at lower amplitudes, and with stable non-resonant motion outside at larger ones.

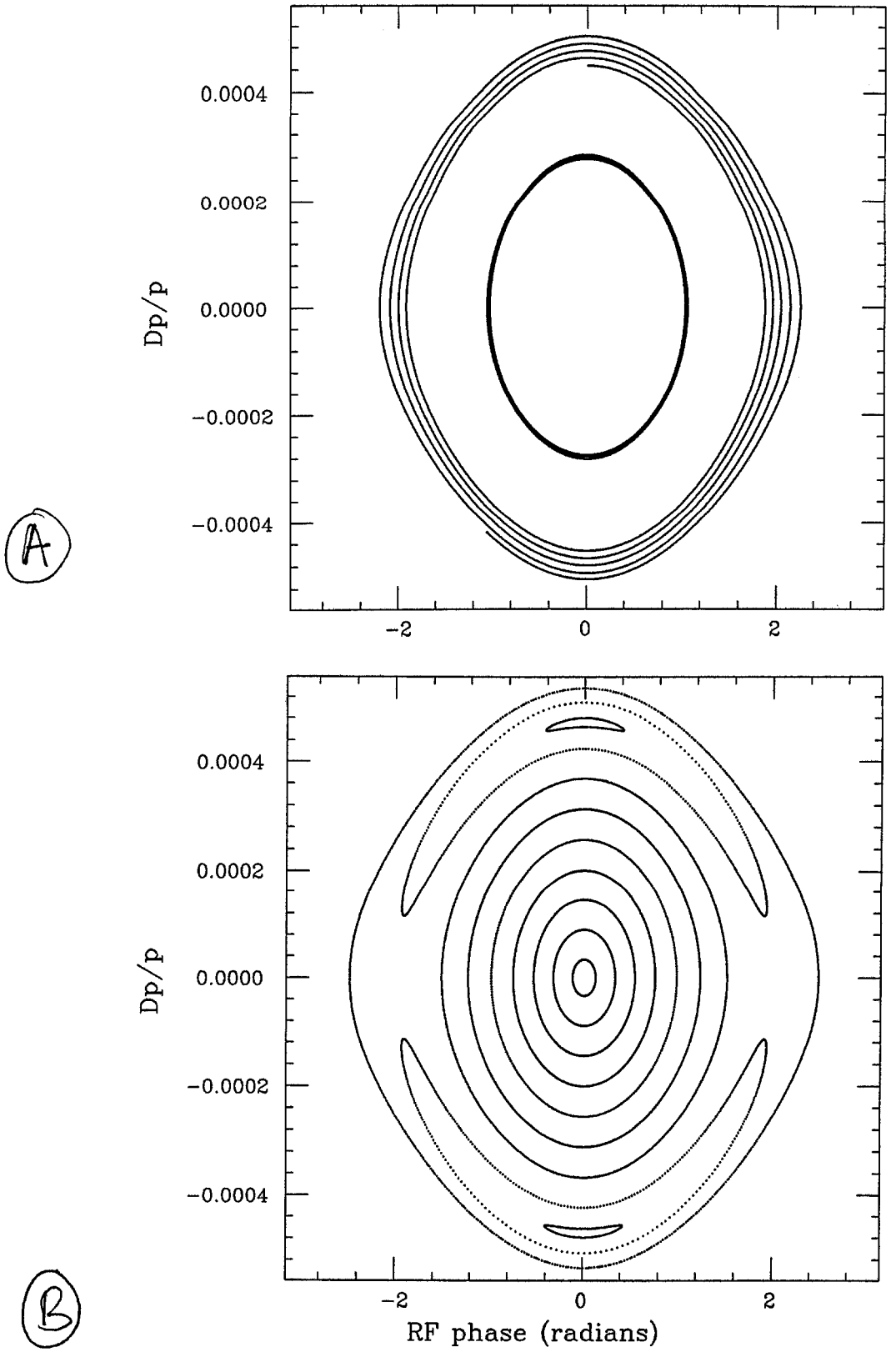

Figure 2 Effect of the drive modulation on phase space. Figure A shows the turn-by-turn motion of 2 particles for 7,000 turns. Figure B shows the Poincare motion of 10 particles for 1,000 modulation periods, each of 750 turns.

Response to both feed and drive modulations

Figure 3 shows what happens as the feed modulation frequency approaches the drive modulation frequency. This crudely simulates the process of first rapidly turning on a feed modulation to resonantly capture protons, and then adiabatically moving them inside the islands to larger amplitudes, before finally delivering them to the drive islands for extraction and channeling.

In the first figure, 3A, the feed period is $T_{feed} = 600$ turns, far from the drive period of $T_{drive} = 750$ turns. Two sets of independent resonances are seen. Since the small oscillation period is $T_{s0} \approx 1127$ turns, the ratio of tunes $Q_{feed}/Q_{s0} \approx 1.89$, and the feed islands are relatively close to the center of the RF bucket, where there is about 5.5% detuning. Only one of the two drive islands is visible, because the Poincare period is now 3,000 turns (the lowest common multiple of T_{feed} and T_{drive}), an even multiple of T_{drive}. Higher harmonic islands of second order in perturbation strength are also seen half way between the two principals. The area of the feed islands is much larger than desirable in practice, due to the relatively flat detuning slope, dQ_s/da, close to the center of the bucket.

The feed islands have moved toward the drive islands in Figure 3B, although regular stable motion still exists inside all the islands, outside them, and between them.

Chaos begins to appear in Figure 3C, due to the incipient overlap of the two resonances. However, motion is still regular and stable inside, outside, and between the resonances. The feed islands appear to be smaller than they really are, due to an unfortunate set of initial conditions. (The same initial conditions are used in all of Figure 3, except for their range limits.)

It appears that there no longer is a non-resonant regular trajectory separating feed and drive islands in Figure 3D . The chaotic region has grown noticeably, although the feed island has not yet been submerged.

The two sets of islands interact strongly in Figure 3E, not only in producing a broad band of chaos, but also in creating high order harmonics around the edges of the drive island. The drive islands still exists stably, although it is impossible to tell how big they are from this picture.

The drive islands appear to have stably absorbed the feed islands in Figure 3F. The presence of large scale chaos clearly signifies that more than one modulation frequency is present, as can be seen by comparing Figures 3F and 2B.

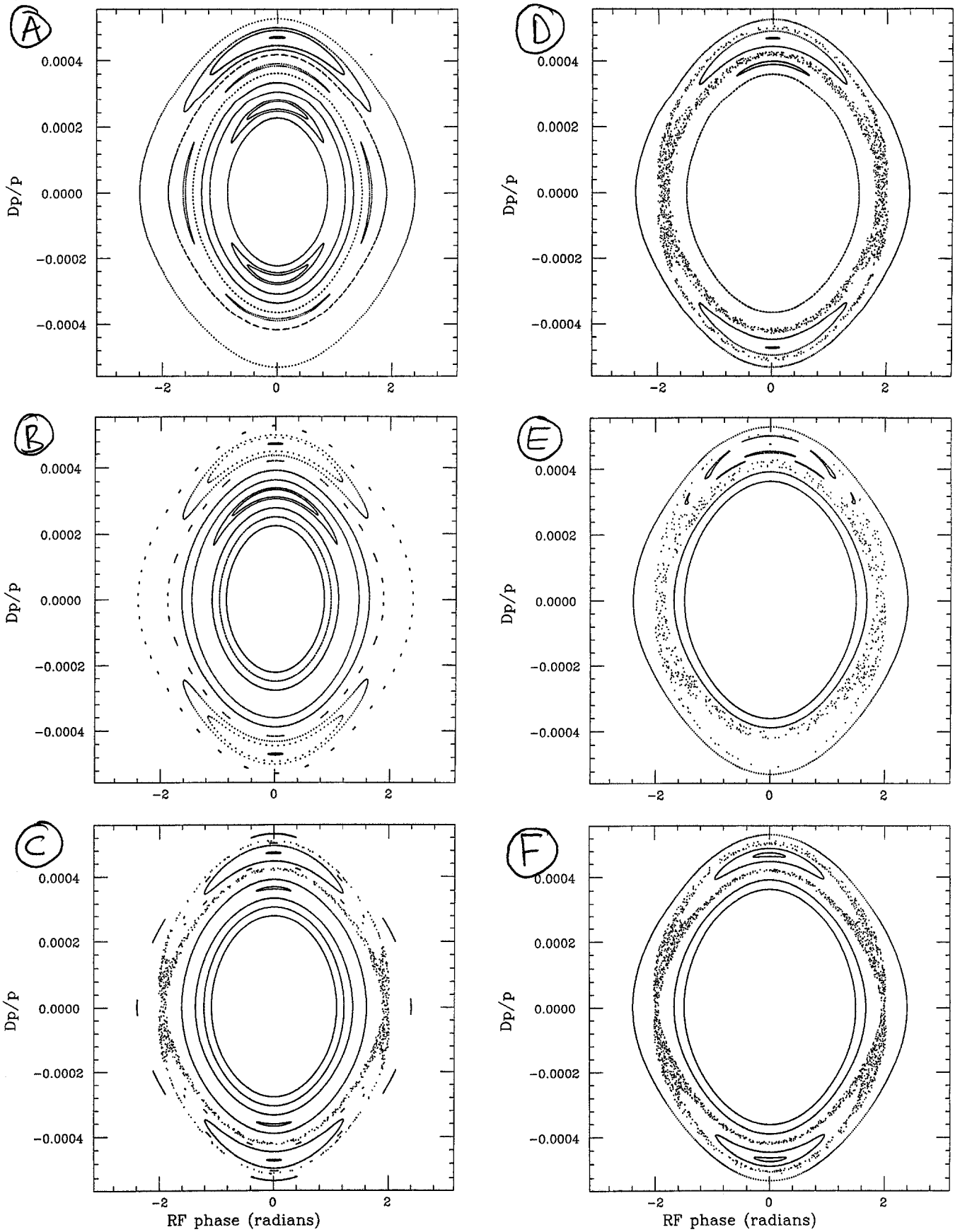

Figure 3 Interaction between drive and feed modulation resonances as the feed period T_d is increased through A) 600, B) 625, C) 650, D) 675, E) 700, and F) 725 turns.

Conclusions

This very preliminary work suggests that amplitude modulation with two or more frequencies might be a powerful technique in the parasitic extraction of protons onto a crystal. Its advantages include

1) direct and strong control of the extraction dynamics with the constant "drive" modulation.
2) sensitive and variable control of the extraction rate, and the location of the source of protons, through the use of the weaker, ramped frequency, "feed" modulation.
3) flexibility in performance from one fill to the next - between different beam distributions.
4) description in terms of a small number of parameters and discrete frequencies, appropriate for understanding the underlying physics, and for designing a realistic feedback system.
5) apparent absence of the center of charge motion often found in phase modulation methods.
6) ease of application to a Tevatron experiment without a crystal inserted. The Tevatron permits voltage modulation.

Of course, many calculations and simulations remain to be done, even ignoring the possibility of early Tevatron experiments. Consider the following questions, for instance

1) how small can the drive amplitude α_{drive} be made?
2) how should $\alpha_{feed}(t)$ and $Q_{feed}(t)$ be programmed as a function of time, to ensure constant island size and efficient pseudo-adiabatic transfer?
3) is the feeding of protons into the drive resonance efficient?
4) what crystal deposition distributions and efficiencies result when betatron motion is included?
5) is there an advantage in including a localized oscillatory orbit distortion, analogous to a coherent betatron oscillation, at a convenient betatron tune of, say, 1/4 ?

The path to follow in performing calculations and simulations to answer these questions is reasonably well trodden, and appears to hide no alligators. Hopefully, it is just a matter of time before these questions are answered - and new ones are posed.

Acknowledgements

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