Fermilab

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ACCELERATOR PHYSICS DEPARTMENT

TITLE: Head-Tail Stability & Linear Coupling

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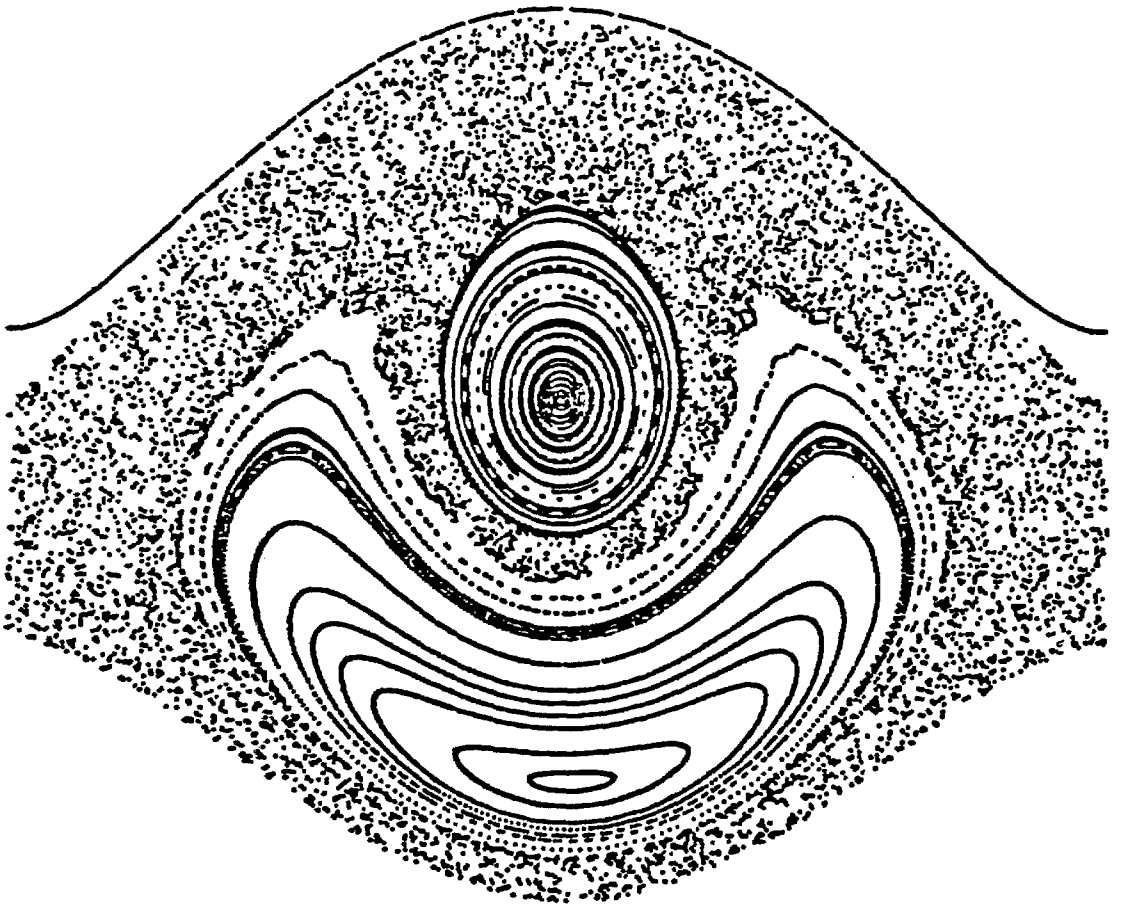

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April 10 '90

Head-Tail stability and linear coupling

The rule of thumb for head-tail stability is that the chromaticities, χ_x and χ_z, are positive. What happens when there is significant linear coupling? What experimental tests of the TeVatron can be made?

In the presence of coupling, the two eigentunes are

$$\boxed{Q_\pm = \frac{1}{2}\left[Q_{x0} + Q_{z0} \pm \left[(Q_{x0} - Q_{z0})^2 + \bar{q}^2 \right]^{\frac{1}{2}} \right]}$$

Q_{x0}, Q_{z0} – uncoupled horz, vert tunes

$\bar{q} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$ – linear coupling vector

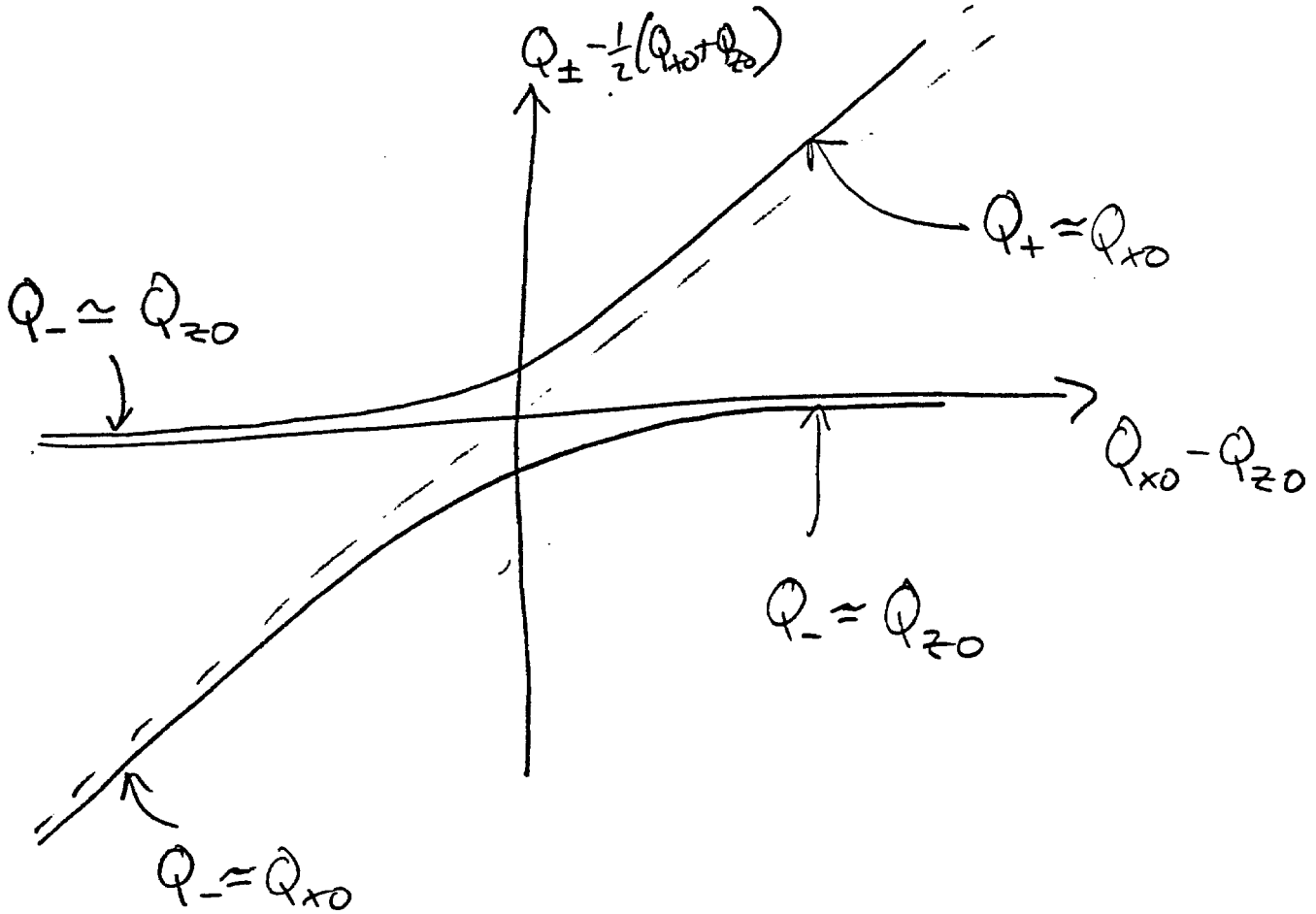
For example

$Q_{x0} > Q_{z0}$, $|Q_{x0} - Q_{z0}| \gg |\bar{q}|$

$Q_+ \simeq Q_{x0}$, $Q_- \simeq Q_{z0}$

or, $Q_{x0} < Q_{z0}$, $|Q_{x0} - Q_{z0}| \gg |\bar{q}|$

$Q_+ \simeq Q_{z0}$, $Q_- \simeq Q_{x0}$



$\boxed{\text{For example}}$

$$Q_{x0} = Q_{z0}, \qquad Q_\pm = \frac{1}{2}\left(Q_{x0}+Q_{z0} \pm |\bar{q}|\right)$$

so $|\bar{q}|$ is closest approach of tunes.

Now, add momentum offset, $\delta = \frac{\Delta p}{p}$

Assume that
$$Q_{x0} \to Q_{x0} + \chi_x \delta$$
$$Q_{z0} \to Q_{z0} + \chi_z \delta$$

where the net chromaticities χ_x and χ_z are of order 1.

But $\bar{q}$ is also a function of momentum

$$\bar{q} \to \bar{q} + \bar{k}\delta$$

For example, as δ changes, the vertical closed orbit changes, through vertical dispersion. This changing offset in chromatic sextupoles causes changes in skew quadrupole fields experienced by the beam.

Now,

$$\boxed{Q_\pm(\delta) = \frac{1}{2}\Big[(Q_{x0}+Q_{z0}) + (\chi_x+\chi_z)\delta \pm \Big[\big(\{Q_{x0}-Q_{z0}\} + \{\chi_x-\chi_z\}\delta\big)^2 + (\bar{q}+\bar{k}\delta)^2\Big]^{\frac{1}{2}}\Big]}$$

$\boxed{\text{For example}}$, $Q_{x0} = Q_{z0} = Q_0$
$\chi_x = \chi_z = \chi$

Then

$$Q_\pm(\delta) = Q_0 + \chi\delta \pm \tfrac{1}{2}|\bar{q} + \bar{k}\delta|$$

$\boxed{\text{Further}}$, suppose that $\bar{q} \to 0$, either through 'perfect' decoupling procedures, or $|\bar{q}| \ll |\bar{k}\,\delta_{typical}|$

Then

$$Q_\pm(\delta) = Q_0 + \chi\delta \pm \tfrac{1}{2}|\bar{K}\delta|$$

Important question: <u>How big is $|\bar{K}|$?</u>

It is <u>consistent</u> with the observation of coherent losses near the coupling resonance that $|\bar{K}| > 2\chi$ so that one effective chromaticity, χ_- goes negative

$$\chi_\pm \simeq \chi \pm \tfrac{1}{2}|\bar{K}|$$

It is also <u>consistent</u> with the reported, apparent, observation that "it is impossible to make $\bar{q}=0$" that $|\bar{K}|\,\delta_{TYP.} \approx 2\times10^{-3}$

An order of magnitude estimate for $|\bar{K}|$ gives

$$|\bar{K}| \approx \frac{\eta_{V\,TYPICAL}}{\eta_{H\,TYPICAL}} \cdot |\chi_{NATURAL}|$$

If vertical dispersion in chromaticity sextupoles causing varying skew quadrupole feedown is the problem.

For example, $\dfrac{\eta_{V\,TYP}}{\eta_{H\,TYP}} \simeq 5\%$ (assuming that coupling from horizontal dispersion into vertical is NOT resonant - this can be tested!)

and $|\chi_{NAT}| \simeq .20$

then $|\bar{k}| \approx 1$

Perhaps $|\overline{\chi_{NAT}}|$ should be much larger, considering persistent current b_2's

Perhaps $|\bar{k}|$ should be much smaller, due to $\sqrt{N}$ effects.

Perhaps the vertical dispersion due to the helix will be a major player

<u>WE CAN MEASURE</u>

1) η_v – vertical dispersion near $Q_{x0} = Q_{z0}$

2) $\bar{q} + \bar{k}\,\delta$, and $|\bar{q} + \bar{k}\,\delta|$

3) Dependence of the coherent losses on χ – should be stable if $\chi \gg |\bar{k}|$

4) $Q_{\pm}(\delta)$