Fermilab

AP-Note-90-005

ACCELERATOR PHYSICS DEPARTMENT

TITLE: MI - Transition Crossing ESME Simulation

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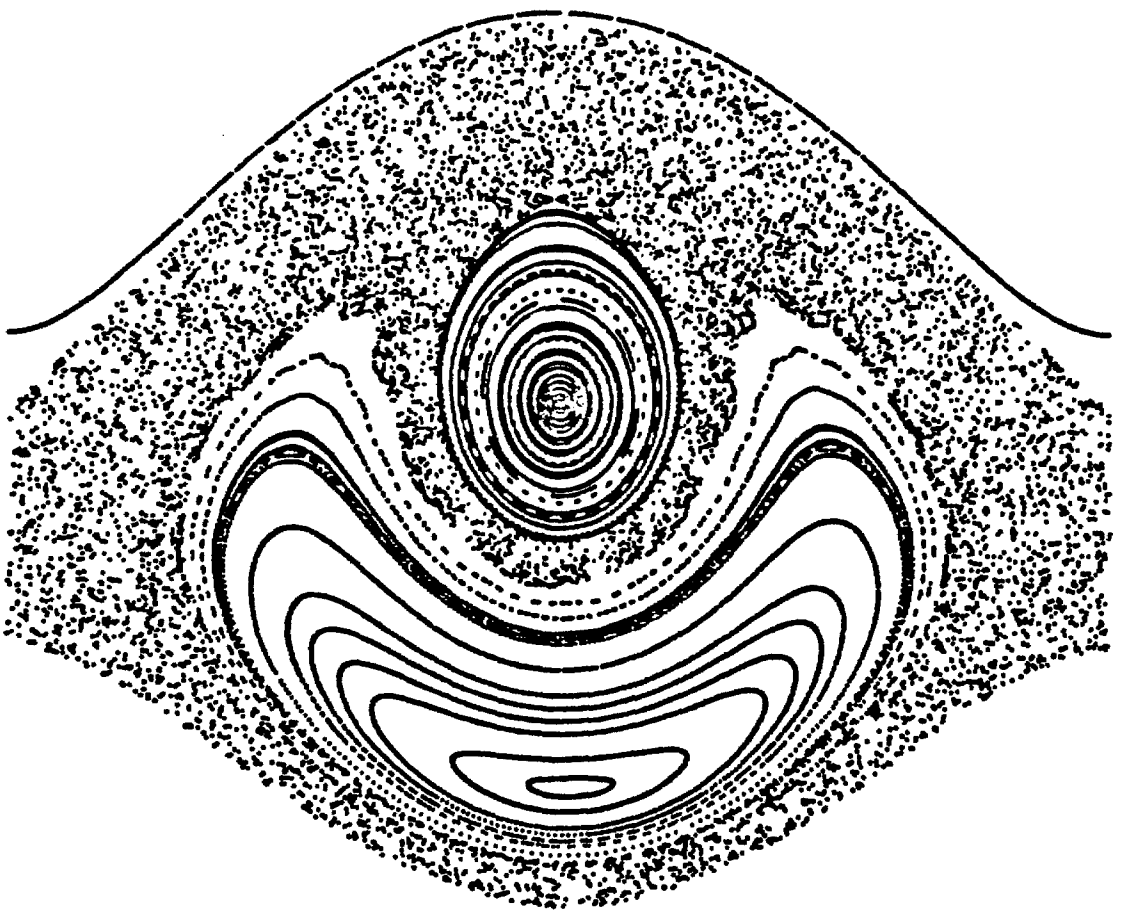
March 16, 1990



3/16/90

MI - Transition Crossing

ESME Simulation

Alex BOGACZ

- Longitudinal phase space eqs. of motion (around transition)

$$\rightarrow \quad \eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$$

$$\frac{1}{\gamma_t^2} = \left(\frac{1}{\gamma_t^0}\right)^2 + \overbrace{4 h r_p R \frac{1}{\beta^2 \gamma^7} \frac{1}{2a^2} \lambda(\theta)}^{\text{Umstätter effect}} - \underbrace{2 j(t) \frac{1}{\gamma^3}}_{\gamma_t \text{ jump}}$$

$$+ \underbrace{\alpha_1 \frac{\Delta p}{p}}_{\text{Johnson effect}}$$

$$\gamma = \gamma_t^0 + \frac{eV}{mc^2} \sin\phi_s \frac{\beta c}{2\pi R} t$$

$$\rightarrow \quad \frac{d}{dt}\left(\frac{\Delta E}{\omega_s}\right) = \frac{eV}{2\pi}\left(\sin\phi - \sin\phi_s\right) + \overbrace{e E_\theta(\theta) R}^{\text{longitudinal space charge}}$$

$$E_\theta(\theta) = \frac{g_0 e h^2}{4\pi\epsilon_0 R^2} \frac{d}{d\theta} \lambda(\theta)$$

- Introducing nonadiabatic time scale T

$$\frac{d}{d\tau}\Theta = \left[\tau - \mu\, j(\tau) + \epsilon\, \lambda(\theta) + \frac{1}{2}\alpha_1 \mu \gamma_t^3 \frac{\Delta\rho}{\rho}\right] P$$

$$\frac{d}{d\tau} P = \Theta + \xi \frac{\partial}{\partial\theta} \lambda(\tau, \theta)$$

$$t = T\tau \qquad T = \frac{R}{c}\left\{\frac{2\pi^2}{h}\left(\frac{mc^2}{eV}\right)^2 \frac{\gamma^4}{\beta} \frac{1}{\sin\phi_s |\cos\phi_s|}\right\}^{1/3}$$

T	μ	$\varepsilon\,\lambda(\theta)$	$\mu\gamma^3\alpha_1\delta$	j	$\frac{d\lambda}{d\theta}\frac{\xi}{\theta}$	$\frac{d\gamma}{dt}$
			MI			
2.3×10^{-3}	5.8	5.6×10^{-1}	4.9	0.94	2.1×10^{-2}	125.
			MR			
3.7×10^{-3}	3.7	4.3×10^{-1}	4.2	1.24	4.2×10^{-2}	68.
			BOOSTER			
2.3×10^{-4}	13.	7.6×10^{-1}	1.6	0.17	2.6×10^{-2}	250.

$$\mu = \left\{\frac{4\pi h}{\beta^2\gamma^4} \frac{mc^2}{eV} \frac{|\cos\phi_s|}{\sin^2\phi_s}\right\}^{1/3}$$

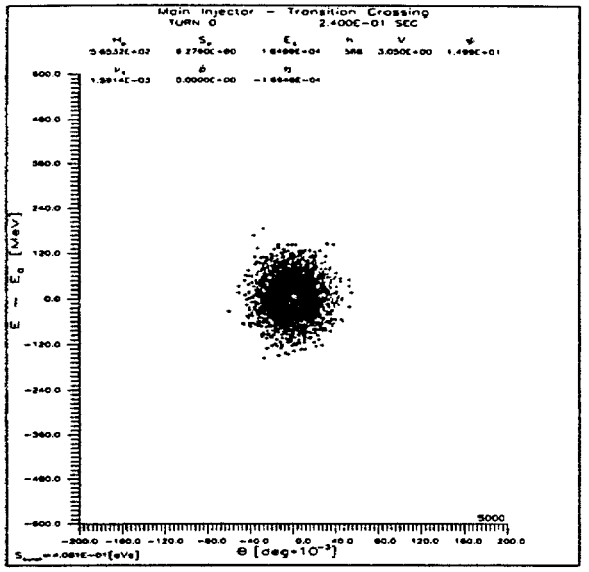
$$\epsilon = 4 h r_p R \frac{1}{2a^2}\left\{\frac{h\pi}{2\beta^8\gamma^{16}} \frac{mc^2}{eV} \frac{|\cos\phi_s|}{\sin^2\phi_s}\right\}^{1/3}$$

$$\xi = \frac{2\pi h^2 g_0}{\gamma^2 |\cos\phi_s|} \frac{r_p}{R} \frac{mc^2}{eV}$$

Relative sensitivities to: γ_t jump (μ)
Umstätter effect (ϵ), Johnson effect ($\mu\alpha_1$), space charge (ξ)

$\alpha_1 = 0$

1



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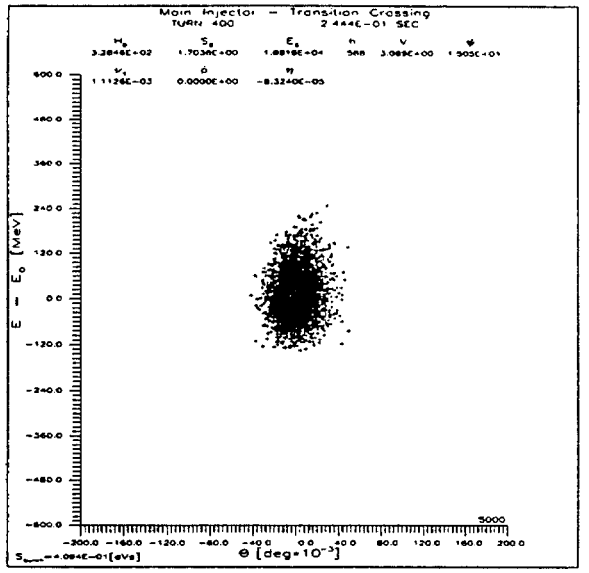

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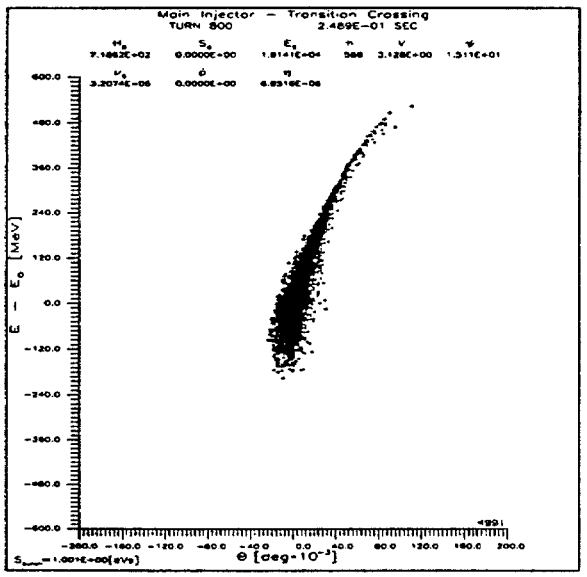

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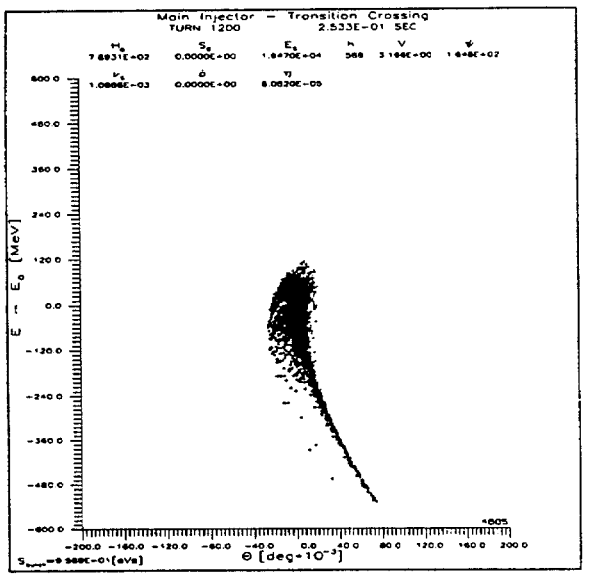

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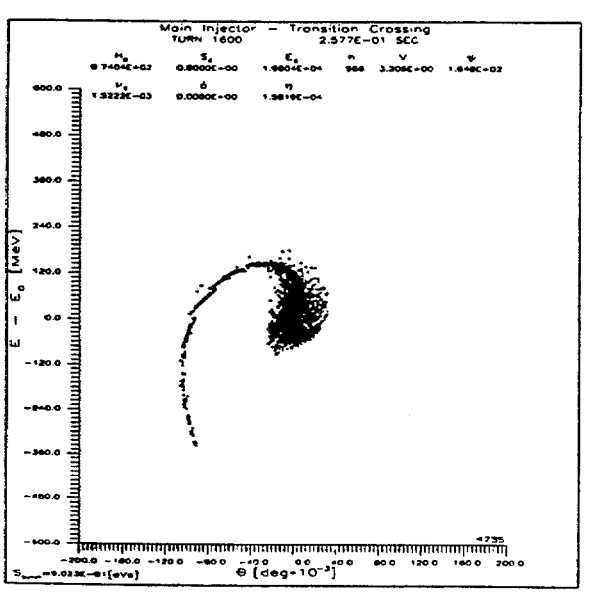

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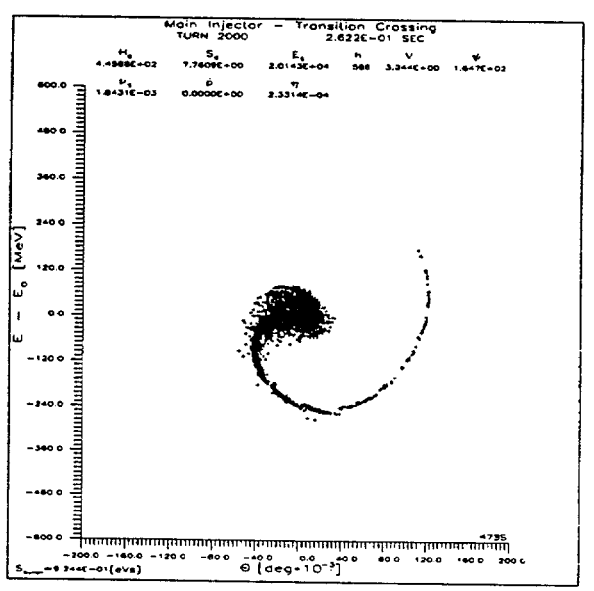
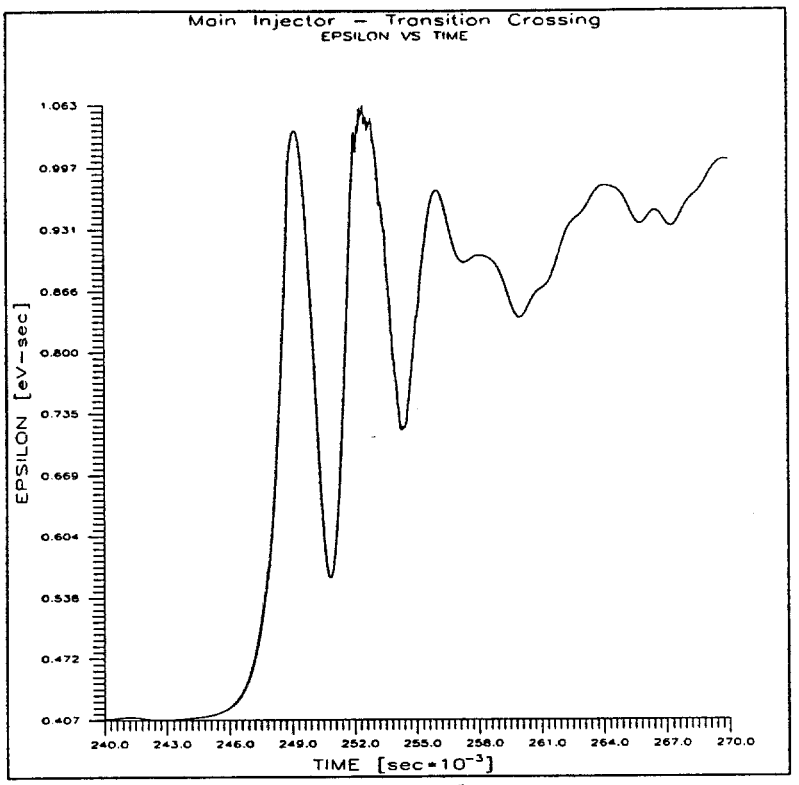
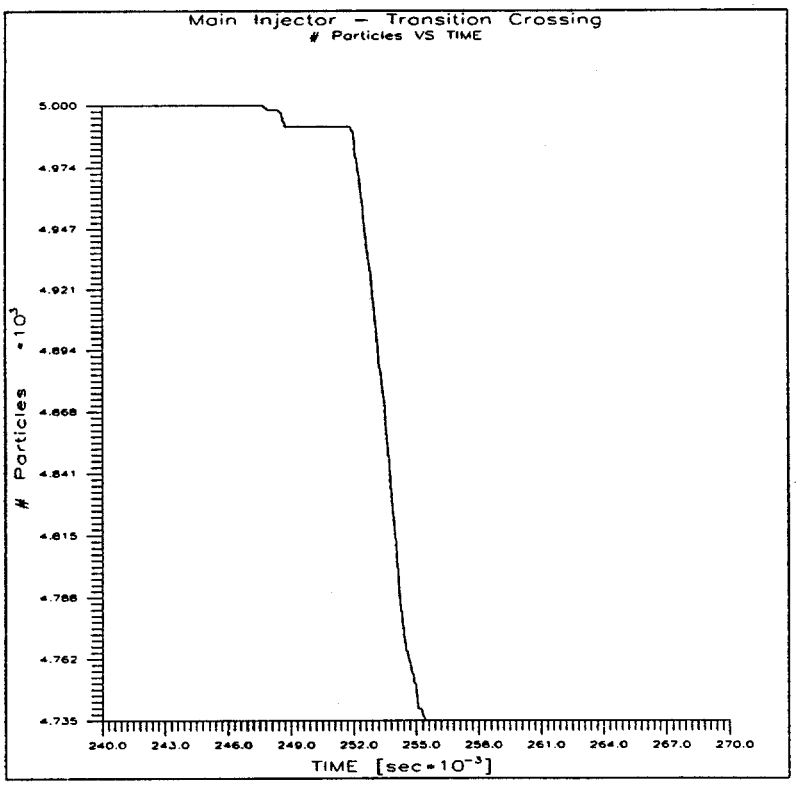


transition @ turn 790



2.5

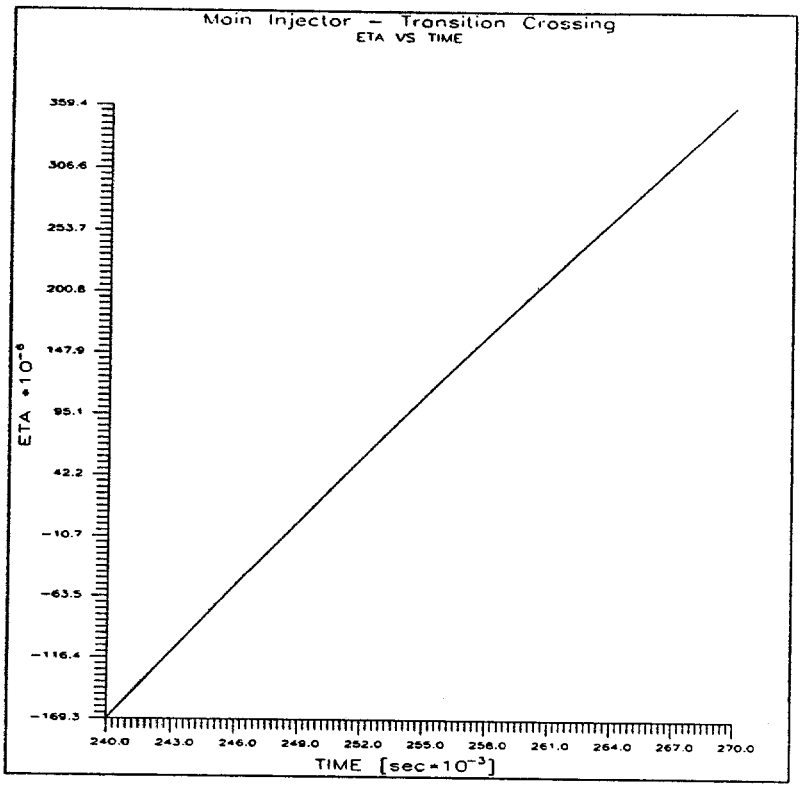
blow - up



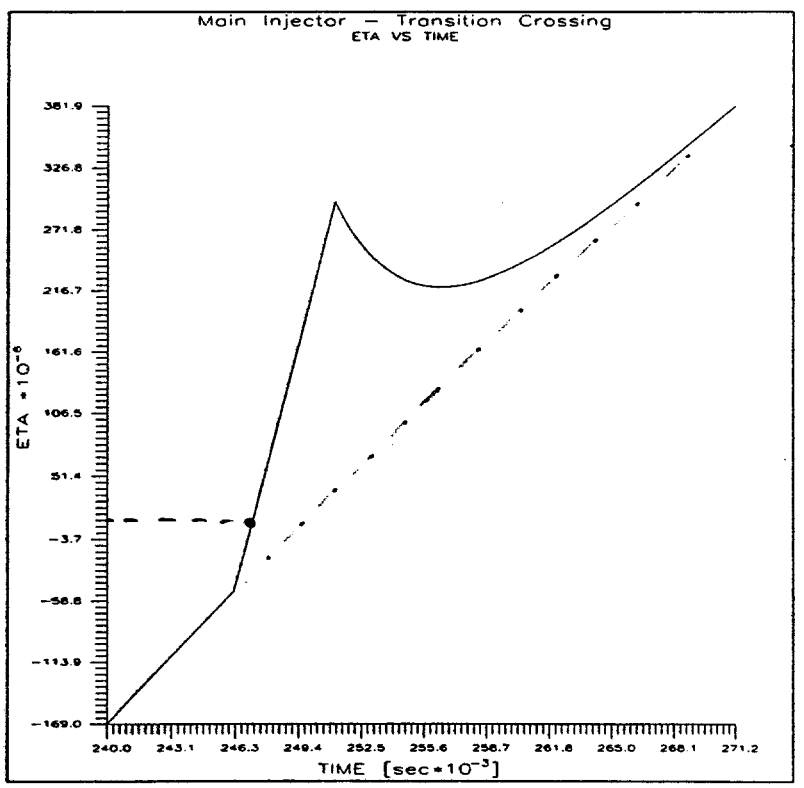
5.5 %

lost

no jump

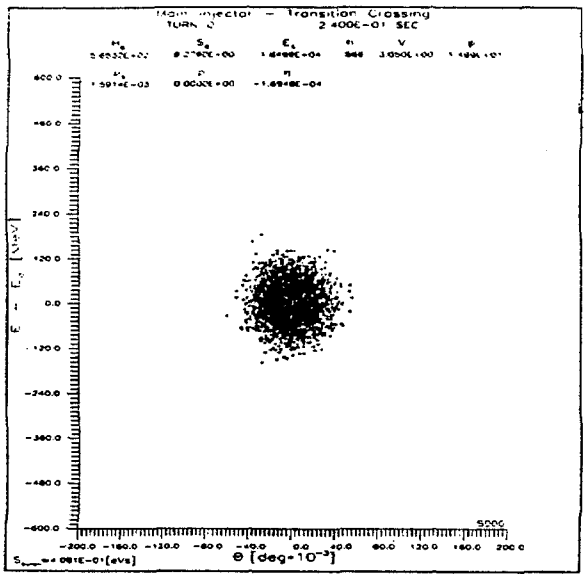


γ_t - jump



$\alpha_1 = 0$ γ_t jump

1



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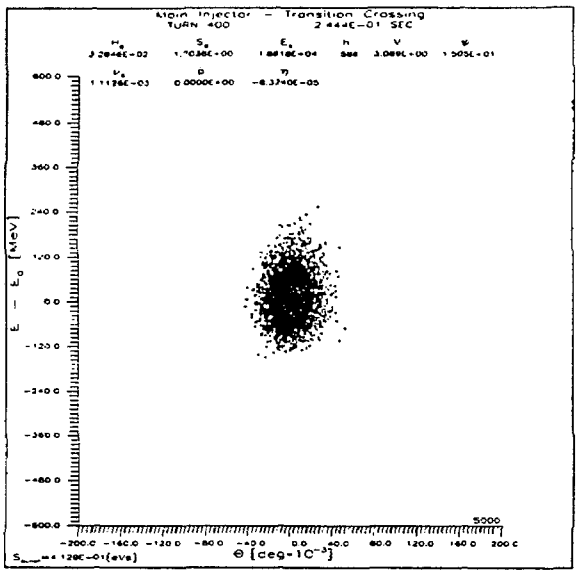

transition @ turn 790

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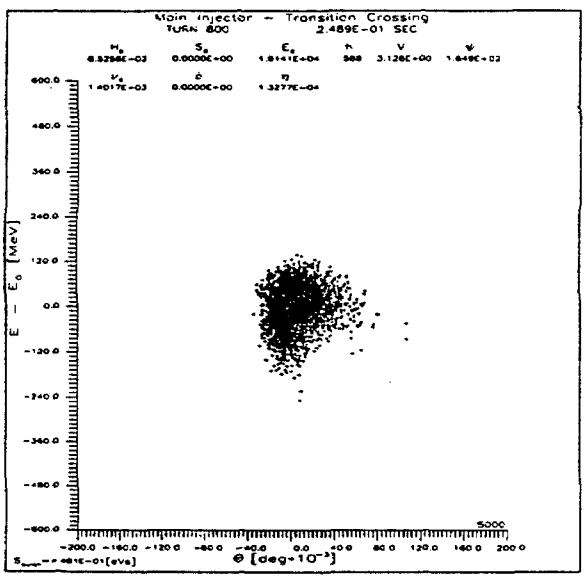

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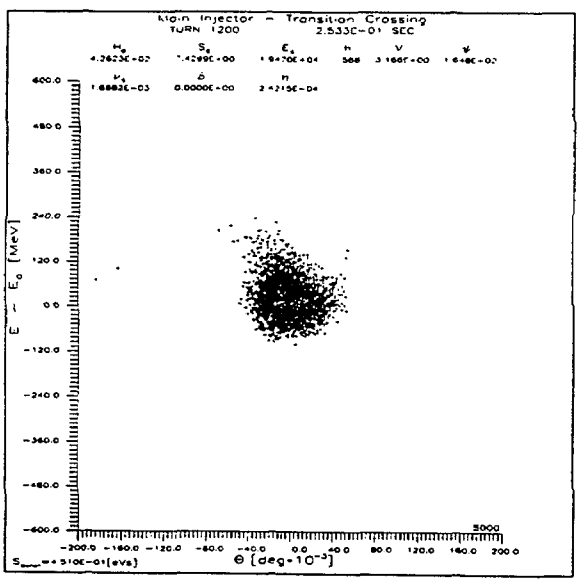

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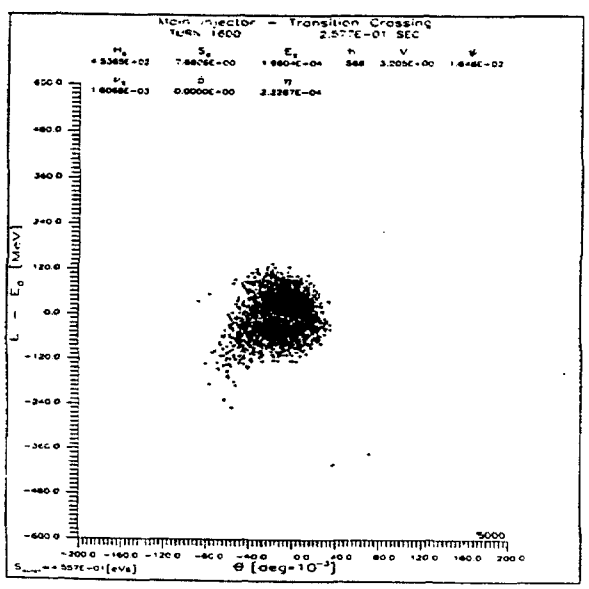

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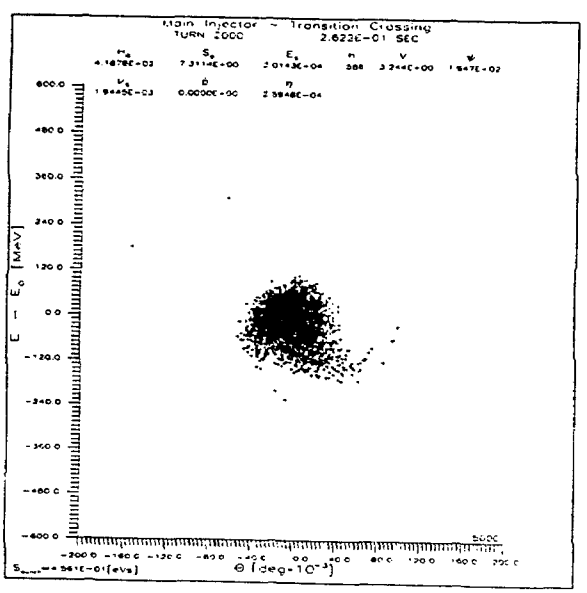
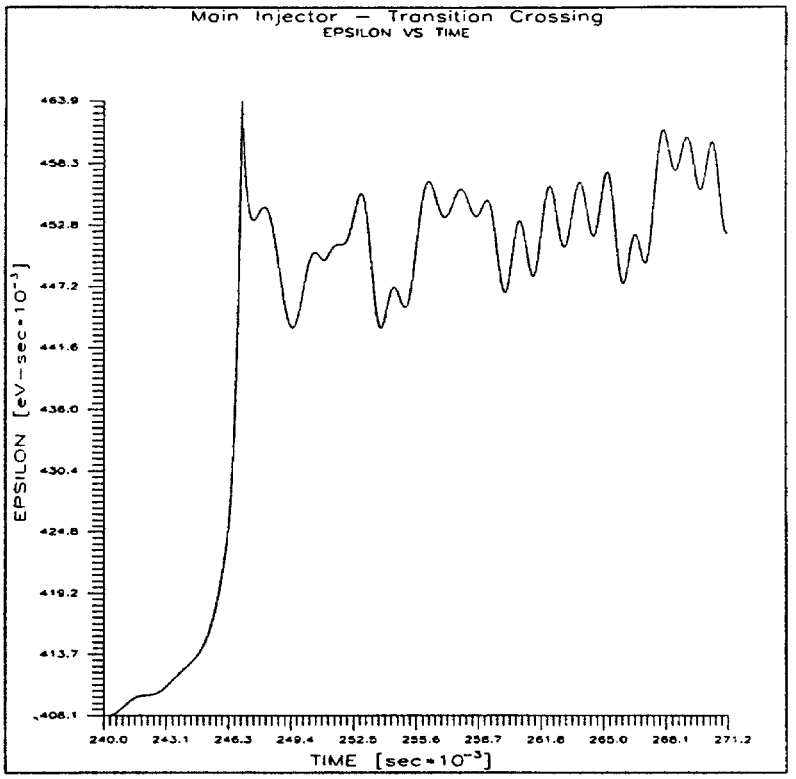
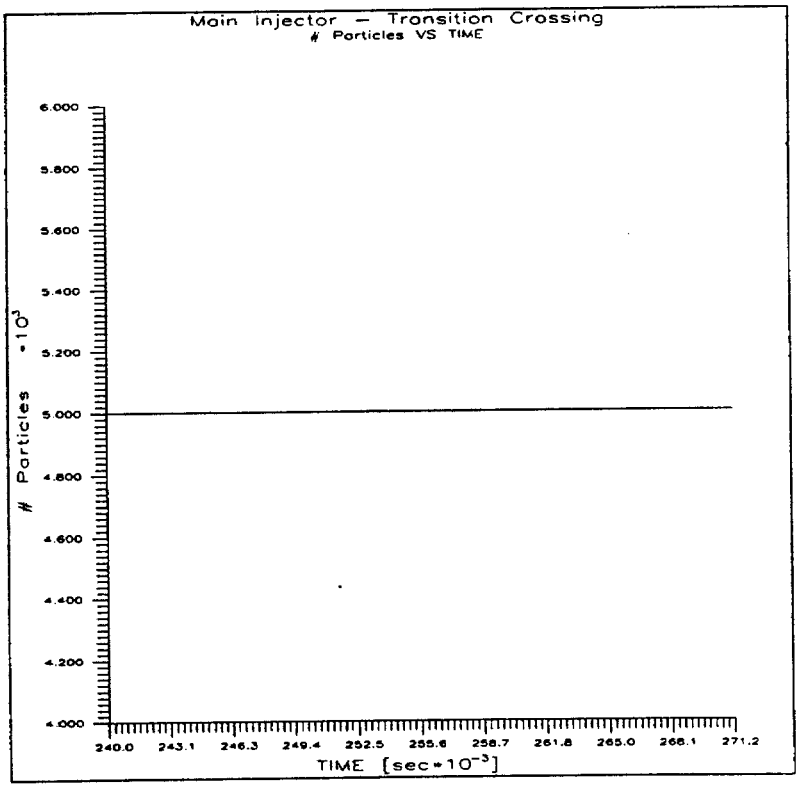




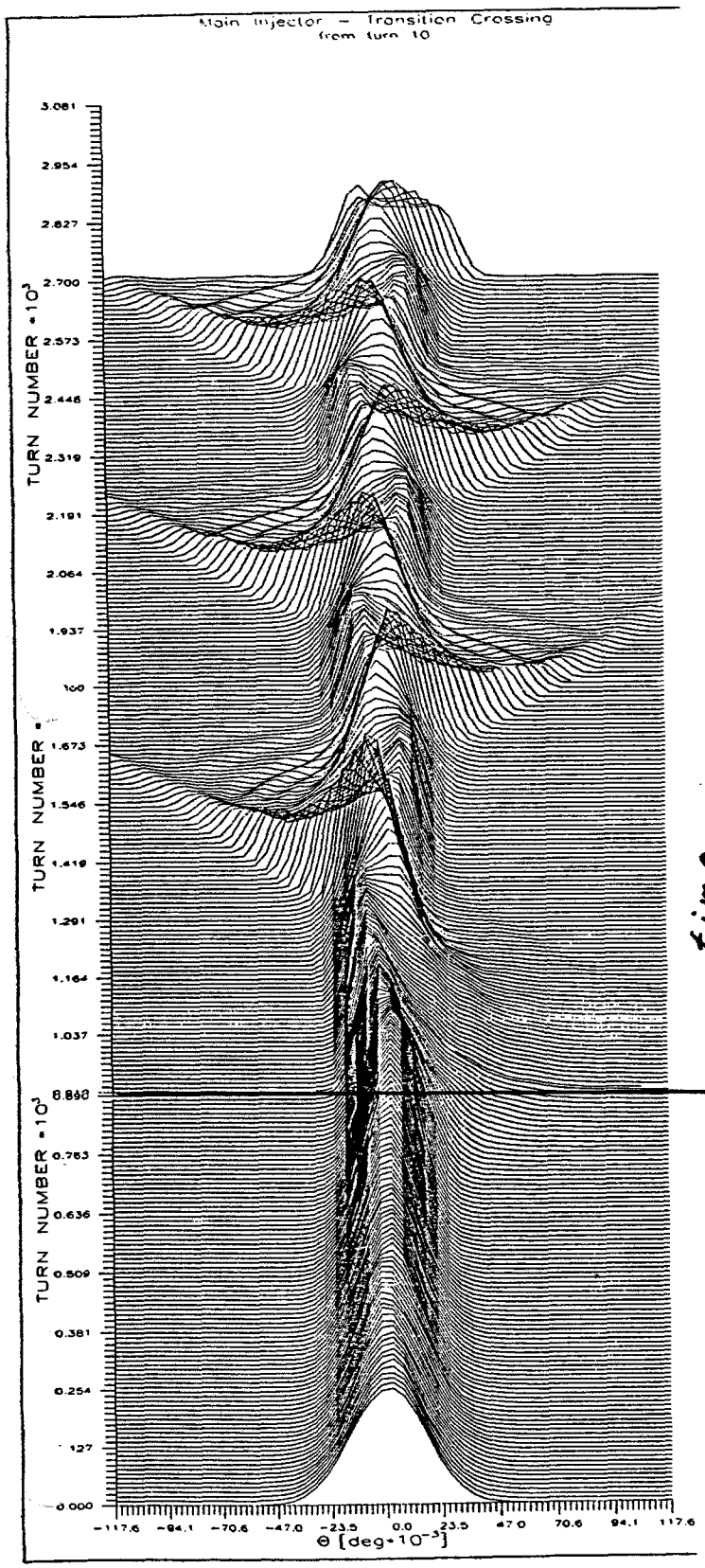
10 %
blow up



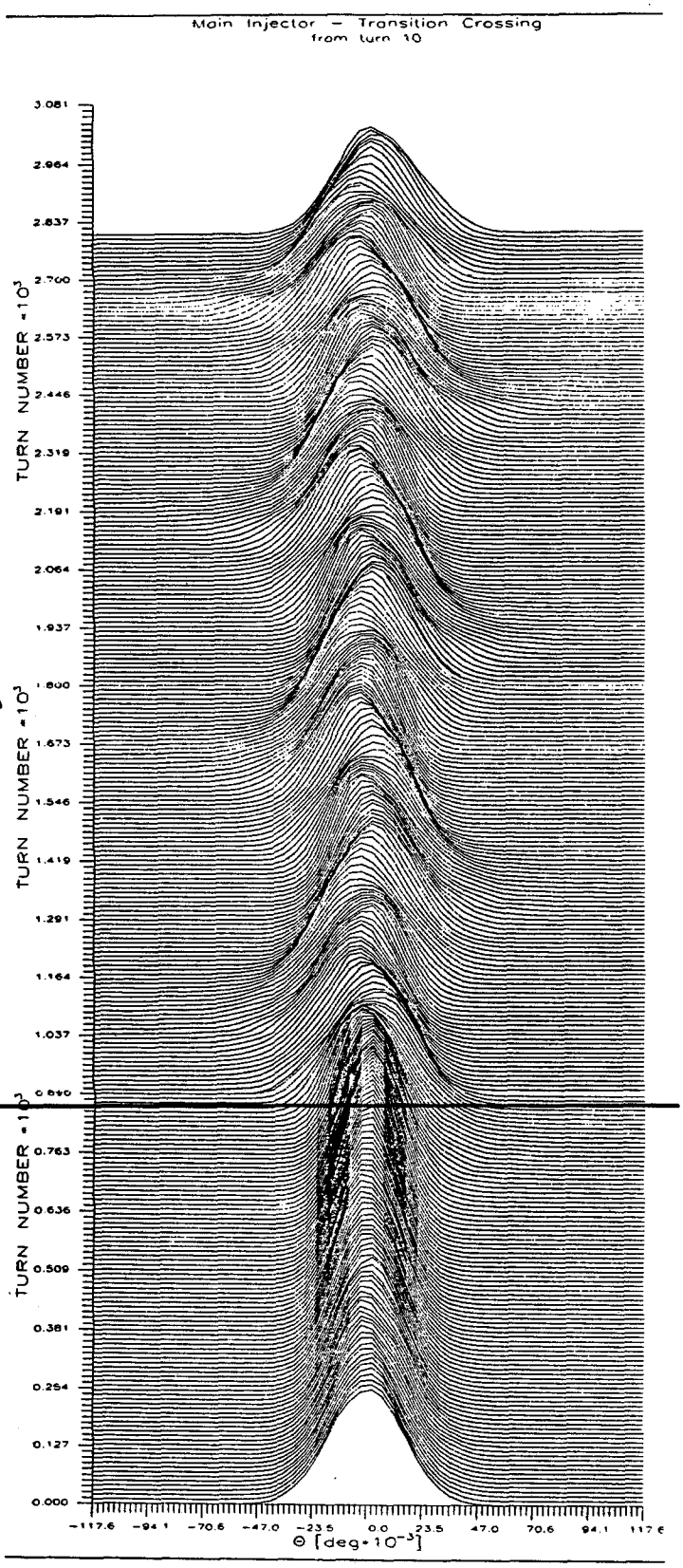
no particle
loss

$\alpha_1 = 0$

no jump



γ_t – jump

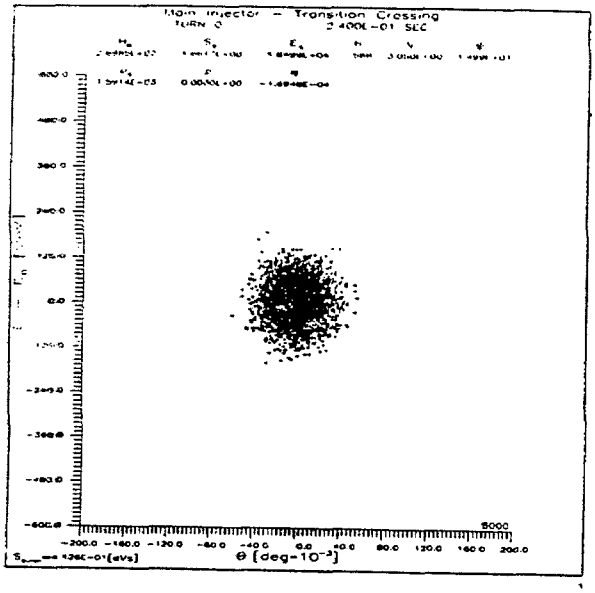


time

$\eta = 0$

$\alpha_1 = 5 \times 10^{-3}$

1



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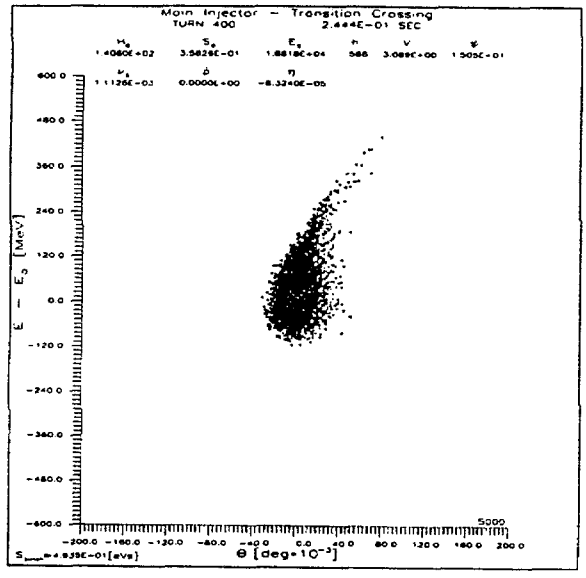

transition @ turn 790

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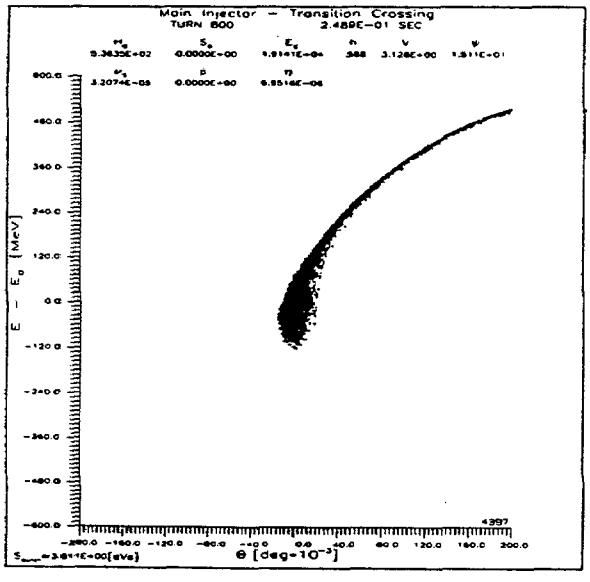

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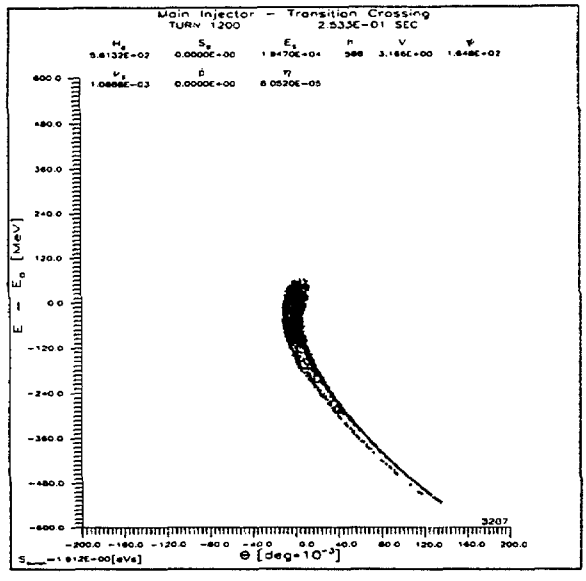

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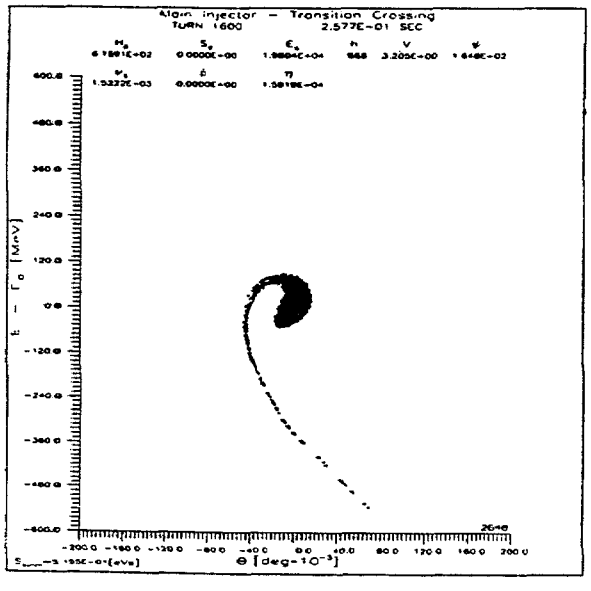

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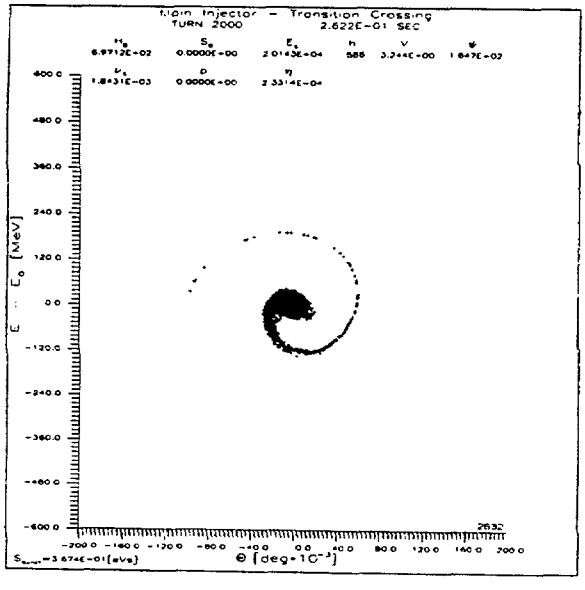
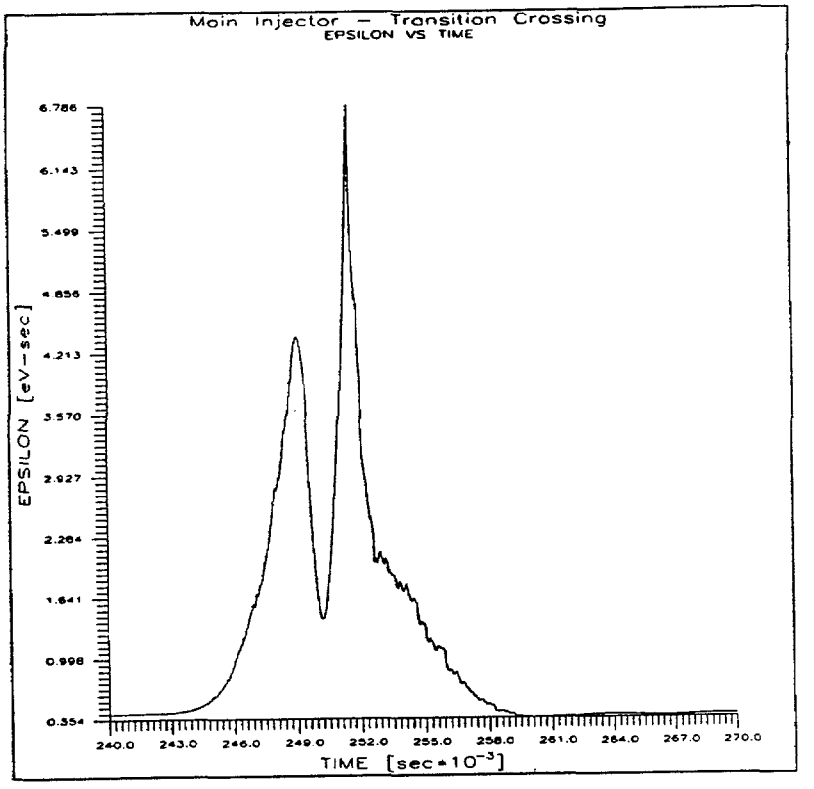
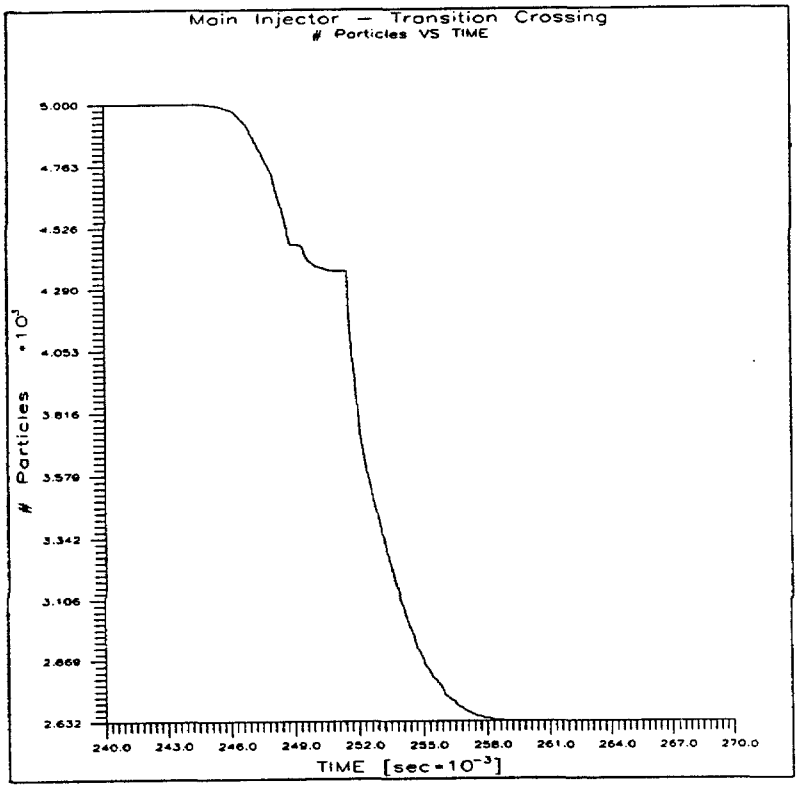




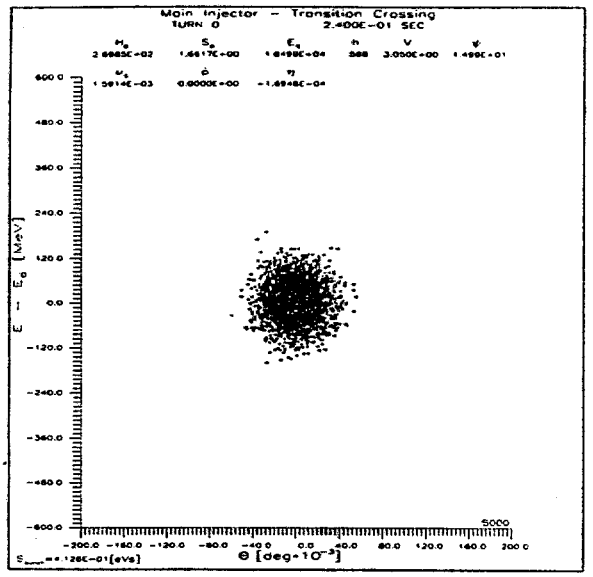
compacted beam



50% beam loss

$\alpha_1 = 5\times10^{-3}$ γ_t jump

1



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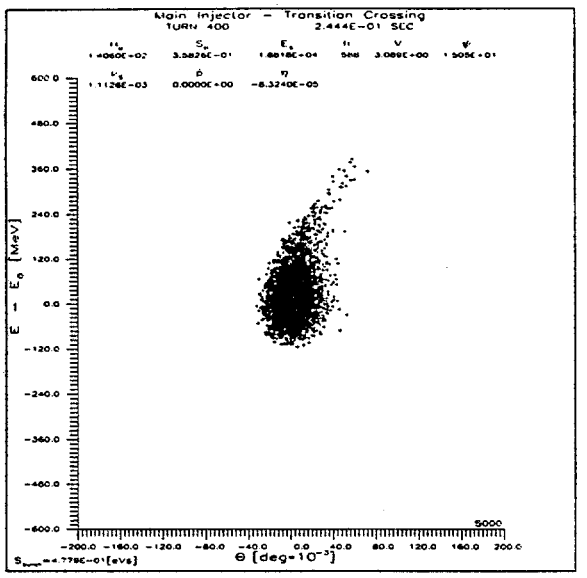

transition @ turn 790

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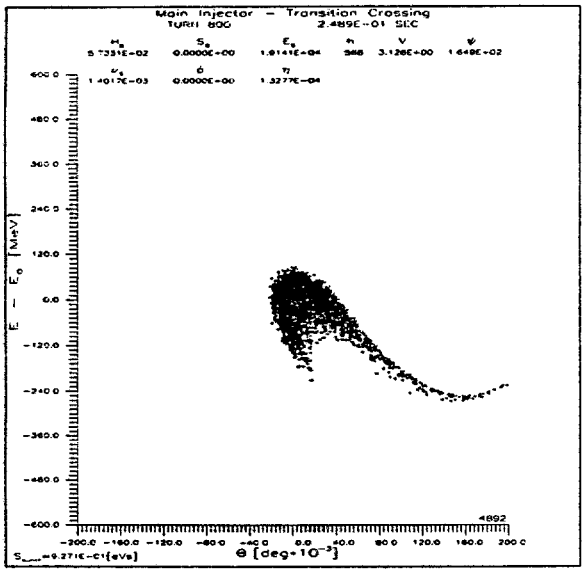

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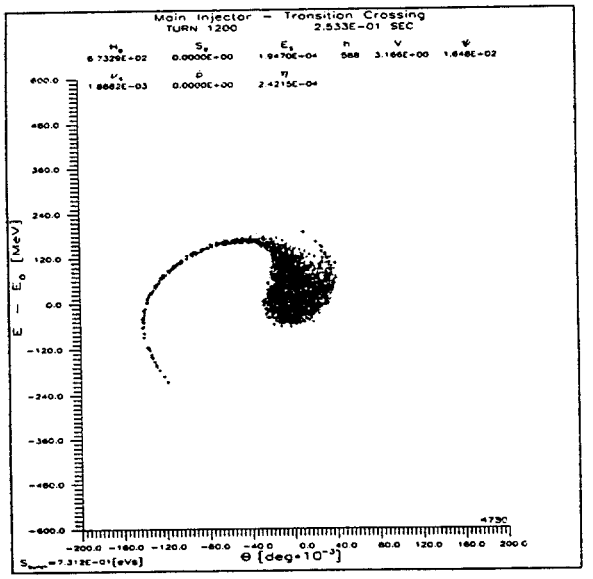

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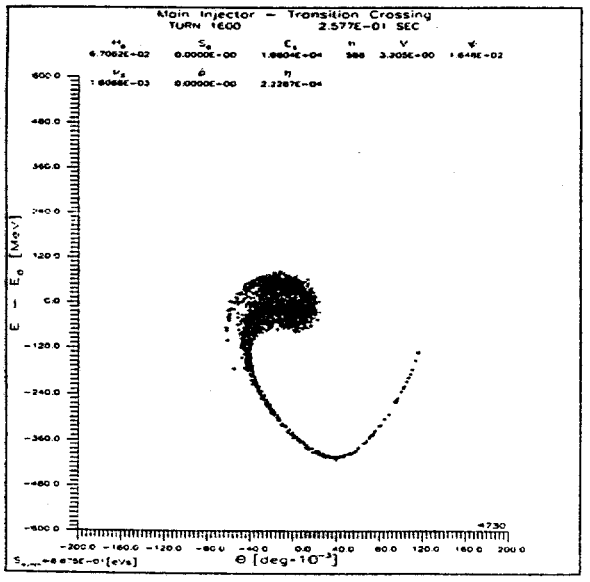

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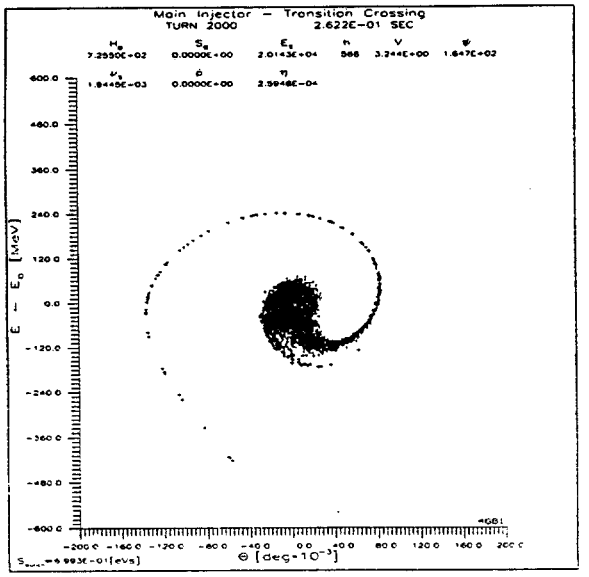
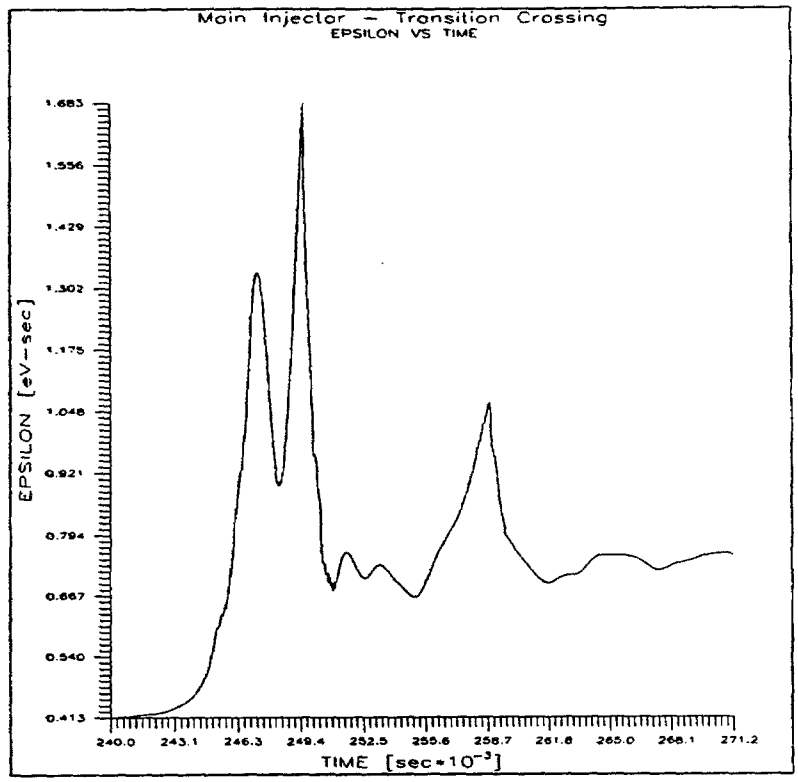
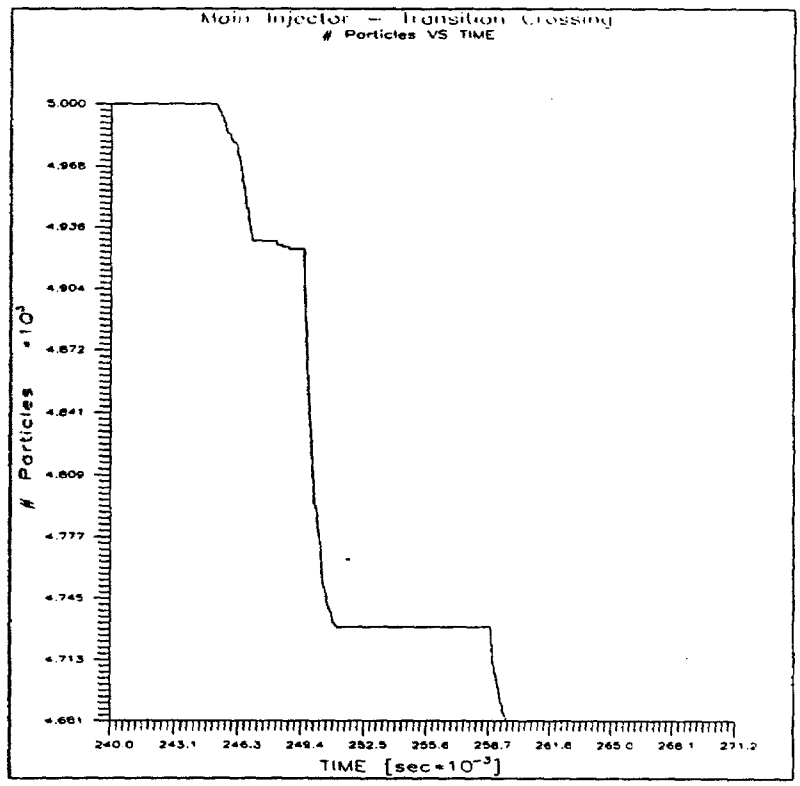




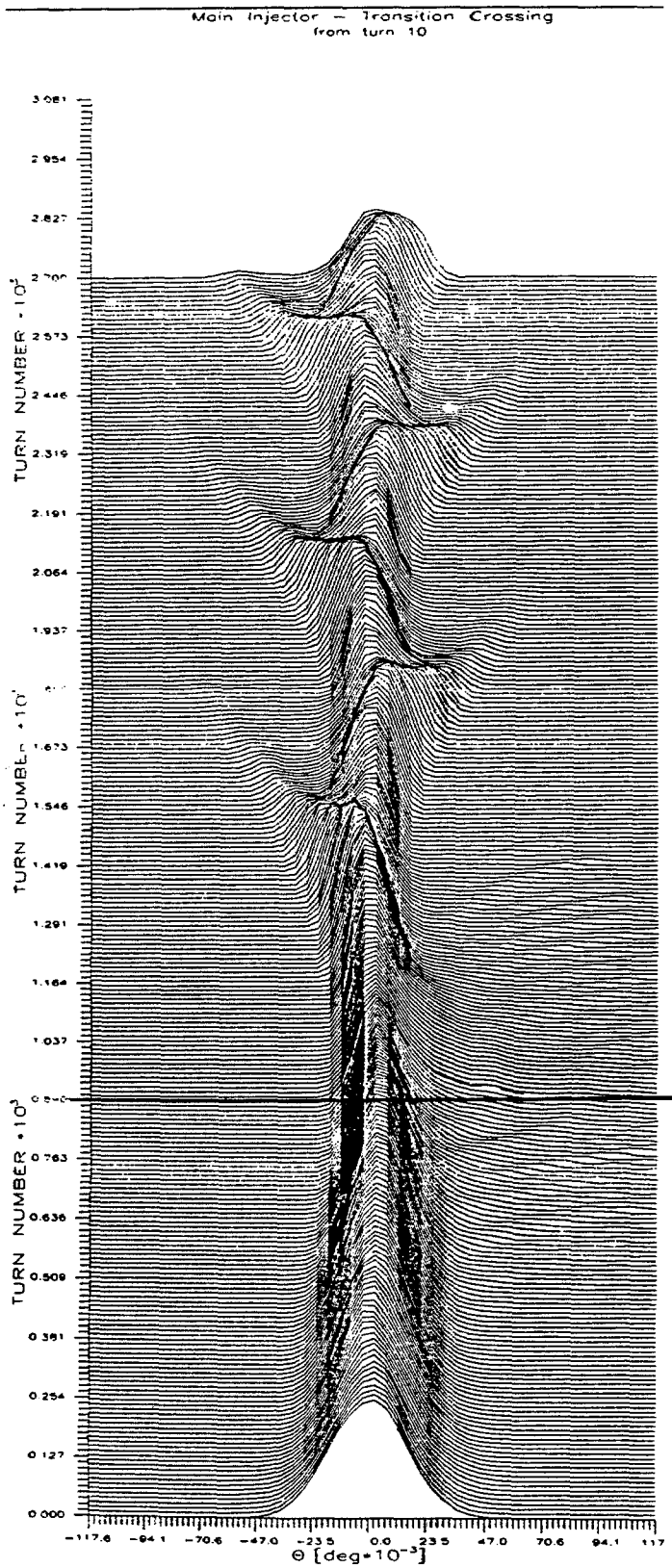
75 % blow up



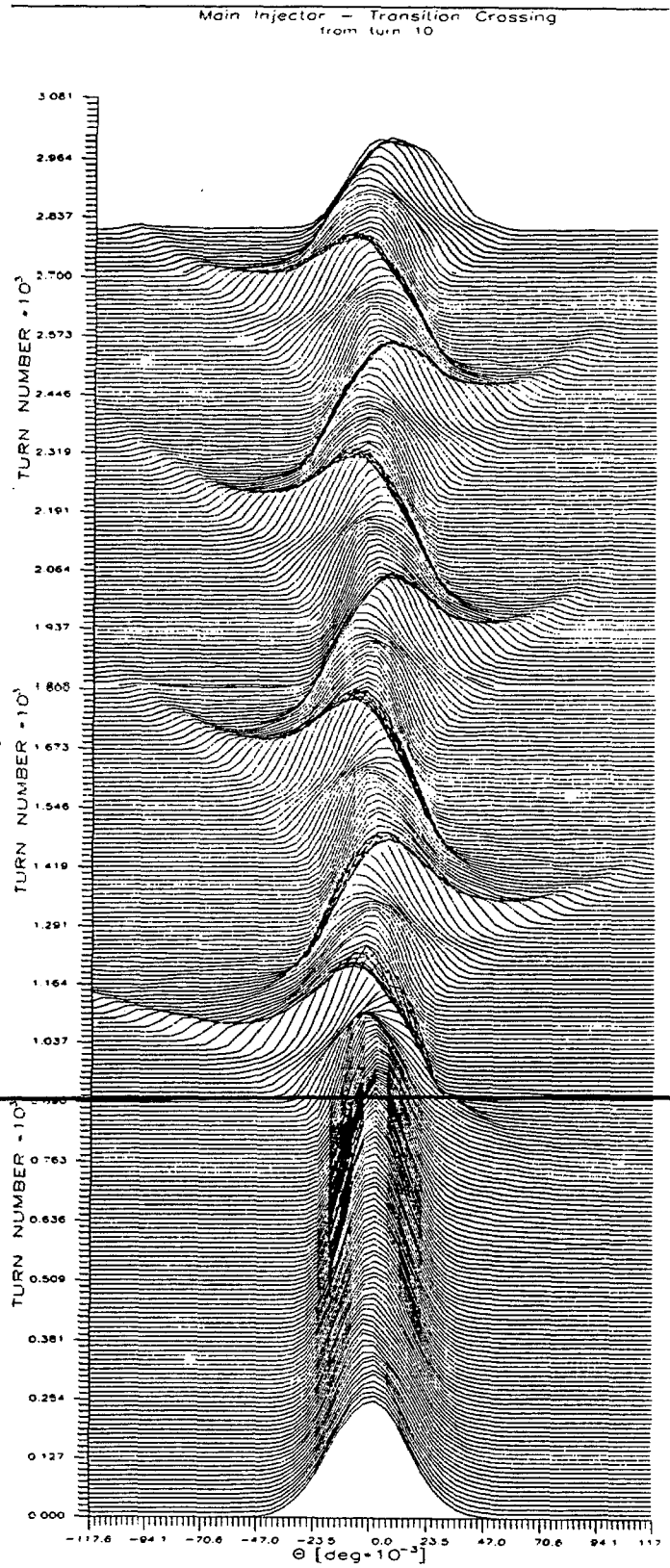
6.5 % particles lost

$\alpha_1 = 5 \times 10^{-3}$

no γ_t jump



γ_t jump



time