DCPT/03/146, DESY 03-184
FERMILAB-Pub-03/239-T
IPPP/03/73
hep-ph/0311148

arXiv:hep-ph/0311148 v1 11 Nov 2003

Precise Prediction for the W-Boson Mass in the Standard Model

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Abstract

The presently most accurate prediction for the W-boson mass in the Standard Model is obtained by combining the complete two-loop result with the known higher-order QCD and electroweak corrections. The numerical impact of the different contributions is analysed in detail. A simple parametrisation of the full result is presented, which approximates the full result for $M_{\rm W}$ to better than 0.5 MeV for 10 GeV $\leq M_{\rm H} \leq$ 1 TeV if the other parameters are varied within their combined 2σ region around their experimental central values. The different sources of remaining theoretical uncertainties are investigated. Their effect on the prediction of $M_{\rm W}$ is estimated to be about 4 MeV for $M_{\rm H} \lesssim 300$ GeV.

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The relation between the W-boson mass, $M_{\rm W}$, the Z-boson mass, $M_{\rm Z}$, the fine structure constant, α, and the Fermi constant, G_μ,

$$M_{\rm W}^2 \left(1 - \frac{M_{\rm W}^2}{M_{\rm Z}^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu}\,(1+\Delta r)\,, \tag{1}$$

is of central importance for precision tests of the electroweak theory. Accordingly, a lot of effort has been devoted over more than two decades to accurately predict the quantity Δr, which summarises the radiative corrections, within the Standard Model (SM) and extensions of it.

The one-loop result [1] can be written as

$$\Delta r^{(\alpha)} = \Delta\alpha - \frac{c_{\rm W}^2}{s_{\rm W}^2}\Delta\rho + \Delta r_{\rm rem}(M_{\rm H}), \tag{2}$$

where $c_{\rm W}^2 = M_{\rm W}^2/M_{\rm Z}^2$, $s_{\rm W}^2 = 1-c_{\rm W}^2$. It involves large fermionic contributions from the shift in the fine structure constant due to light fermions, $\Delta\alpha \propto \log m_f$, and from the leading contribution to the ρ parameter, $\Delta\rho$. The latter is quadratically dependent on the top-quark mass, $m_{\rm t}$, as a consequence of the large mass splitting in the isospin doublet [2]. The remainder part, $\Delta r_{\rm rem}$, contains in particular the dependence on the Higgs-boson mass, $M_{\rm H}$. Higher-order QCD corrections to Δr are known at $\mathcal{O}(\alpha\alpha_{\rm s})$ [3] and $\mathcal{O}(\alpha\alpha_{\rm s}^2)$ [4,5].

Recently the full electroweak two-loop result for Δr has been completed. It consists of the fermionic contribution [6,7,8], which involves diagrams with one or two closed fermion loops, and the purely bosonic two-loop contribution [9].

Beyond two-loop order the results for the pure fermion-loop corrections (i.e. contributions containing n fermion loops at n-loop order) are known up to four-loop order [10]. They contain in particular the leading contributions in $\Delta\alpha$ and $\Delta\rho$. Most recently results for the leading three-loop contributions of $\mathcal{O}(G_\mu^3 m_{\rm t}^6)$ and $\mathcal{O}(G_\mu^2\alpha_{\rm s}m_{\rm t}^4)$ have been obtained for arbitrary values of $M_{\rm H}$ (by means of expansions around $M_{\rm H} = m_{\rm t}$ and for $M_{\rm H} \gg m_{\rm t}$) [11], generalising a previous result which was obtained in the limit $M_{\rm H} = 0$ [12].

Eq. (1) is usually employed for predicting the W-boson mass,

$$M_{\rm W}^2 = M_{\rm Z}^2 \left\{\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha}{\sqrt{2}G_\mu M_{\rm Z}^2}\Big[1+\Delta r(M_{\rm W}, M_{\rm Z}, M_{\rm H}, m_{\rm t}, \ldots)\Big]}\right\}, \tag{3}$$

which is done by an iterative procedure since Δr itself depends on $M_{\rm W}$. Comparison of the prediction for $M_{\rm W}$ within the SM with the experimental value allows to obtain indirect constraints on the Higgs-boson mass. These constraints are affected both by the experimental error of $M_{\rm W}$ and by the uncertainty of the theory prediction. The current experimental error of the W-boson mass is $\delta M_{\rm W}^{\rm exp} = 34$ MeV [13]. The accuracy in the measurement of the W-boson mass is expected to improve to about $\delta M_{\rm W}^{\rm exp,Tev/LHC} = 15$ MeV [14] from the measurements at RunII of the Tevatron and the LHC, and to about $\delta M_{\rm W}^{\rm exp,LC} = 7$ MeV at a future Linear Collider (LC) running at the WW threshold [15]. The uncertainty of the theory prediction is caused by the experimental errors of the input parameters, e.g. $m_{\rm t}$, and by the uncertainty from unknown higher-order corrections. In the global SM fit to all data [16] the highest sensitivity to $M_{\rm H}$ arises from the predictions for $M_{\rm W}$ and the effective weak mixing angle at the Z-boson resonance, $\sin^2\theta_{\rm eff}$.

In the present paper we combine the various pieces being relevant for the prediction of $M_{\rm W}$ into a common result and analyse the numerical impact of the different contributions. Since in particular the electroweak two-loop result is very lengthy and involves numerical integrations of two-loop scalar integrals, it is not possible to present the full result in a compact analytic form. We therefore provide a simple parametrisation of the full result which is easy to implement and should be accurate enough for practical applications. We discuss the sources of the remaining theoretical uncertainties and obtain an estimate for the uncertainty from unknown higher-order corrections.

We incorporate the following contributions into the result for Δr,

$$\Delta r = \Delta r^{(\alpha)} + \Delta r^{(\alpha\alpha_{\rm s})} + \Delta r^{(\alpha\alpha_{\rm s}^2)} + \Delta r^{(\alpha^2)}_{\rm ferm} + \Delta r^{(\alpha^2)}_{\rm bos} + \Delta r^{(G_\mu^2\alpha_{\rm s}m_{\rm t}^4)} + \Delta r^{(G_\mu^3 m_{\rm t}^6)}, \qquad (4)$$

where $\Delta r^{(\alpha)}$ is the one-loop result, eq. (2), $\Delta r^{(\alpha\alpha_{\rm s})}$ and $\Delta r^{(\alpha\alpha_{\rm s}^2)}$ are the two-loop [3] and three-loop [4, 5] QCD corrections, and $\Delta r^{(\alpha^2)}_{\rm ferm}$ [6, 7, 8] and $\Delta r^{(\alpha^2)}_{\rm bos}$ [9] are the fermionic and purely bosonic electroweak two-loop corrections, respectively. The contributions $\Delta r^{(G_\mu^2\alpha_{\rm s}m_{\rm t}^4)}$ and $\Delta r^{(G_\mu^3 m_{\rm t}^6)}$ have been obtained from the leading three-loop contributions to $\Delta\rho$ given in Ref. [11].

We have not included the pure fermion-loop contributions at three-loop and four-loop order obtained in Ref. [10], because their contribution turned out to be small as a consequence of accidental numerical cancellations, with a net effect of only about 1 MeV in $M_{\rm W}$ (using the real-pole definition of the gauge-boson masses). Since the result given in Ref. [10] contains the leading contributions involving powers of $\Delta\alpha$ and $\Delta\rho$ beyond two-loop order, we do not make use of resummations of $\Delta\alpha$ and $\Delta\rho$ as it was often done in the literature in the past (see e.g. Refs. [17]). Accordingly, the quantity Δr appears in eq. (3) in fully expanded form. This means, for instance, that we do not include the $\mathcal{O}(\alpha^3)$ term $3(\Delta\alpha)^2\Delta r^{(\alpha)}_{\rm bos}$, which can be inferred from the electric charge renormalisation. It affects the prediction for $M_{\rm W}$ by about 1.5 MeV. This shift is however expected to partially cancel with the corresponding contributions proportional to $(\Delta\alpha)(\Delta\rho)\Delta r^{(\alpha)}_{\rm bos}$ and $(\Delta\rho)^2\Delta r^{(\alpha)}_{\rm bos}$ in an analogous way as for the pure fermion-loop contributions.

In Table 1 the numerical values of the different contributions to Δr are given for $M_{\rm W} = 80.426$ GeV [13]. The other input parameters that we use in this paper are [13]

$$\begin{aligned} &m_{\rm t} = 174.3 \text{ GeV}, \quad m_{\rm b} = 4.7 \text{ GeV}, \quad M_{\rm Z} = 91.1875 \text{ GeV}, \quad \Gamma_{\rm Z} = 2.4952 \text{ GeV}, \\ &\alpha^{-1} = 137.03599976, \quad \Delta\alpha = 0.05907, \quad \alpha_{\rm s}(M_{\rm Z}) = 0.119, \\ &G_\mu = 1.16637 \times 10^{-5} \text{ GeV}^{-2}, \end{aligned} \qquad (5)$$

where $\Delta\alpha \equiv \Delta\alpha_{\rm lept} + \Delta\alpha^{(5)}_{\rm had}$, and $\Delta\alpha_{\rm lept} = 0.0314977$ [18]. For $\Delta\alpha^{(5)}_{\rm had}$ we use the value given in Ref. [19], $\Delta\alpha^{(5)}_{\rm had} = 0.027572 \pm 0.000359$. The total width of the Z boson, $\Gamma_{\rm Z}$, appears as an input parameter since the experimental value of $M_{\rm Z}$ in eq. (5), corresponding to a Breit–Wigner parametrisation with running width, needs to be transformed in our calculation into the mass parameter defined according to the real part of the complex pole, which corresponds to a Breit–Wigner parametrisation with a constant decay width, see Ref. [7]. It is understood that $M_{\rm W}$ in this paper always refers to the conventional definition according to a Breit–Wigner parametrisation with running width. The change

$M_{\rm H}$/ GeV	$\Delta r^{(\alpha)}$	$\Delta r^{(\alpha\alpha_s)}$	$\Delta r^{(\alpha\alpha_s^2)}$	$\Delta r^{(\alpha^2)}_{\rm ferm}$	$\Delta r^{(\alpha^2)}_{\rm bos}$	$\Delta r^{(G_\mu^2\alpha_s m_t^4)}$	$\Delta r^{(G_\mu^3 m_t^6)}$
100	283.41	35.89	7.23	28.56	0.64	−1.27	−0.16
200	307.35	35.89	7.23	30.02	0.35	−2.11	−0.09
300	323.27	35.89	7.23	31.10	0.23	−2.77	−0.03
600	353.01	35.89	7.23	32.68	0.05	−4.10	−0.09
1000	376.27	35.89	7.23	32.36	−0.41	−5.04	−1.04

Table 1: The numerical values ($\times 10^4$) of the different contributions to Δr specified in eq. (1) are given for different values of $M_{\rm H}$ and $M_{\rm W} = 80.426$ GeV (the W and Z masses have been transformed so as to correspond to the real part of the complex pole). The other input parameters are listed in eq. (5).

of parametrisations is achieved with the one loop QCD corrected value of the W-boson width as described in Ref. [7].

Table 1 shows that the two-loop QCD correction, $\Delta r^{(\alpha\alpha_s)}$, and the fermionic electroweak two-loop correction, $\Delta r^{(\alpha^2)}_{\rm ferm}$ are of similar size. They both amount to about 10% of the one-loop contribution, $\Delta r^{(\alpha)}$, entering with the same sign. The most important correction beyond these contributions is the three-loop QCD correction, $\Delta r^{(\alpha\alpha_s^2)}$, which leads to a shift in $M_{\rm W}$ of about -11 MeV. For large values of $M_{\rm H}$ also the contribution $\Delta r^{(G_\mu^2\alpha_s m_t^4)}$ becomes sizable. The purely bosonic two-loop contribution, $\Delta r^{(\alpha^2)}_{\rm bos}$, and the leading electroweak three-loop correction, $\Delta r^{(G_\mu^3 m_t^6)}$, give rise to shifts in $M_{\rm W}$ which are significantly smaller than the experimental error envisaged for a future Linear Collider, $\delta M_{\rm W}^{\rm exp,LC} = 7$ MeV [15].

Since Δr is evaluated in Table 1 for a fixed value of $M_{\rm W}$, the contributions $\Delta r^{(\alpha\alpha_s)}$ and $\Delta r^{(\alpha\alpha_s^2)}$ are $M_{\rm H}$-independent. In the iterative procedure for evaluating $M_{\rm W}$ according to eq. (3), on the other hand, also these contributions become $M_{\rm H}$-dependent through the $M_{\rm H}$-dependence of the inserted $M_{\rm W}$ value.

The result for $M_{\rm W}$ based on eqs. (3), (4) can be approximated by the following simple parametrisation (see Ref. [20] for an earlier parametrisation of $M_{\rm W}$),

$$\begin{aligned} M_{\rm W} &= M_{\rm W}^0 - c_1\,{\rm dH} - c_2\,{\rm dH}^2 + c_3\,{\rm dH}^4 + c_4({\rm dh}-1) - c_5\,{\rm d}\alpha + c_6\,{\rm dt} - c_7\,{\rm dt}^2 \\ &\quad - c_8\,{\rm dH}\,{\rm dt} + c_9\,{\rm dh}\,{\rm dt} - c_{10}\,{\rm d}\alpha_{\rm s} + c_{11}\,{\rm dZ}, \end{aligned} \tag{6}$$

where

$$\begin{aligned} {\rm dH} &= \ln\left(\frac{M_{\rm H}}{100\ {\rm GeV}}\right), \quad {\rm dh} = \left(\frac{M_{\rm H}}{100\ {\rm GeV}}\right)^2, \quad {\rm dt} = \left(\frac{m_{\rm t}}{174.3\ {\rm GeV}}\right)^2 - 1, \\ {\rm dZ} &= \frac{M_{\rm Z}}{91.1875\ {\rm GeV}} - 1, \quad {\rm d}\alpha = \frac{\Delta\alpha}{0.05907} - 1, \quad {\rm d}\alpha_{\rm s} = \frac{\alpha_{\rm s}(M_{\rm Z})}{0.119} - 1, \end{aligned} \tag{7}$$

	$\delta M_{\rm W}$(full result)/ MeV	$\delta M_{\rm W}$(eqs. (6)–(8))/ MeV
$\delta M_{\rm H} = 100$ GeV	-41.3	-41.4
$\delta m_{\rm t} = 5.1$ GeV	31.0	31.0
$\delta M_{\rm Z} = 2.1$ MeV	2.6	2.6
$\delta\left(\Delta\alpha_{\rm had}^{(5)}\right) = 0.00036$	-6.5	-6.5
$\delta\alpha_{\rm s}(M_{\rm Z}) = 0.0027$	-1.7	-1.7

Table 2: Shifts in $M_{\rm W}$ caused by varying $M_{\rm H}$ by 100 GeV and the other input parameters by 1σ around their experimental central values [13]. The first column shows the full result for $M_{\rm W}$, while the second column is based on the simple parametrisation of eqs. (6)–(8). The shifts $\delta M_{\rm W}$ are relative to the value $M_{\rm W} = 80.3799$ GeV which is the result for $M_{\rm H} = 100$ GeV and the central values of the other input parameters as specified in eq. (5).

and the coefficients $M_{\rm W}^0, c_1, \ldots, c_{11}$ take the following values

$$\begin{aligned} M_{\rm W}^0 &= 80.3799 \text{ GeV}, & c_1 &= 0.05429 \text{ GeV}, & c_2 &= 0.008939 \text{ GeV}, \\ c_3 &= 0.0000890 \text{ GeV}, & c_4 &= 0.000161 \text{ GeV}, & c_5 &= 1.070 \text{ GeV}, \\ c_6 &= 0.5256 \text{ GeV}, & c_7 &= 0.0678 \text{ GeV}, & c_8 &= 0.00179 \text{ GeV}, \\ c_9 &= 0.0000659 \text{ GeV}, & c_{10} &= 0.0737 \text{ GeV}, & c_{11} &= 114.9 \text{ GeV}. \end{aligned} \tag{8}$$

The parametrisation given in eqs. (6)–(8) approximates the full result for $M_{\rm W}$ to better than 0.5 MeV over the whole range of 10 GeV $\leq M_{\rm H} \leq$ 1 TeV if all other experimental input values vary within their combined 2σ region around their central values given in eq. (7).

In Table 2 the full result for $M_{\rm W}$ and the parametrisation of eqs. (6)–(8) are compared with each other. The table shows the shifts in $M_{\rm W}$ (relative to the value $M_{\rm W} = 80.3799$ GeV, which is the result for $M_{\rm H} = 100$ GeV and the central values of the other input parameters as specified in eq. (5)) induced by varying $M_{\rm H}$ by 100 GeV and the other input parameters by 1σ around their experimental central values [13]. In the example of Table 2, where only one parameter has been varied in each row and all others have been kept at their central values, the maximum deviation between the full result for $M_{\rm W}$ and the parametrisation of eqs. (6)–(8) is below 0.1 MeV.

The parametrisation of eqs. (6)–(8) yields a good approximation of the full result for $M_{\rm W}$ even for values of $M_{\rm H}$ much smaller than the experimental 95% C.L. lower bound on the Higgs-boson mass, $M_{\rm H} = 114.4$ GeV [21]. If one restricts to the region $M_{\rm H} >$ 100 GeV, a slight readjustment of the coefficients in eq. (8) yields an even more accurate parametrisation of the full result. If eqs. (6), (7) are used together with the following

values of the coefficients,

$$\begin{aligned} M_{\rm W}^0 &= 80.3800 \text{ GeV}, & c_1 &= 0.05253 \text{ GeV}, & c_2 &= 0.010345 \text{ GeV}, \\ c_3 &= 0.001021 \text{ GeV}, & c_4 &= -0.000070 \text{ GeV}, & c_5 &= 1.077 \text{ GeV}, \\ c_6 &= 0.5270 \text{ GeV}, & c_7 &= 0.0698 \text{ GeV}, & c_8 &= 0.004055 \text{ GeV}, \\ c_9 &= 0.000110 \text{ GeV}, & c_{10} &= 0.0716 \text{ GeV}, & c_{11} &= 115.0 \text{ GeV}, \end{aligned} \tag{9}$$

the full result for $M_{\rm W}$ is approximated to better than 0.2 MeV over the range of 100 GeV $\leq M_{\rm H} \leq$ 1 TeV if all other experimental input values vary within their combined 2σ region around their central values given in eq. (7).

From Table 2 one can read off the parametric theoretical uncertainties in the prediction for $M_{\rm W}$ being caused by the experimental errors of the input parameters. The dominant parametric uncertainty at present (besides the dependence on $M_{\rm H}$) is induced by the experimental error of the top-quark mass. It is almost as large as the current experimental error of the W-boson mass, $\delta M_{\rm W}^{\rm exp} = 34$ MeV [13]. The uncertainty caused by the experimental error of $m_{\rm t}$ will remain the dominant source of theoretical uncertainty in the prediction for $M_{\rm W}$ even at the LHC, where the error on $m_{\rm t}$ will be reduced to $\delta m_{\rm t} = 1$–2 GeV [22]. A further improvement of the parametric uncertainty of $M_{\rm W}$ will require the precise measurement of $m_{\rm t}$ at a future Linear Collider [23], where an accuracy of about $\delta m_{\rm t} = 0.1$ GeV will be achievable [15].

We now turn to the second source of theoretical uncertainties in the prediction for $M_{\rm W}$, namely the uncertainties from unknown higher-order corrections. Different approaches have been used in the literature for estimating the possible size of uncalculated higher-order corrections [24, 25, 26, 7]. The "traditional Blue Band method" is based on the fact that the results of calculations employing different renormalisation schemes or different prescriptions for including non-leading contributions in resummed or expanded form differ from each other by higher-order corrections. The deviations between the results of different codes in which the same corrections have been organised in a somewhat different way are used in this method as a measure for the size of unknown higher-order corrections [24]. In applying this method it is not easy to quantify how big the variety of different "options" and different codes should be in order to obtain a reasonable estimate of the higher-order uncertainties. As the method cannot account for genuine effects of irreducible higher-order corrections, it may lead to an underestimate of the theoretical uncertainties if at an uncalculated order a new source of potentially large corrections appears, e.g. a certain enhancement factor.

In Ref. [26] a different prescription has been proposed, in which for each type of unknown corrections the relevant enhancement factors are identified and the remaining coefficient arising from the actual loop integrals is set to unity. In Ref. [7] higher-order QCD corrections have been estimated in two different ways, from the renormalisation scale dependence (in particular taking into account the effect of switching from the on-shell to the $\overline{\rm MS}$ definition of the top-quark mass) and from assuming that, for instance, the ratio of the $\mathcal{O}(\alpha^2\alpha_{\rm s})$ and $\mathcal{O}(\alpha^2)$ corrections is of the same size as the ratio of the $\mathcal{O}(\alpha\alpha_{\rm s})$ and $\mathcal{O}(\alpha)$ corrections.

Several of the corrections whose possible size had been estimated in Refs. [25, 26, 7] have meanwhile been calculated [9, 11], and it turned out that the estimates agree rea-

sonably well with the actual size of the corrections. This adds confidence to applying the same kind of methods also for an estimate of the remaining higher-order uncertainties.

There are three sources of remaining uncertainties in the prediction for $M_{\rm W}$ from unknown higher-order corrections:

- The corrections at $\mathcal{O}(\alpha^2\alpha_{\rm s})$ beyond the known contribution of $\mathcal{O}(G_\mu^2\alpha_{\rm s}m_{\rm t}^4)$:
  The numerical effect of the $\mathcal{O}(G_\mu^2\alpha_{\rm s}m_{\rm t}^4)$ correction was found to be up to 5 MeV in $M_{\rm W}$ for a light Higgs-boson mass, $M_{\rm H} \lesssim 300$ GeV [11]. This contribution represents the leading term in an expansion for asymptotically large values of $m_{\rm t}$. In the calculation of the electroweak two-loop corrections it was found that the formally next-to-leading order term of $\mathcal{O}(G_\mu^2 m_{\rm t}^2 M_{\rm Z}^2)$ has approximately the same numerical effect as the formally leading term of $\mathcal{O}(G_\mu^2 m_{\rm t}^4)$ [27]. It can therefore be expected that also the formally next-to-leading order term of $\mathcal{O}(G_\mu^2\alpha_{\rm s}m_{\rm t}^2 M_{\rm Z}^2)$ may be of similar size as the leading $\mathcal{O}(G_\mu^2\alpha_{\rm s}m_{\rm t}^4)$ term. We therefore assign an uncertainty of about 3 MeV to the remaining theoretical uncertainties at $\mathcal{O}(\alpha^2\alpha_{\rm s})$ (for $M_{\rm H} \lesssim 300$ GeV).

- The unknown electroweak three-loop corrections:
  The numerical effect of the $\mathcal{O}(G_\mu^3 m_{\rm t}^6)$ contribution was found to be small [11], shifting $M_{\rm W}$ by less than 0.3 MeV for $M_{\rm H} \lesssim 300$ GeV. This shift is significantly smaller than the estimate in Ref. [26]. The pure fermion-loop corrections at three-loop order were found in Ref. [10] to shift $M_{\rm W}$ by about 1 MeV, which however involved an accidental numerical cancellation. It thus doesn't seem to be justified to assume that all other electroweak three-loop corrections are completely negligible. In Ref. [7] it was pointed out that reparametrising the W-boson width, which enters the prediction for $M_{\rm W}$ at the two-loop level, by G_μ instead of α shifts the prediction for $M_{\rm W}$ by about 1 MeV, which is formally an effect of $\mathcal{O}(\alpha^3)$. In order to take into account uncertainties of this kind (see also the discussion below eq. (4)) we assign an uncertainty of 1–2 MeV to the unknown corrections at $\mathcal{O}(\alpha^3)$.

- The four-loop QCD corrections of $\mathcal{O}(\alpha\alpha_{\rm s}^3)$:
  The possible effect of the leading term of $\mathcal{O}(G_\mu\alpha_{\rm s}^3 m_{\rm t}^2)$ was estimated in Ref. [26] to be about 1.3 MeV. Employing the known results at lower order of $\alpha_{\rm s}$ and assuming a geometric progression yields a slightly larger result. We thus assign an uncertainty of 1–2 MeV for the $\mathcal{O}(\alpha\alpha_{\rm s}^3)$ corrections.

Adding the above estimates for the different kinds of unknown higher-order corrections in quadrature, we find as estimate of the remaining theoretical uncertainties from unknown higher-order corrections

$$\delta M_{\rm W}^{\rm theo} \approx 4 \text{ MeV}. \tag{10}$$

This estimate holds for a relatively light Higgs boson, $M_{\rm H} \lesssim 300$ GeV. For a heavy Higgs boson, i.e. $M_{\rm H}$ close to the TeV scale, the remaining theoretical uncertainty is significantly larger.

In Fig. 1 we have updated the comparison between the theory prediction for $M_{\rm W}$ within the SM and the experimental value, using the currently most accurate theory prediction based on eqs. (3), (4) and the most up-to-date experimental data [13]. For the theoretical uncertainty the estimate of eq. (10) and the parametric uncertainties corresponding to 1σ

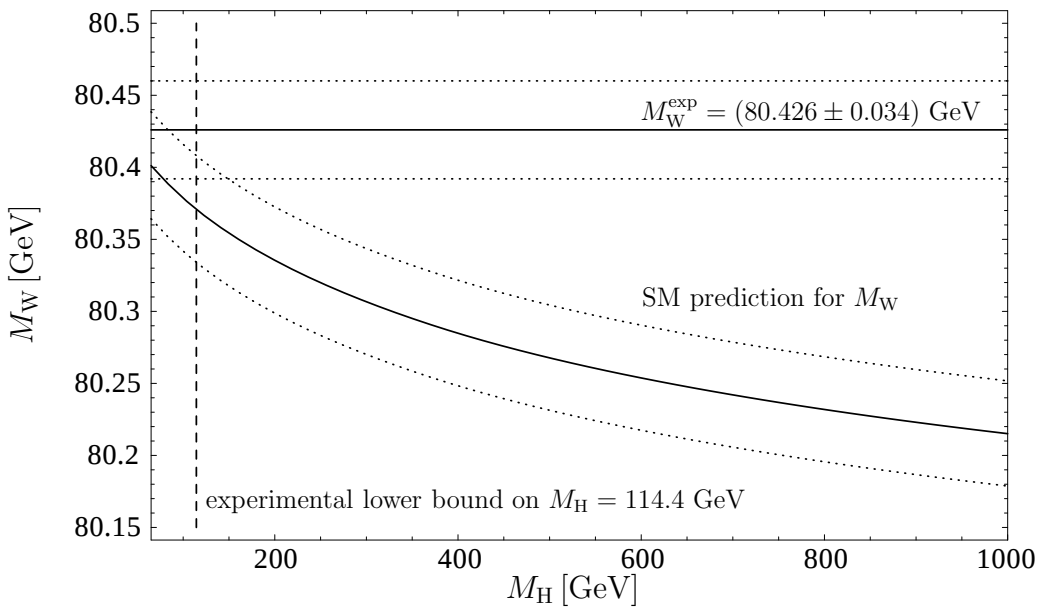

Figure 1: Prediction for M_W in the SM as a function of M_H for $m_t = 174.3 \pm 5.1$ GeV. The current experimental value, $M_W^{exp} = 80.426 \pm 0.034$ GeV [13], and the experimental 95% C.L. lower bound on the Higgs-boson mass, $M_H = 114.4$ GeV [21], are also indicated.

variations of the input parameters (see Table 2) have been used. As discussed above, at present the theoretical uncertainty is dominated by the effect of the experimental error of the top-quark mass.

Fig. 1 confirms the well-known preference for a light Higgs-boson mass within the SM. If the 95% exclusion bound from the direct search for the SM Higgs is taken into account [21], the 1σ bands corresponding to the theory prediction and the experimental result for M_W show only a marginal overlap.

In summary, we have presented the currently most accurate prediction for M_W in the Standard Model. We have discussed the relative importance of the complete one-loop and two-loop contributions as well as the known corrections beyond two-loop order. We have summarised the present status of the theoretical uncertainties of M_W from the experimental errors of the input parameters, and we have obtained an estimate for the remaining theoretical uncertainties from unknown higher-order corrections. In the region of Higgs-mass values preferred by the electroweak precision data, $M_H \lesssim 300$ GeV, the uncertainty from unknown higher-order corrections amounts to about 4 MeV. This is much smaller than the present experimental error of M_W and even below the envisaged future experimental error at the next generation of colliders. Having reached this level of theoretical precision of M_W is important, however, for the precision test of the electroweak theory, in particular in view of the fact that M_W can be used as an input for calculating the effective weak mixing angle at the Z resonance, $\sin^2\theta_{eff}$.

We have furthermore presented a simple parametrisation of the full result containing all relevant corrections, which should be sufficiently accurate for practical applications. It approximates the full result for M_W to better than 0.5 MeV over the whole range of

10 GeV $\leq M_{\rm H} \leq$ 1 TeV if all other experimental input values vary within their combined 2σ region around their experimental central values. In view of the experimental exclusion bound on the Higgs-boson mass of $M_{\rm H} >$ 114.4 GeV it will normally be sufficient to restrict to the smaller range of 100 GeV $\leq M_{\rm H} \leq$ 1 TeV. For this case we provide a simple parametrisation which approximates the full result for $M_{\rm W}$ even within 0.2 MeV.

Acknowledgements

This work was supported in part by TMR, EC-Contracts No. HPRN-CT-2002-00311 (EURIDICE) and HPRN-CT-2000-00149 (Physics at Colliders), and the KBN Grants 2P03B01025 and 5P03B09320. M.A., M.C. and A.F. would like to thank the Institute for Particle Physics Phenomenology of the University of Durham for warm hospitality during the period when part of this work was completed.

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