

Higgs as a Holographic Pseudo-Goldstone Boson

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Abstract

The AdS/CFT correspondence allows to relate 4D strongly coupled theories to weakly coupled theories in 5D AdS. We use this correspondence to study a scenario in which the Higgs appears as a composite pseudo-Goldstone boson (PGB) of a strongly coupled theory. We show how a non-linearly realized global symmetry protects the Higgs mass and guarantees the absence of quadratic divergences at any loop order. The gauge and Yukawa interactions for the PGB Higgs are introduced in a simple way in the 5D AdS theory, and their one-loop contributions to the Higgs potential are calculated using perturbation theory. These contributions are finite, giving a squared-mass to the Higgs which is one-loop smaller than the mass of the first Kaluza-Klein state. We also show that if the symmetry breaking is caused by boundary conditions in the extra dimension, the PGB Higgs corresponds to the fifth component of the bulk gauge boson. To make the model fully realistic, a tree-level Higgs quartic coupling must be induced. We present a possible mechanism to generate it and discuss the conditions under which an unwanted large Higgs mass term is avoided.

1 Introduction

Despite its tremendous phenomenological success, the standard model is almost certainly not a fundamental theory of nature. Quantum instabilities of the Higgs potential strongly suggest that it must be replaced by some other theory at energies not much higher than the electroweak scale. Such a theory, for example, can be a supersymmetric theory [1], a strongly coupled gauge theory [2], or a theory of quantum gravity [3]. In these theories the quadratic divergence of the Higgs squared-mass parameter is cut off either by embedding the Higgs boson into some larger multiplet or by giving it an internal structure. A physical scale then exists, Λ_{NP} , at which many new particles appear revealing the underlying symmetry or dynamics.

In this paper we consider a class of theories where the standard-model Higgs boson arises as a composite particle of some strongly coupled dynamics. The dynamical scale Λ_{NP} then must be parametrically larger than the scale of the Higgs mass in order to avoid strong constraints coming from direct and indirect searches of new particles at colliders. This suggests that the Higgs mass must be protected by some (approximate) symmetry even below the scale Λ_{NP} . A natural candidate for such a symmetry is an internal global symmetry under which the Higgs boson transforms non-linearly: the Higgs is a pseudo-Goldstone boson (PGB) of the broken global symmetry. This situation is somewhat similar to that of the pion in QCD, although for the Higgs there are additional requirements. It must have a sizable quartic self-coupling and appropriate Yukawa couplings to the quarks and leptons. In this paper we aim to build theories of this kind, which are well under control as effective field theories, and in which some quantities are even calculable despite the strongly interacting dynamics.

The basic observation is the following. Suppose we have a strongly coupled gauge theory that produces the Higgs boson as a composite state. In general, it is quite difficult to obtain quantitative low-energy predictions in such a theory because of non-perturbative effects; one can at best derive estimates by using certain scaling arguments. This is indeed the case if the gauge interaction is asymptotically free, as in QCD. However, it is not necessarily true if the theory remains strongly coupled in the UV and approaches a non-trivial conformal fixed point. In this case it is possible that the theory, in the limit of large number of “colors”, has an equivalent description in terms of a weakly coupled 5D theory defined on the truncated anti de-Sitter (AdS) space [4, 5]. This allows us to construct theories where the Higgs boson is interpreted as a composite state of a strong dynamics and yet some physical quantities such as the Higgs potential can be computed using perturbation theory.

The actual implementation of the above idea is quite simple, as far as gauge and Yukawa interactions are concerned. These interactions explicitly break the global symmetry, but, as we will see, they do not induce quadratic divergences for the Higgs

mass at any loop order. The global symmetry protects the Higgs mass parameter at high energies and predicts it to be a one-loop factor smaller than the mass of the lightest resonance $\sim \Lambda_{\text{NP}}$. This property makes this class of theories quite interesting.

To make the model fully realistic, however, we must generate a sizable Higgs quartic coupling. This is crucial not only to obtain a large enough physical Higgs mass, but also to obtain the needed mass gap between the electroweak scale and Λ_{NP} (as suggested by experiments). We will propose a mechanism that allows to generate a Higgs quartic coupling at tree level. This mechanism needs specific assumptions on the form of the explicit breaking of the global symmetry. However, these are assumptions on the underlying physics around the Planck scale, and not on the TeV-scale physics which yields the Higgs boson as a composite particle. Therefore, once the particular pattern of breaking is assumed (which is not quite unnatural from the viewpoint of the 4D picture), we can compute the Higgs potential generated at loop level through the explicit symmetry-breaking effects.

Since the cutoff of our theory is around the Planck scale, there is no obstacle in extending the theory beyond Λ_{NP} , up to very high energies. In this respect, our framework may be viewed as a way to provide a UV completion for “little Higgs” theories [6] in which realistic models of the PGB Higgs have been constructed. It might be interesting to construct a UV completion of the existing little Higgs models [7] using a warped spacetime as outlined in the present paper.

In the next section we start by defining the framework in more detail. We describe the basic structure of our theories in terms of both 4D and 5D pictures. An explicit model is given in section 3, in which the Higgs boson is identified with a PGB arising from a scalar field located on the infrared brane. We present a possible mechanism to obtain a sizable Higgs quartic coupling and discuss the size of quantum corrections to the Higgs potential, which trigger electroweak symmetry breaking. In section 4 we consider theories where the Higgs boson arises from the extra-dimensional component of a gauge field in a warped 5D spacetime. We point out that also in this case the Higgs is interpreted as a composite PGB in the 4D picture, and we present several realistic models. Conclusions are given in section 6.

2 Higgs as a Composite Pseudo-Goldstone Boson

In this section we describe a class of theories where the standard-model Higgs arises as a composite PGB of a strong interaction, and in which the presence of a weakly coupled dual description allows the computation of certain quantities. We begin with the 4D description of our theory, which we also refer to as the holographic theory. In this picture the theory consists of two sectors. One is a sector of elementary particles that correspond to the standard-model gauge bosons and (some of the) quarks and

leptons. The other is a strongly coupled conformal field theory (CFT) sector, where the conformal symmetry is broken at low energies $1/L_1 \ll M_{\text{Pl}}$. This sector produces CFT bound states due to the strong dynamics at the scale $1/L_1$. The Higgs will be one of these bound states.

To have a little hierarchy between $1/L_1$ and the Higgs mass, we require the Higgs to be a Goldstone boson of the CFT sector. For this purpose, we assume that the CFT has a global symmetry larger than the standard-model electroweak gauge group $SU(2)_L \times U(1)_Y$. We find that it must be at least $SU(3)$. If a global $SU(3)_L$ symmetry is spontaneously broken to $SU(2)_L$ by the CFT strong dynamics, then 5 Goldstone bosons appear, a doublet and a singlet under $SU(2)_L$. The doublet will be associated with the Higgs boson. The $SU(3)_L$ is not a symmetry of the standard-model gauge and matter fields which belong to the elementary sector, so that the couplings of these fields with the CFT explicitly break the global $SU(3)_L$ invariance. A mass for the Higgs boson is generated at loop level, which, if negative, will trigger electroweak symmetry breaking. The loop factor appearing in the Higgs mass can give a rationale for the little hierarchy between the electroweak scale and the compositeness scale $1/L_1$, although to perform quantitative computations we must go to the weakly coupled dual description of the theory.

The AdS/CFT correspondence [4], as applied to a spontaneously broken CFT with a UV cutoff [5], allows us to relate the above scenario to a theory in a slice of 5D AdS. In this AdS picture the theory is weakly coupled and we can perform explicit calculations. The metric of the spacetime is given by [8]

$$ds^2 = \frac{1}{(kz)^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) \equiv g_{MN} dx^M dx^N. \quad (1)$$

The 5D coordinates are labeled by capital Latin letters, $M = (\mu, 5)$ where $\mu = 0, \dots, 3$; $z = x^5$ represents the coordinate for the fifth dimension and $1/k$ is the AdS curvature radius. This spacetime has two boundaries at $z = L_0 \equiv 1/k \sim 1/M_{\text{Pl}}$ (Planck brane) and $z = L_1 \sim 1/\text{TeV}$ (TeV brane): the theory is defined on the line segment $L_0 \leq z \leq L_1$.

The global symmetry of the 4D CFT is realized as a bulk gauge symmetry in the 5D picture. In the case of a 4D theory where the CFT sector has a global $SU(3)_L$ invariance, the dual theory is a 5D $SU(3)_L$ gauge theory. This $SU(3)_L$ is spontaneously broken by two scalars. One living on the Planck brane and the other on the TeV brane. Being separated in space, these scalars do not “see” each other at the classical level, so that the theory contains, in the gaugeless limit, an enlarged global $SU(3) \times SU(3)$ symmetry. By giving vacuum expectation values (VEVs) to the two scalars, the global symmetry is spontaneously broken, $SU(3) \times SU(3) \rightarrow SU(2) \times SU(2)$, delivering two sets of 5 Goldstone bosons. When we gauge the $SU(3)_L$ subgroup of $SU(3) \times SU(3)$, the Goldstone bosons which parametrize the

$SU(3)_L/SU(2)_L$ space are true Goldstone bosons and are eaten to form massive gauge vectors. The remaining ones are PGBs, since gauge interactions do not respect the full global $SU(3) \times SU(3)$. In a slice of warped space the scalar living on the Planck brane corresponds, to a very good approximation, to the true Goldstone boson. This is because its VEV will be naturally of order M_{Pl} , much larger than the VEV of the scalar on the TeV brane. Therefore, the scalar on the TeV brane corresponds to the PGB, which we identify as the standard-model Higgs boson.

The holographic dual of this 5D setup is thus the 4D theory we described before: the Higgs corresponds to the composite Goldstone boson of a CFT sector whose global $SU(3)_L$ invariance is spontaneously broken down to $SU(2)_L$ by the strong dynamics. An explicit breaking of the global CFT invariance is communicated by the interactions with the elementary sector, and a mass for the Goldstone bosons is generated at one loop. The spontaneous symmetry breaking of the CFT sector corresponds to the TeV-brane breaking of the 5D theory, while the explicit breaking given by the elementary sector is associated with the Planck-brane dynamics. This means that any process of the holographic theory where the explicit breaking is communicated from the elementary sector to the CFT, will correspond in the AdS picture to some kind of transmission from the Planck brane to the TeV brane. The mass of the PGB is an important example: non-locality in the 5D theory implies that it is a calculable and finite effect. This is a crucial ingredient of our class of theories, which gives a rationale for explaining the little hierarchy. We will come back to this point in the next section, where we compute the Higgs mass at one loop.

The scalar on the Planck brane can be replaced by boundary conditions that break $SU(3)_L$ to $SU(2)_L$ on the Planck brane. The breaking on the TeV brane can also be realized by boundary conditions. In this case, the PGB corresponds to the fifth component of the gauge boson as we will see in section 4.

A realistic theory must have Yukawa couplings between the Higgs and quarks and leptons. It is simple to incorporate this feature in our theories. The fermions must be put in the bulk (at least one of their chiralities) in representations of $SU(3)_L$ and be coupled to the Higgs on the TeV brane. After the $SU(3)_L$ breaking on the Planck brane, we can obtain the standard-model quarks and leptons as the only massless fermions (before electroweak symmetry breaking). Large enough Yukawa couplings are obtained if the bulk fermion masses are in a certain range such that they probe the $SU(3)_L$ breaking effect on the Planck brane. These Yukawa couplings then induce a negative one-loop contribution to the Higgs mass term, which can trigger electroweak symmetry breaking.

Models with the Higgs as a PGB face, however, a significant phenomenological challenge. The Higgs quartic coupling λ_H is induced only at one-loop level, giving a physical Higgs mass, $m_h^2 = 2\lambda_H \langle H \rangle^2$, below the experimental bound of $m_h \gtrsim 114$ GeV. This is one of the major obstacles for the realization of the Higgs as a

PGB. A realistic model has to induce λ_H at tree level. The challenge is to induce this coupling without generating a tree-level mass term; otherwise the little hierarchy between the electroweak scale and the compositeness scale is lost. Below we present a possible mechanism, although, as we will explain, it requires further assumptions about the symmetry breaking physics at high energies to keep the PGB massless at tree level.

3 A Model

In this section we present an explicit model that leads to the standard model with the Higgs boson as a PGB. This represents a concrete example for the general theories introduced in the previous section. We discuss an alternative possibility in the next section, where the Higgs is identified with the extra-dimensional component of the gauge boson, A_5 .

We consider a 5D theory in a slice of AdS with a gauge symmetry $SU(3)_L \times U(1)_X$, which contains the electroweak $SU(2)_L \times U(1)_Y$ as a subgroup. The extra $U(1)_X$ is introduced to give the correct hypercharges to the fermions. All the gauge bosons are assumed to have Neumann boundary conditions at both branes (we will work in the unitary gauge $A_5 = 0$). The matter content of the model is the following (we only consider the third-generation quark sector for simplicity, but the extension to the full standard model is straightforward). We introduce two bulk fermions Q and D which transform as $\mathbf{3}_{1/3}^*$ and $\mathbf{3}_0$ under $SU(3)_L \times U(1)_X$. Since the bulk fermions Ψ are in the Dirac representation, they are decomposed into the left-handed, Ψ_L , and right-handed, Ψ_R , components in terms of the 4D irreducible representation (Weyl fermion): $Q = Q_L + Q_R$ and $D = D_L + D_R$. The boundary conditions for these fields are chosen such that Q_L and D_R (Q_R and D_L) obey Neumann (Dirichlet) boundary conditions at the Planck and TeV branes. This implies that only Q_L and D_R have massless zero modes. We also introduce the boundary fermion U_R on the TeV brane, which transforms as $\mathbf{1}_{2/3}$ under $SU(3)_L \times U(1)_X$.

We assume that $SU(3)_L \times U(1)_X$ is broken on the Planck brane to the standard-model group $SU(2)_L \times U(1)_Y$, with $Y = T_8/\sqrt{3} + X$. This breaking can be caused, for example, by a scalar S on the Planck brane with quantum numbers $\mathbf{3}_{1/3}$ under $SU(3)_L \times U(1)_X$. The bulk fermion fields are decomposed under the standard-model group as

$$Q(\mathbf{3}_{1/3}^*) = Q_L^{(D)}(\mathbf{2}_{1/6}^*) + Q_L^{(S)}(\mathbf{1}_{2/3}) + Q_R^{(D)}(\mathbf{2}_{1/6}^*) + Q_R^{(S)}(\mathbf{1}_{2/3}),$$

$$D(\mathbf{3}_0) = D_L^{(D)}(\mathbf{2}_{1/6}) + D_L^{(S)}(\mathbf{1}_{-1/3}) + D_R^{(D)}(\mathbf{2}_{1/6}) + D_R^{(S)}(\mathbf{1}_{-1/3}).$$

We assume that the symmetry breaking dynamics at the Planck brane is such that only $Q_L^{(D)}$ and $D_R^{(S)}$ are left as 4D massless fields, and we identify these fields with

the standard-model doublet quark q_L and singlet bottom quark b_R .¹ The standard-model singlet top quark, t_R , is identified with the brane field U_R . When the Yukawa couplings on the TeV brane are included, however, q_L will be a mixture of $Q_L^{(D)}$ and $D_L^{(D)}$ states, while t_R will be a mixture of U_R and $Q_R^{(S)}$ states, as we will see below. The color $SU(3)_C$ can be introduced in a straightforward way: Q , D and U all transform as $\mathbf{3}$ of $SU(3)_C$.

What about the Higgs field? To have the Higgs as a PGB, we introduce a scalar field Σ on the TeV brane transforming as $\mathbf{3}_{1/3}$ under $SU(3)_L \times U(1)_X$. We assume that Σ gets a VEV, say by the potential $\mathcal{L} = \delta(z - L_1) \sqrt{-g_{\text{ind}}} (\Sigma^\dagger \Sigma - \tilde{v}^2)^2$ where g_{ind} is the induced metric on the brane. We can then parametrize the scalar field Σ as

$$\Sigma(\mathbf{3}_{1/3}) = e^{iT^a G^a} \begin{pmatrix} 0 \\ 0 \\ \tilde{v} + \varphi \end{pmatrix}, \quad T^a G^a = \frac{1}{\tilde{v}} \begin{pmatrix} 0 & H \\ H^\dagger & \eta \end{pmatrix}, \quad (2)$$

where the fields H and η are PGBs that transform respectively as doublet and singlet of $SU(2)_L$, and φ is a real scalar field; \tilde{v} is the VEV of Σ in terms of the 5D metric. We also define, for later convenience, the VEV in terms of the 4D metric $v \equiv \tilde{v} L_0 / L_1$, which takes a value of the order of the local cutoff on the TeV brane, $\Lambda_{\text{IR}} \sim \text{TeV}$. The unbroken group under the Σ VEV can naturally be aligned with that under the S VEV, due to possible radiative effects relating them. We then find that the PGB field H has the appropriate $SU(2)_L \times U(1)_Y$ quantum numbers, $\mathbf{2}_{1/2}$, to be identified as the standard-model Higgs boson.

We now proceed to the interaction terms of the theory. The 5D theory has mass terms for the bulk fermions:

$$\mathcal{L} = \sqrt{g} [\mathcal{L}_{\text{kin}} - M_Q \bar{Q}_L Q_R - M_D \bar{D}_L D_R], \quad (3)$$

where \mathcal{L}_{kin} are the kinetic terms (see Eq. (37) for the explicit expression). These mass terms control the shape of the wavefunctions of the Q_L and D_R zero modes, and hence the size of the various low-energy 4D couplings. In addition, we introduce the Yukawa couplings for the matter fields on the TeV brane:

$$\mathcal{L} = \delta(z - L_1) \sqrt{-g_{\text{ind}}} [\lambda_U \Sigma^\dagger \bar{Q}_L U_R + \lambda_D \Sigma^i \bar{Q}_L^i D_R^k \epsilon^{ijk} + \text{h.c.}], \quad (4)$$

where i, j, k represent the $SU(3)_L$ index. After integrating out the Kaluza-Klein (KK) states, we obtain the following effective Lagrangian:

$$\begin{aligned} \mathcal{L}_{4\text{D}} = & Z_H |D_\mu H|^2 + i Z_Q \bar{q}_L \not{\partial} q_L + i \bar{b}_R \not{\partial} b_R + i Z_U \bar{t}_R \not{\partial} t_R \\ & - i \lambda_U f_Q H^\dagger \bar{q}_L t_R + i \lambda_D f_Q f_D H \bar{q}_L b_R + \text{h.c.} \end{aligned} \quad (5)$$

¹This situation can be realized, for example, by adding the following couplings on the Planck brane: $S^\dagger \bar{Q}_L U'_R + M \bar{D}_R D'_L + S \bar{D}'_L D''_R$, where $U'_R(\mathbf{1}_2/3)$, $D'_L(\mathbf{3}_0)$ and $D''_R(\mathbf{1}_{-1/3})$ are extra quarks that marry with the unwanted $SU(3)_L$ partners of the zero modes of $Q_L^{(D)}$ and $D_R^{(S)}$. This symmetry breaking pattern can be more easily obtained if we choose to break $SU(3)_L$ through boundary conditions. The details of this breaking, however, are not relevant for physics at the TeV scale.

Here, f_Q (f_D) denotes the value of the zero-mode wavefunction of $Q_L^{(D)}$ ($D_R^{(S)}$) at the TeV brane; one has

$$f_i = \frac{1}{N_0} \left(\frac{L_1}{L_0} \right)^{(1/2 \mp c_i)}, \quad N_0^2 = \frac{L_0}{(1 \mp 2c_i)} \left[\left(\frac{L_1}{L_0} \right)^{(1 \mp 2c_i)} - 1 \right], \quad (6)$$

where the \mp sign holds for the zero mode of a left (right) handed field and $c_i = M_i/k$ for $i = Q, D$. The factors Z_H , Z_Q and Z_U arise respectively due to the mixing of H with the heavy KK gauge bosons, the mixing of the zero mode of $Q_L^{(D)}$ with the KK states of $D_R^{(D)}$, and the mixing of U_R with the KK states of $Q_L^{(S)}$. These mixings appear when Σ gets a VEV, and we find

$$Z_H^{-1} = 1 + \frac{(g_5 \tilde{v})^2}{4k}, \quad Z_Q = 1 + |f_Q \lambda_D \tilde{v}|^2 G_{D_R^{(D)}}, \quad Z_U = 1 + |\lambda_U \tilde{v}|^2 G_{Q_L^{(S)}}. \quad (7)$$

Here, $G_{Q_L^{(S)}}$ ($G_{D_R^{(D)}}$) is the 5D propagator of $Q_L^{(S)}$ ($D_R^{(D)}$) evaluated on the TeV brane at zero 4D momentum: $G_{Q_L^{(S)}} = \hat{G}_L^{(-,+)}(L_1, L_1; 0)$ and $G_{D_R^{(D)}} = \hat{G}_R^{(-,+)}(L_1, L_1; 0)$, where the propagators $\hat{G}_{L,R}^{(-,+)}(z, z'; p)$ can be found in the Appendix. After canonically normalizing the fields we obtain the top and bottom Yukawa couplings

$$h_t = -\frac{i\lambda_U f_Q}{\sqrt{Z_H Z_Q Z_U}}, \quad h_b = \frac{i\lambda_D f_Q f_D}{\sqrt{Z_H Z_Q}}. \quad (8)$$

The Yukawa couplings strongly depend on M_Q and M_D . For example, the dependence of the top Yukawa coupling on M_Q is given by

$$h_t \sim \begin{cases} (L_0/L_1)^{|c_Q|-1/2} & \text{for } |c_Q| > 1/2, \\ [\log(L_1/L_0)]^{-1/2} & \text{for } |c_Q| = 1/2, \\ \mathcal{O}(1) & \text{for } |c_Q| < 1/2, \end{cases} \quad (9)$$

and therefore the theory can be realistic only if $|M_Q| \lesssim k/2$. It can also be shown that $|M_D| \lesssim k/2$ is also needed to obtain realistic Yukawa couplings.

The holographic picture offers a simple explanation for the behavior of Eq. (9). For $c_Q > 1/2$ the holographic theory consists of a CFT sector coupled to a left-handed dynamical ‘‘source’’ χ_L which transforms as a doublet of $SU(2)_L$ [9]:

$$\mathcal{L} = \mathcal{L}_{\text{CFT}} + i\bar{\chi}_L \not{\partial} \chi_L + \lambda k^{1/2-c_Q} \bar{\chi}_L \cdot \mathcal{O}_R + \text{h.c.} + \dots \quad (10)$$

Here, λ is a dimensionless coupling, and the ellipses stand for higher order operators (M_{Pl} -suppressed), and gauge and bottom interactions. The coupling $\bar{\chi}_L \cdot \mathcal{O}_R$ induces a tree-level mixing between the elementary source and the CFT bound states; its strength is determined by the AdS/CFT correspondence, which relates the dimension

of \mathcal{O}_R with the value of c_Q : $\dim[\mathcal{O}_R] = 3/2 + |c_Q + 1/2|$. For $c_Q > 1/2$ the coupling is always irrelevant, so that the physical massless eigenstate q_L , to be identified with the standard-model quark doublet, is almost the elementary state χ_L . The quark singlet t_R appears in the theory as a composite CFT state. For $c_Q < -1/2$ the holographic picture is very different. We find that the 4D theory must contain a right-handed dynamical source χ_R which is a singlet of $SU(2)_L$ [9]:

$$\mathcal{L} = \mathcal{L}_{\text{CFT}} + i\bar{\chi}_R \not{\partial} \chi_R + \lambda k^{1/2+c_Q} \bar{\chi}_R \cdot \mathcal{O}_L + \text{h.c.} + \dots \quad (11)$$

The AdS/CFT correspondence requires $\dim[\mathcal{O}_L] = 3/2 + |c_Q - 1/2|$, and the coupling between the elementary source and the CFT is again irrelevant for $c_Q < -1/2$. The physical q_L appears now as a composite CFT state, while t_R is almost the elementary state χ_R .

How does the Yukawa coupling between the composite Higgs H and the physical quarks q_L and t_R arise? As is clear from Eqs. (10) and (11), the global $SU(3)_L$ invariance of the CFT is not a symmetry of the elementary sector, and the explicit breaking is communicated to the conformal sector through its coupling with the source χ_L or χ_R . The Yukawa coupling is then generated only through the insertion of the composite operators $\mathcal{O}_{L,R}$, since the process must involve the elementary source. This implies that the top Yukawa coupling must be proportional to $\lambda k^{1/2-|c_Q|}$, and therefore $h_t \sim \lambda (L_1/L_0)^{1/2-|c_Q|}$ as obtained in the 5D picture, Eq. (9).

For $-1/2 \leq c_Q \leq 1/2$ either of the two descriptions, Eq. (10) or Eq. (11), is valid. In this case the coupling of the CFT to the elementary sector is relevant (marginal for $c_Q = \pm 1/2$). This implies that the both physical quarks q_L and t_R are almost composite states. The only way to generate the top Yukawa is then through the virtual exchange of the source. The source has now a relevant coupling with the CFT, so that the CFT correction to the elementary propagator becomes important. By resumming the infinite series of CFT insertions, one can express this correction as a renormalization of the coupling λ ; for example, in the holographic picture of Eq. (10), the conformal invariance gives

$$\lambda^2(\mu) \sim \frac{\lambda^2(k)}{1 + \lambda^2(k) (\mu^2/k^2)^{c_Q-1/2}}. \quad (12)$$

Therefore, for $|c_Q| < 1/2$ and $\mu \ll k$, one has $\lambda(\mu) \sim (\mu/k)^{1/2-|c_Q|}$, so that the top Yukawa coupling, proportional to $\lambda k^{1/2-|c_Q|}$, is always of order one. The particular case $|c_Q| = 1/2$ gives a logarithmic suppression $h_t \propto [\ln(L_1/L_0)]^{-1/2}$.

Summarizing, we have obtained an $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory with a massless Higgs boson, H , a massless quark doublet, q_L , and two massless singlet quarks, t_R and b_R , which have the Yukawa couplings of Eq. (8).² The fermion content

²In the model presented here, there is also an extra singlet PGB η , which obtains a mass at one-loop level.



Figure 1: *One-loop corrections to the PGB mass in the holographic theory from the gauge field (a) and an elementary fermion (b). If the coupling of the source χ with the conformal sector is relevant, then the fermion propagator in the diagram (b) must be intended as corrected by an infinite series of CFT insertions.*

thus reproduces the third generation quark sector of the standard model. To complete the standard-model structure, however, we have to discuss the Higgs potential. We address this remaining issue in the next two subsections.

3.1 One-loop contribution to the Higgs potential

At tree level, the Higgs potential vanishes due to the $SU(3)_L$ symmetry on the TeV brane. At one-loop level, however, an effective potential for H will be induced. In the holographic picture this comes from the interactions between the CFT and the elementary fields, which explicitly break the global $SU(3)_L$ symmetry. At one loop the relevant diagrams for the Higgs mass term are those sketched in Fig. 1. The situation is quite similar to QCD, where the charged pion gets a mass at one loop due to the fact that the coupling to the photon explicitly breaks the global chiral symmetry. In the 5D picture the effect comes from loops of bulk fields that propagate from the TeV brane, where H lives, to the Planck brane, where we have the breaking of $SU(3)_L$. This is a non-local effect and therefore is finite. The one-loop effective potential is similar to that calculated in Ref. [10].

The gauge contribution to the Higgs potential arises from loops of $SU(3)_L$ gauge bosons. This is given by (the $U(1)_X$ contribution can be obtained in a similar way)

$$\begin{aligned} \sqrt{-g_{\text{ind}}} V_{\text{gauge}}(\Sigma) &= \frac{3}{2} \sum_{n=1}^{\infty} \text{Tr} \int_0^{\infty} \frac{dp}{8\pi^2} p^3 \frac{(-1)^{n+1}}{n} \left[\mathcal{G} \cdot \mathcal{M}^2(\Sigma) \right]^n \\ &= \frac{3}{2} \text{Tr} \int_0^{\infty} \frac{dp}{8\pi^2} p^3 \ln \left[1 + \mathcal{G} \cdot \mathcal{M}^2(\Sigma) \right]. \end{aligned} \quad (13)$$

Here, \mathcal{M}^2 is the boundary squared-mass matrix of the gauge boson fields in the background Σ :

$$\mathcal{M}_{ab}^2(\Sigma) = 2g_5^2 \Sigma^\dagger T_a T_b \Sigma, \quad (14)$$

and \mathcal{G} is a matrix propagator $\mathcal{G}_{ab} = G_a(p)\delta_{ab}$, where $G_a(p)$ are the propagators of the $SU(3)_L$ gauge boson ($a = 1, \dots, 8$) evaluated on the TeV brane with 4D momentum

p : $G_a(p) = \hat{G}(L_1, L_1; p)$ where $\hat{G}(z, z'; p)$ is the rescaled gauge boson propagator given in the Appendix with $m_1 = 0$ and $m_0 \sim M_{\text{Pl}} (m_0 = 0)$ for the gauge bosons of the broken (unbroken) generators on the Planck brane. The effective potential of Eq. (13) has the Coleman-Weinberg potential form, with the only difference that 4D propagators have been replaced by 5D propagators. Plugging Eq. (2) into Eq. (13), we can obtain the gauge contribution to the effective potential of H . In particular, the mass of H for $vL_1 \ll 1$ is given by

$$m_H^2 = \frac{9g_5^2}{4} \left(\frac{L_1}{L_0} \right)^2 \int_0^\infty \frac{dp}{8\pi^2} p^3 \left[G_{\text{III}}(p) - \frac{2}{3}G_{\text{II}}(p) - \frac{1}{3}G_{\text{I}}(p) \right], \quad (15)$$

where $G_{\text{III,II,I}}(p)$ are the $SU(2)_L$ triplet, doublet and singlet components of the $SU(3)_L$ gauge boson propagators. The integrand in Eq. (15) reaches its maximum at $p \sim 1/L_1$ and is exponentially suppressed for $p > 1/L_1$. Therefore, m_H^2 is finite and very insensitive to physics at energies above $1/L_1$. Notice that, contrary to supersymmetry, the cancellation of quadratic divergences in Eq. (15) is due to particles of equal spin. Eq. (15) yields

$$m_H^2 \simeq \left(\frac{0.12}{L_1} \right)^2. \quad (16)$$

Hence, the PGB mass turns out to be an order of magnitude smaller than the scale $1/L_1$, which is also smaller than Λ_{IR} . This mass gap is even larger if m_H is compared with the mass of the first KK state, $\Lambda_{\text{NP}} \approx \pi/L_1$. The contribution of Eq. (16) is positive and cannot trigger electroweak symmetry breaking by itself.

The top contribution to the effective potential is given by

$$\sqrt{-g_{\text{ind}}} V_{\text{top}}(\Sigma) = -6 \int_0^\infty \frac{dp}{8\pi^2} p^3 \ln \left[1 + G_i(p) m_i^2(\Sigma) \right], \quad (17)$$

where m_i^2 is the boundary squared-mass of $Q_L^{(i)}$ in the background Σ :

$$m_i^2(\Sigma) = |\lambda_U|^2 \Sigma_i^\dagger \Sigma_i, \quad (18)$$

and $G_i(p)$ are the propagators of $Q_L^{(i)}$ ($i = D, S$) evaluated on the TeV brane at 4D momentum p . These are $G_D(p) = \hat{G}_L^{(+,+)}(L_1, L_1; p)$ and $G_S(p) = \hat{G}_L^{(-,+)}(L_1, L_1; p)$, where $\hat{G}_L^{(\pm,+)}(z, z'; p)$ can be found in the Appendix. This gives a contribution to the Higgs mass of order

$$m_H^2 \sim -\frac{h_i^2}{\pi^2} \frac{1}{L_1^2}, \quad (19)$$

where the exact value depends on M_Q . The top contribution is negative and, for certain values of M_Q , is larger than the gauge contribution. The top contribution can then be responsible for the breaking of the electroweak symmetry.

Despite the occurrence of electroweak symmetry breaking, the one-loop effective potential calculated above cannot by itself lead to a realistic scenario of electroweak symmetry breaking. A tree-level Higgs quartic coupling is necessary to obtain a large enough physical Higgs mass and to generate a Higgs VEV a loop factor smaller than $1/L_1$. In the next subsection we will provide a mechanism to generate a Higgs quartic coupling, which together with the one-loop gauge and top contributions can lead to a realistic theory of electroweak symmetry breaking.

Before concluding this subsection, it is interesting to analyze the absence of quadratic divergences in this class of models from a 4D perspective. In the standard model, the dominant contribution to the Higgs mass term comes from the loop of the top quark and it is quadratically divergent. The cancellation of this divergence then must arise from a loop of some extra fields. This is indeed what happens in our case. The top quark is realized as the zero mode of a bulk fermion and it corresponds, in the holographic picture, to a mixture between the elementary source and the CFT bound states. The physical spectrum then consists of a massless top quark, plus a series of CFT bound states that form a complete multiplet of the global $SU(3)_L$. It is the contribution of this tower of additional states that cancels the divergence of the top loop. This is, therefore, a cancellation involving an infinite numbers of 4D fields.

The picture described above, however, is not quite illuminating to understand the finiteness of the top contribution to the Higgs mass. To understand it better, we can perform a change of basis going from the mass eigenstate basis to the “interaction basis”, where we separate an elementary source field from the tower of composite CFT states (the physical top quark is a mixture of these states). The whole contribution to the Higgs mass then arises from an exchange of the elementary field (Fig. 1), because only the elementary sector feels the explicit breaking directly. Now comes the most important point. Since the elementary field couples only linearly to the CFT sector, the correction to the Higgs mass *cannot* proceed through a loop of elementary modes directly coupled to the Higgs, but it must necessarily involve the strong CFT dynamics. Therefore, a large momentum circulating in the top loop always flows into the CFT cloud, and consequently the resulting Higgs mass always involves a form factor $F(q^2)$ which encodes the non-perturbative effects of the CFT. Since $F(q^2)$ is suppressed for q larger than the compositeness scale, the loop momentum integral is cut off above that scale. This is the reason why the quadratic divergences are absent in our theory. In this picture, no cancellation between different divergent contributions is necessary. The Higgs mass correction is finite simply because of the form factor suppressing the contribution from large virtual momenta: at high energies the constituents of the composite Higgs become transparent to the short wavelength probe of the elementary fermion, so that $F(q^2) \rightarrow 0$ for $q^2 \rightarrow \infty$. As the explicit computation reveals, the damping is exponential. This strong damping can be understood by recalling that in the 5D theory the mass correction arises as a finite non-local effect. As such, it involves a brane to brane propagation, and that explains

the exponential suppression for internal momenta larger than $\sim 1/L_1$.

A similar scenario, which does not have quadratic divergences for the Higgs, was considered in Ref. [10]. In that model too, the Higgs is a bound state of the CFT, and its tree-level mass vanishes since the conformal sector is invariant under a global supersymmetry. This invariance is explicitly broken by the interactions with the elementary sector, so that the Higgs is expected to acquire a mass at one loop. The 5D realization of this scenario consists of a warped supersymmetric setup, where matter and gauge fields propagate in the bulk of AdS and the Higgs field is localized on the TeV brane. If supersymmetry breaking is triggered on the Planck brane, the correction to the Higgs mass will be a finite non-local effect. The corresponding holographic description is almost the same as in our case: the Higgs mass is generated through the exchange of some elementary field and it is finite because the CFT strong dynamics exponentially suppresses contributions from large virtual momenta.

3.2 A mechanism for the quartic coupling

Although the Higgs potential is generated by radiative corrections, it is not sufficient to guarantee a successful phenomenology. We need a large $\sim O(1)$ quartic coupling while keeping the quadratic term smaller than the effective cutoff scale, *i.e.*, the scale that suppresses higher dimensional terms in the Higgs potential. Here we present an example of dynamics providing such a situation.

The basic idea is simpler to understand in the 4D CFT picture. The explicit breaking of $SU(3)_L$ comes from the elementary sector. Hence, in order to generate a quartic coupling at tree level, the Higgs must mix with some elementary scalar φ . Since the mass of the elementary scalar is not protected by any symmetry, it is expected to be of order the cutoff scale $\approx k$. This implies that, if we want to generate an unsuppressed $SU(3)_L$ breaking effect in the Higgs sector, the elementary scalar φ must be coupled to the CFT with a coupling linear in k : $\mathcal{L}_{\text{int}} = \lambda k \varphi \cdot \mathcal{O}_\varphi$, *i.e.*, the operator \mathcal{O}_φ must have dimension 2. If $\langle 0 | \mathcal{O}_\varphi \mathcal{O}_\Sigma^2 | 0 \rangle \neq 0$, where \mathcal{O}_Σ is an operator that creates Σ , the Higgs will have non-trivial interactions with the scalar φ , and an explicit breaking of $SU(3)_L$ will appear in the Higgs potential at tree level. For example, if φ is a **6** of $SU(3)_L$, which decomposes under $SU(2)_L$ as a triplet (φ_T), a doublet (φ_D) and a singlet (φ_S), a Higgs quartic coupling is generated from the diagrams of Fig. 2. We are assuming here that the different $SU(2)_L$ components of φ have different masses due to the $SU(3)_L$ breaking in the elementary sector; otherwise the quartic coupling is zero because of the non-linearly realized $SU(3)_L$ symmetry. In this theory, however, there is the danger of also generating a quadratic term for H . This comes from the diagrams of Fig. 3. A way to avoid the generation of a Higgs squared-mass is to assume that the breaking of $SU(3)_L$ in the elementary φ affects only φ_T and not the other $SU(2)_L$ components, φ_D and φ_S . It is clear that in this

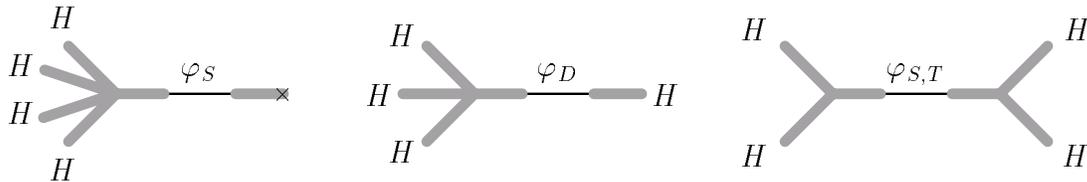


Figure 2: *Contributions to the Higgs quartic coupling. The CFT dynamics is represented as a thick gray line, while a thin black line represents the propagator of the elementary scalar φ . A cross \times indicates an $SU(3)_L$ breaking by the CFT.*



Figure 3: *Contributions to the Higgs mass. The CFT dynamics is represented as a thick gray line, while a thin black line represents the propagator of the elementary scalar φ . A cross \times indicates an $SU(3)_L$ breaking by the CFT.*

case, the diagrams of Fig. 3 respect $SU(3)_L$ and vanish. Therefore, a mechanism of this type must require a certain pattern of $SU(3)_L$ breaking in the elementary φ sector.³

Let us see how this idea is implemented in the 5D AdS picture. By AdS/CFT, a scalar operator of dimension 2 corresponds to a bulk scalar Φ of squared-mass $M_\Phi^2 = -4k^2$. Note that as long as $M_\Phi \geq -4k^2$ (and certain conditions for the brane masses are met), Φ is not tachyonic and does not develop a VEV. The 5D scalar Φ is responsible for communicating the $SU(3)_L$ breaking *on the Planck brane* to the TeV brane where the Higgs lives.⁴ The field Φ must be coupled to the Higgs on the TeV brane. We thus require Φ to transform as $\mathbf{6}_{2/3}$ under $SU(3)_L \times U(1)_X$ and have the following coupling:

$$\mathcal{L} = \delta(z - L_1) \sqrt{-g_{\text{ind}}} [\lambda_\Phi \Sigma \Phi^\dagger \Sigma + \text{h.c.}] , \quad (20)$$

where λ_Φ is a coupling of mass dimensions 1/2. A small deviation from $M_\Phi^2 = -4k^2$, as arising for example from the one-loop correction to the bulk mass of Φ , will not spoil our mechanism.

By integrating out the Φ field, we find a tree-level Higgs potential in the low-

³We could have a Higgs quartic coupling without quadratic term if only φ_T is present. Nevertheless, in this case the quartic coupling turns out to be negative.

⁴Another possibility, which could avoid introducing Φ , is to have a Higgs with a profile in the extra dimension.

energy effective theory,

$$V_{H,\text{tree}} = m_H^2 |H|^2 + \frac{\lambda_H}{2} |H|^4. \quad (21)$$

m_H^2 and λ_H are generated by 5D diagrams similar to those in Fig. 3 and Fig. 2 respectively, where the internal propagators now represent the 5D field Φ with the appropriate $SU(2)_L$ quantum numbers. We find

$$m_H^2 = 2\lambda_\Phi^2 v^2 \left[\hat{G}_S - \hat{G}_D \right], \quad (22)$$

$$\lambda_H = \frac{2}{3}\lambda_\Phi^2 \left[8\hat{G}_D - 5\hat{G}_S - 3\hat{G}_T \right], \quad (23)$$

where \hat{G}_S , \hat{G}_D and \hat{G}_T are the rescaled propagators of the $SU(2)_L$ singlet, doublet and triplet components of Φ with end points on the TeV brane, evaluated at zero 4D momentum. The explicit form of these propagators can be found in the Appendix. For $M_\Phi^2 \simeq -4k^2$, which we assume here, they are given by

$$\hat{G}_a \simeq \frac{1}{r_a} \left[1 + (m_{0,a} L_0 - 2) \ln \left(\frac{L_1}{L_0} \right) \right], \quad (24)$$

where $r_a \equiv m_{0,a} + m_1 + L_0(m_{0,a} - 2L_0^{-1})(m_1 + 2L_0^{-1}) \ln(L_1/L_0)$ and $a = S, D, T$. We introduced a common scalar mass m_1 on the TeV brane for the $SU(2)_L$ singlet, doublet and triplet components of Φ field, but allowed different masses $m_{0,a}$ on the Planck brane. The values of the masses $m_{0,a}$ are determined by the high-energy $SU(3)_L$ -breaking dynamics on the Planck brane. Here we do not specify a particular pattern for this symmetry breaking, but rather we look for the parameter region of $m_{0,a}$ (and m_1) where the successful Higgs phenomenology is obtained.

Inserting Eq. (24) into Eq. (22), we obtain

$$m_H^2 \simeq \frac{2\lambda_\Phi^2}{r_D r_S} (m_{0,D} - m_{0,S}) v^2. \quad (25)$$

We see that, as expected, if the doublet and singlet components of Φ do not feel the breaking of $SU(3)_L$ on the Planck brane, $m_{0,D} = m_{0,S}$, the resulting Higgs squared-mass parameter is zero. Assuming this, the quartic coupling, Eq. (23), is given by

$$\lambda_H \simeq \frac{2\lambda_\Phi^2}{r_T r_S} (m_{0,T} - m_{0,S}). \quad (26)$$

Therefore, for $m_{0,T} \gtrsim m_{0,S}$, we obtain a sufficiently large Higgs quartic coupling.

We have seen that the desirable Higgs potential is obtained for $m_{0,T} \gtrsim m_{0,S} \simeq m_{0,D}$.⁵ Although this may appear a rather ad hoc hypothesis, we stress that it is

⁵An alternative parameter region is $m_{0,D}, m_{0,S} \lesssim k \lesssim m_{0,T}, m_1$, in which case the tree-level Higgs potential is given by $m_H^2 \simeq \lambda_\Phi^2 (m_{0,D} - m_{0,S}) (2 \ln(L_1/L_0)^2 m_1^2)^{-1} v^2$ and $\lambda_H \simeq 2\lambda_\Phi^2 m_{0,T} / (m_1 r_T)$ so that we can have $m_H^2 \ll v^2$, $\lambda_H \sim 1$.

an assumption about the underlying physics at the Planck scale which is responsible for the symmetry breaking. We do not explicitly address this sector here, but we expect that there are some mechanisms realizing (approximately) the situation discussed above. We must say, however, that even if $m_{0,S} = m_{0,D}$ at tree level due to some specific $SU(3)_L$ breaking pattern on the Planck brane, quantum effects (coming, for example, from gauge interactions) will modify this relation. Therefore, a Higgs squared-mass will be induced from Eq. (25), at least, at the quantum level. This contribution is difficult to calculate since it depends on the Planck-brane fields that break $SU(3)_L$, but it can be estimated to be one-loop factor smaller than v^2 . We can then conclude that the Higgs potential Eq. (21) together with the one-loop contributions of Eqs. (13) and (17) can lead to a successful electroweak symmetry breaking with a VEV for the Higgs a loop factor smaller than $1/L_1$, and a physical Higgs mass larger than the experimental bound. The precise determination of $\langle H \rangle$ and m_h is, however, not possible here due to the dependence of λ_H on the unknown free parameters of the model.

4 Pseudo-Goldstone Bosons as Holograms of A_5

In this section we consider the possibility of breaking the gauge symmetry in the warped extra dimension by boundary conditions, and not by scalar fields on the two branes. We will show that the holographic picture is almost the same as before and that the holographic PGB in this case corresponds in the 5D theory to the zero mode of the fifth component of the gauge boson, A_5 . We note that the model presented in the previous section and the one presented here must coincide in the limit $v \gg \Lambda_{\text{IR}}$, since the breaking of $SU(3)_L$ by the scalar Σ is equivalent to a breaking by boundary conditions in the limit $v \rightarrow \infty$ [11].

Let us consider the most general case of a bulk gauge group G reduced to the subgroups H_0 and H_1 on the Planck and TeV branes, respectively. This corresponds to assigning the following boundary conditions to the gauge bosons at the Planck and TeV branes:

$$\begin{aligned}
A_\mu^a (+, +) & \quad T^a \in \text{Alg}\{H\}, \\
A_\mu^{\bar{a}} (+, -) & \quad T^{\bar{a}} \in \text{Alg}\{H_0/H\}, \\
A_\mu^{\hat{a}} (-, +) & \quad T^{\hat{a}} \in \text{Alg}\{H_1/H\}, \\
A_\mu^{\hat{a}} (-, -) & \quad .
\end{aligned} \tag{27}$$

Here, by $+$ ($-$) we denote the Neumann (Dirichlet) boundary condition, and $H = H_0 \cap H_1$. The A_5 's have the opposite boundary conditions to those of the corresponding A_μ 's.

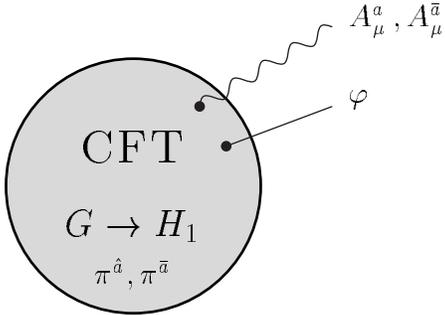


Figure 4: *The holographic theory consists of a CFT interacting with an elementary sector represented here by the gauge fields $A_\mu^a, A_\mu^{\bar{a}}$ and a generic field φ . The Goldstone bosons $\pi^{\hat{a}}$ are eaten by the gauge fields $A_\mu^{\bar{a}}$ to form massive vectors; the remaining $\pi^{\hat{a}}$ are PGBs.*

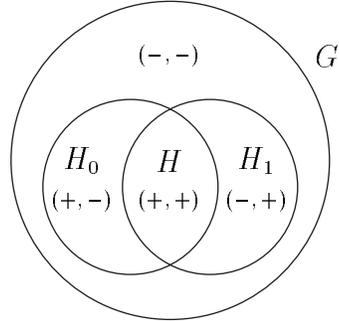


Figure 5: *The pattern of symmetry breaking.*

The holographic 4D theory consists of a CFT sector whose global invariance G is spontaneously broken down to H_1 by strong dynamics, with an order parameter of $\mathcal{O}(\text{TeV})$. External gauge fields weakly gauge the subgroup H_0 of G :

$$\mathcal{L} = \mathcal{L}_{\text{CFT}} - \frac{1}{4g^2} (F_{\mu\nu}^\alpha)^2 + A_\mu^\alpha J^{\mu\alpha}, \quad \alpha = a, \bar{a}. \quad (28)$$

This situation is somewhat different from the model of section 3, where the whole G was gauged in the 4D theory and Higgsed down to H_0 at high energies. The two scenarios, however, are indistinguishable from low-energy observers. The gauging of only a subgroup of the global symmetry G is experienced by the CFT as an explicit breaking of G .

Let us count the number of PGBs present in the theory. The spontaneous breaking in the CFT sector delivers $n = \dim(G/H_1)$ Goldstone bosons. However, the gauging of H_0 , Eq. (28), makes part of them, $m = \dim(H_0/H)$, being eaten by the gauge bosons $A_\mu^{\bar{a}}$. The remaining $n - m$ are PGBs; they are massless at tree level, but they acquire masses from radiative corrections due to the explicit breaking of G by the interaction terms of Eq. (28). Only the gauge bosons associated to the symmetry subgroup $H = H_0 \cap H_1$ are exactly massless. Figs. 4 and 5 give a pictorial representation of the holographic theory and of the symmetry breaking pattern.

In general, any interaction with the elementary sector can communicate the explicit breaking of G to the CFT sector. The AdS/CFT correspondence prescribes that adding a generic field in the bulk of AdS with Neumann boundary conditions on the Planck brane corresponds to modifying the CFT content and adding some elementary 4D field φ which couples to the conformal sector through a coupling $\varphi \cdot \mathcal{O}_\varphi$

(see Fig. 4). Since φ will come in a representation of the group H_0 (rather than G), which is the symmetry of the elementary sector, the coupling $\varphi \cdot \mathcal{O}_\varphi$ is not G -invariant. As the only source of explicit breaking is represented by those interactions, this necessarily implies that the mass terms for the PGBs are generated only through processes where elementary fields are exchanged (Fig. 1).

Since the 4D holographic picture and the 5D AdS setup describe the same physics, they must exhibit the same physical spectrum. Therefore, there must be $n - m$ massless scalars in the 5D theory after KK reduction. We now see in detail how these massless scalars appear. The 5D gauge Lagrangian is given by

$$\mathcal{L}_{\text{gauge}} = \sqrt{g} \left[-\frac{1}{4g_5^2} g^{KM} g^{LN} F_{KL} F_{MN} + \mathcal{L}_{\text{GF}} \right], \quad (29)$$

where the metric g_{MN} is that of Eq. (1) and \mathcal{L}_{GF} is the gauge-fixing term. A convenient choice for \mathcal{L}_{GF} is

$$\mathcal{L}_{\text{GF}} = -\frac{1}{2\xi g_5^2} \left[g^{\mu\nu} \partial_\mu A_\nu + z \xi g^{55} \partial_z (A_5/z) \right]^2, \quad (30)$$

with which all mixing terms between A_μ and A_5 cancel [12]. Eq. (29) can be written as

$$\begin{aligned} \mathcal{L}_{\text{gauge}} = \frac{1}{2g_5^2 k z} \left\{ A_\mu \left[\eta^{\mu\nu} \partial_\rho \partial^\rho - (1 - 1/\xi) \partial^\mu \partial^\nu \right] A_\nu + (\partial_z A_\nu)^2 \right. \\ \left. + (\partial_\mu A_5)^2 - \xi z^2 \left(\partial_z \frac{A_5}{z} \right)^2 \right\} + \dots, \end{aligned} \quad (31)$$

where the ellipses stand for interaction terms; 4D indices are raised/lowered with $\eta_{\mu\nu}$. We now perform a KK reduction. We are interested only in the massless scalar spectrum. This comes from the zero modes of the fifth components of the gauge bosons, $A_5(x, z) = f_0(z) A_5^{(0)}(x) + \dots$, where $f_0(z)$ satisfies

$$\partial_z \left(\frac{f_0(z)}{z} \right) = 0. \quad (32)$$

The solution to this equation is only compatible with $(+, +)$ boundary conditions, in which case we obtain

$$f_0(z) = \frac{z}{N_0}, \quad \text{where } N_0 = \sqrt{\frac{L_1^2 - L_0^2}{2}}. \quad (33)$$

The components of A_5 with $(+, +)$ boundary conditions are $A_5^\hat{a}$. There are $\dim[G/H_1] - \dim[H_0/H]$ of them, as expected from the 4D dual picture. Therefore the massless modes of $A_5^\hat{a}$ must correspond to the PGBs of the 4D CFT [13]. A further check of

this correspondence comes from their wavefunction. It is peaked towards the TeV brane as expected if the holographic PGBs are really bound states of the CFT. The excited KK modes of A_5 can be eliminated from the spectrum by going to the unitary gauge $\xi = \infty$.

It is clear from the 5D point of view that a tree-level potential for A_5 is absent, because it is forbidden by gauge invariance. An effective potential is then generated radiatively as a function of the non-local, gauge invariant Wilson line $W = \text{Tr } \mathcal{P} \exp(i \int_{L_1}^{L_0} dz A_5)$. This implies that A_5 will get a mass at one loop, which, by non-locality, is finite and cutoff insensitive. In general, the energy is minimized at a nonzero background value of A_5 , triggering spontaneous breaking of the symmetry H [14]. As in flat space, different values of the background $\mathcal{A}_5 = f_0(z) c^{\hat{a}} T^{\hat{a}}$, with $c^{\hat{a}} = \text{const.}$, define physically inequivalent vacua. This is because one cannot find a continuous gauge parameter $\theta(z) = \theta^{\hat{a}}(z) T^{\hat{a}}$ with the defined boundary conditions $(-, -)$ that eliminates \mathcal{A}_5 by the use of the gauge transformation $\mathcal{A}_5 \rightarrow U \mathcal{A}_5 U^\dagger + U \partial_z \theta U^\dagger$. Nevertheless, by relaxing one of the two boundary conditions, *e.g.* that on the Planck brane, it is possible to “gauge away” \mathcal{A}_5 by the transformation with

$$\theta(z) = - \int_{L_1}^z dz \mathcal{A}_5. \quad (34)$$

Under this gauge transformation, however, the charged bulk fields are also transformed: $\Phi(x, z) \rightarrow e^{i\theta(z)} \Phi(x, z)$. This implies that the theory with $\mathcal{A}_5 = 0$ is equivalent to that with nonzero background \mathcal{A}_5 , but only if we use the redefined bulk fields $e^{i\theta(z)} \Phi \equiv \Phi'$ instead of Φ . The Planck-brane boundary conditions of the fields Φ' are then different from those of the fields Φ , since Φ and Φ' differ at $z = L_0$ by a non-trivial gauge phase:

$$\Phi'(L_0) = e^{-i\theta(L_0)} \Phi(L_0). \quad (35)$$

This phase is the Wilson line.

The one-loop contributions to the effective potential of A_5 are easily estimated as follows. The appearance of the Wilson line in the vacuum energy requires a contribution of a bulk field that propagates from one brane (at L_0) to the other (at L_1). The energy involved in this contribution is then of the order of the inverse of the conformal distance between the branes, $E \sim 1 / \int_{L_0}^{L_1} dz \sim 1/L_1$. Therefore, the mass of A_5 is estimated to be $m_{A_5}^2 \sim g_5^2 (L_1/L_0)^2 E^4 \hat{G}(L_0, L_1; p)|_{E \sim p \sim 1/L_1}$, where $\hat{G}(z, z'; p)$ is the propagator of the bulk field given in the Appendix. It is interesting to look at the limit $L_0 \rightarrow 0$. In this case the propagators from L_0 to L_1 for the gauge boson and the graviton vanish, implying that no effective potential is induced for A_5 . We thus find that A_5 's are massless in this limit at all loop orders. Notice that, contrary to the flat space case, the zero modes of A_5 are still normalizable modes even though the extra dimension is infinite [see Eq. (33)], and thus remain in the theory as massless scalars. This is in fact what we expect from holography. In the

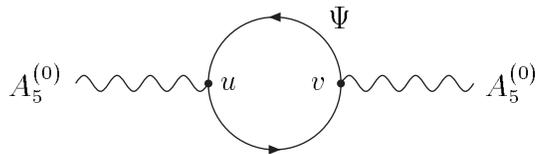


Figure 6: *One-loop correction in AdS to the A_5 zero-mode mass term.*

4D picture, the limit $L_0 \rightarrow 0$ corresponds to sending the UV cutoff to infinity. This implies that the 4D low-energy gauge coupling becomes zero (gaugeless limit), and the gauge bosons that explicitly break G decouple from the theory, making the PGBs true Goldstone bosons. Note that in the gaugeless limit the number of Goldstone bosons is $n = \dim(G/H_1)$ instead of $n - m$. In the 5D AdS the m extra massless scalars come from $A_5^{\bar{a}}$; they have $(-, +)$ boundary conditions and admit zero modes for $L_0 = 0$.

The limit $L_0 \rightarrow 0$ is subtler when other interactions are present. The point is that interactions between the CFT and the elementary sector that proceed through a *relevant* coupling are not expected to die off when the UV cutoff goes to infinity. This is the case, for example, of the interaction between an elementary chiral fermion χ and a CFT operator, $\mathcal{L}_{\text{int}} = \bar{\chi}\mathcal{O}$, for $\dim[\mathcal{O}] < 5/2$. By AdS/CFT this corresponds to a bulk fermion with mass $|M_\Psi| < k/2$. In the 5D picture the non-decoupling is evident from the fact that the brane to brane fermionic propagator does not go to zero in the limit $L_0 \rightarrow 0$ for $|M_\Psi| < k/2$.

4.1 The mass of A_5 at one loop

We present here the calculation of the mass of $A_5^{(0)}$ at one-loop level, which confirms the statements made above. We consider the simple case of Eq. (27) with $H_0 = H_1 = H$, and concentrate on the contribution from a bulk fermion with a 5D mass M_Ψ and the following boundary conditions:

$$\begin{aligned} A_\mu^a(+, +), \quad A_5^a(-, -), \\ A_\mu^{\hat{a}}(-, -), \quad A_5^{\hat{a}}(+, +), \end{aligned} \quad \Psi = \begin{bmatrix} \psi_L^i(+, +) & \psi_R^i(-, -) \\ \psi_L^{\hat{i}}(-, -) & \psi_R^{\hat{i}}(+, +) \end{bmatrix}. \quad (36)$$

The relevant diagram at one loop is depicted in Fig. 6. The contribution from other particles can be easily derived from this result.

The Lagrangian of a 5D fermion with a constant bulk mass M_Ψ is

$$\mathcal{L} = \sqrt{g} \left[\frac{i}{2} \bar{\Psi} e_A^M \Gamma^A D_M \Psi - \frac{i}{2} (D_M \Psi)^\dagger \Gamma^0 e_A^M \Gamma^A D_M \Psi - M_\Psi \bar{\Psi} \Psi \right], \quad (37)$$

with $e_A^M = kz \delta_A^M$ the inverse vielbein and $\Gamma^M = \{\gamma^\mu, -i\gamma^5\}$ the 5D Dirac matrices. The covariant derivative is

$$D_M = \partial_M + \frac{1}{8} \omega_{MAB} [\Gamma^A, \Gamma^B] - iA_M, \quad (38)$$

where the only non-vanishing entries in the spin connection ω_M^{AB} are $\omega_{\mu a5} = -\eta_{\mu a}/z$. The mass correction to the zero mode of A_5 is written as

$$m_{A_5}^2 = -(g_5^2 k) C(r) \int \frac{d^4 p}{(2\pi)^4} \int_{L_0}^{L_1} du \frac{1}{(ku)^4} \int_{L_0}^{L_1} dv \frac{1}{(kv)^4} f_0(u) f_0(v) \times \text{Tr} [\gamma^5 iS^{(+,+)}(v, u; p) \gamma^5 iS^{(-,-)}(u, v; p)], \quad (39)$$

where $C(r)$ is the Dynkin index $\text{Tr}(T^\alpha T^\beta) = C(r) \delta^{\alpha\beta}$ for a fermion in the representation r . Here, f_0 is the A_5 zero-mode wavefunction, Eq. (33), and $S^{(\pm, \pm)}(z, z'; p)$ denotes the propagator of a 5D fermion, with boundary conditions (\pm, \pm) and 4D momentum p , between the two points z and z' along the fifth dimension.

Using the fermion propagator given in the Appendix, one can obtain an expression for $m_{A_5}^2$ in terms of integrals of Bessel functions. In the particular case of integer values of M_Ψ/k , the Bessel functions reduce to trigonometric functions so that the integrals greatly simplify. For example, when $M_\Psi = 0$ Eq. (39) becomes, after some algebra:

$$m_{A_5}^2 = -\frac{C(r)}{\pi^2} (g_5^2 k) \int_{L_0}^{L_1} dv f_0(v) \int_{L_0}^v du f_0(u) \int_0^\infty dp \frac{p^3}{\sinh[p(L_1 - L_0)]}. \quad (40)$$

The integrals are convergent and the result is finite. The momentum integral involves the brane to brane propagator, as expected from the previous discussion, and it converges exponentially. Performing the integrals one obtains the result

$$m_{A_5}^2 = -\frac{C(r)}{\pi^2} (g_5^2 k) \frac{1}{L_1^2 - L_0^2} F(L_0/L_1), \quad (41)$$

where the function $F(x)$ is given, for example, by

$$F(x)|_{M_\Psi=0} = \frac{3}{8} \zeta(3) \frac{(1+x)^2}{(1-x)^2}, \quad (42)$$

$$F(x)|_{M_\Psi=k} = \frac{x(1+x)^2}{4(1-x)^2} \int_0^\infty dt \frac{t^5}{\sinh t} \frac{1}{[(x-1)^2 t \cosh t + (-1+x(2-x+t^2)) \sinh t]} \simeq 1.67x + \mathcal{O}(x^2), \quad (43)$$

$$\begin{aligned}
F(x)|_{M_\Psi=2k} &= \frac{x^3(1+x)^2}{4(1-x)^2} \int_0^\infty dt \frac{t^9}{[(x-1)^2 t \cosh t + (-1+x(2-x+t^2)) \sinh t]} \\
&\quad \times \left[3(1-x)^2 (t^3 + 6t - 3xt - 3) t \cosh t \right. \\
&\quad \left. + (9(1-x)^4 + 3(1-x)^2(1+x^2 - 3x)t^2 + x^2 t^4) \sinh t \right]^{-1} \\
&\simeq 12.4 x^3 + \mathcal{O}(x^4),
\end{aligned} \tag{44}$$

for the case of $M_\Psi/k = 0, 1, 2$, respectively. We find that the A_5 mass correction is $\mathcal{O}(1/L_1^2)$ for $|M_\Psi| < k/2$, while it receives a strong suppression for $|M_\Psi| > k/2$ ($F(x)|_{M_\Psi=ck} \propto x^{2|c|-1}$ for $|c| > 1/2$). This is in agreement with the holographic picture where for $|M_\Psi| > k/2$ the CFT operator coupled to the elementary fermion becomes irrelevant and the Yukawa coupling becomes small as shown in Eq. (9).⁶

It is interesting to notice that $m_{A_5}^2$ is even under a change of the sign of M_Ψ . From a 5D perspective this is expected, as a change of the sign in the bulk fermion mass is equivalent to inverting the chirality, $L \leftrightarrow R$. Given the assignment for the boundary conditions of the fermion, Eq. (36), this chirality inversion corresponds to exchanging the i superscript with \hat{i} , an operation which leaves Eq. (41) invariant. From a 4D holographic perspective, on the other hand, the fact that the result does not depend on the sign of M_Ψ arises as a consequence of the requirement that the two CFT descriptions in terms of the left-handed and right-handed sources, Eq. (10) and Eq. (11), are equivalent for $-k/2 \leq M_\Psi \leq k/2$.

4.2 The standard-model Higgs as a hologram of A_5

It has already been noticed [6] that the Higgs as a PGB can be realized as the discretization of the Wilson-loop in a deconstructed fifth dimension. We have shown here that the connection can be even more strict: the PGB can be the holographic image of the fifth component of the gauge field that lives in a warped extra dimension. In fact, such a Higgs happens to be a composite bound state of a strongly interacting (conformal) sector, so that different ideas, which previously seemed distinct, merge together into a single scenario. Moreover, if the bulk group G is simple, one could

⁶ Apparently, Eq. (41) does not seem to give a one-loop suppression of $m_{A_5}^2$ compared with $1/L_1^2$ for $M_\Psi < k/2$, as $g_5^2 k$ is expected to be large $\sim \ln(L_1/L_0)$ in realistic cases. This is because the 4D Yukawa coupling is large $\sim \sqrt{g_5^2 k}$ for $M_\Psi < k/2$, canceling the suppression from the loop factor. On the other hand, for $M_\Psi = k/2$ the 4D Yukawa coupling is given by $\sqrt{g_5^2 k / \ln(L_1/L_0)} = O(1)$, so that $m_{A_5}^2$ is one-loop suppressed compared with $1/L_1^2$. In general, written in terms of the 4D Yukawa coupling h_t , Eq. (41) always yields the result Eq. (19) regardless of the value of M_Ψ , i.e., the value of h_t .

also pursue the idea of unification of the standard-model electroweak interactions. If boundary gauge kinetic terms do not play a major role, the renormalization-group flow of the gauge couplings, in the general scenario of Eq. (27), will follow the pattern:

$$\begin{array}{ccccc}
 H = H_0 \cap H_1 & \longrightarrow & H_1 & \longrightarrow & G \\
 \text{at low} & & \text{exact unification} & & \text{apparent "unification"} \\
 \text{energy} & & \text{at } E \sim 1/L_1 & & \text{at } E \sim 1/L_0
 \end{array}$$

Strictly speaking, the gauge couplings of the holographic theory never become G -symmetric at high energies. Nevertheless, low-energy observers still find unification predictions from G as if unification occurred at $E \sim 1/L_0$ [15], because the gauge couplings in the holographic theory become strong at $E \sim 1/L_0$ and thus their low-energy values are insensitive to the initial values at high energies.

The most economical choice for a simple electroweak group is $SU(3)_L$. In this case it is known that the hypercharge normalization does not come out correct: embedding the Higgs in an adjoint of $SU(3)_L$ gives a prediction $\sin^2 \theta_W = 3/4$ [16], which cannot be accommodated neither with a unification at TeV nor at M_{Pl} . This difficulty, however, is avoided if we introduce brane-localized gauge kinetic terms such that the low-energy gauge coupling values are correctly reproduced. In this case the above picture is modified by large threshold corrections arising either at $E \sim 1/L_0$ or $1/L_1$, depending on where we put the brane-localized kinetic terms. The situation is similar in the case where one tries to embed also the standard-model $SU(3)_C$ into the bulk simple group. The simplest possibility in this case is $SU(6)$. Although a naive prediction from the $SU(6)$ group theory does not yield the observed values of the gauge couplings, we can always adjust them by appropriately choosing the coefficients of brane-localized gauge kinetic terms, which do not necessarily respect $SU(6)$. Therefore, although these theories are not as predictive as 4D supersymmetric unified theories, they are not in contradiction with the observed value of the low-energy gauge couplings.

An important issue for any realistic theory with the Higgs as A_5 is to have correct Yukawa couplings between matter and the Higgs. This issue was studied in flat space in Ref. [17], and the mechanism considered there can be applied in our warped case without any essential modification. For example, we can adopt the $SU(6)$ model of [17] (either supersymmetric or non-supersymmetric) with the gauge group reduced to $SU(5) \times U(1)_X$ on the Planck brane and to $SU(4)_C \times SU(2)_L \times U(1)$ on the TeV brane. Below we explain things in the supersymmetric case. The matter fields are introduced as hypermultiplets, transforming as $\mathcal{D}(\mathbf{15})$, $\mathcal{U}(\mathbf{20})$ and $\mathcal{E}(\mathbf{15})$ (and $\mathcal{N}(\mathbf{6})$ for right-handed neutrinos) under $SU(6)$, together with some brane fields. [In the non-supersymmetric case, these fields are bulk and brane fermions.] Realistic Yukawa matrices are then reproduced by appropriately choosing the bulk masses for these fields: $c \equiv M/k \simeq 1/2$ for the third generation and $c > 1/2$ for the first two

generations (the size of the 4D Yukawa coupling is given by $\approx \sqrt{\ln(L_1/L_0)} g$, $\approx g$, and $\approx |c|\sqrt{\ln(L_1/L_0)} g (L_0/L_1)^{(|c|-1/2)}$ for $|c| < 1/2$, $= 1/2$, and $> 1/2$, respectively, where g is the 4D gauge coupling). An additional ingredient to the flat space case is that some components of the bulk multiplets become exponentially light for $c > 1/2$. These are the fields with the following boundary conditions: $\Phi(+, -)$ and $\Phi^c(-, +)$ (in the 4D superfield notation) and give unwanted vector-like matter with masses much lighter than the TeV scale. We can, however, make these fields heavy by introducing appropriate fields $\bar{\Phi}$ and $\bar{\Phi}^c$ on the Planck and TeV branes, respectively, and by coupling them to Φ and Φ^c through the brane mass terms $\delta(z - L_0)[\bar{\Phi}\Phi]_{\theta^2}$ and $\delta(z - L_1)[\bar{\Phi}^c\Phi^c]_{\theta^2}$. [The corresponding terms in a non-supersymmetric theory are the brane fermion masses.] Proton decay is potentially dangerous in this theory, because $SU(6)/(SU(3)_C \times SU(2)_L \times U(1)_Y)$ gauge fields have masses of order TeV, which could mediate rapid proton decay. However, the structure of the theory allows us to impose the baryon number: $\mathcal{D}(1)$, $\mathcal{U}(1)$, $\mathcal{E}(0)$, $\mathcal{N}(0)$ with appropriate charges for the brane fields. Therefore, we can make proton absolutely stable. An important difference with respect to the flat space model is that, in the absence of brane-localized gauge kinetic terms, the gauge couplings in the present model should unify into $SU(5) \times U(1)_X$ at the TeV scale. As was explained before, this unwanted prediction is avoided if we introduce TeV-brane localized gauge kinetic terms such that the low-energy gauge coupling values are correctly reproduced,⁷ although it implies a loss of any quantitative prediction about gauge coupling unification. Small neutrino masses are obtained either by exponential suppressions of the neutrino Yukawa couplings or by the seesaw mechanism operated on the Planck brane in the case of large and small bulk right-handed neutrino masses, respectively.

The last issue toward a realistic theory of the PGB Higgs is the quartic coupling. If the theory is supersymmetric, as the one described above, the $\mathcal{O}(1)$ quartic coupling is generated through the supersymmetric gauge potential. Supersymmetric theories have two Higgs doublets at low energies, one of which is the PGB of the global symmetry. The tree-level potential takes the form of $V_H \propto (|H_1|^2 - |H_2|^2)^2$, and the PGB Higgs corresponds to the direction $H_1 = H_2$. Supersymmetry can be broken either at the Planck brane [10] or at the TeV brane [19]. In the former case, the holographic theory is essentially a non-supersymmetric theory. Supersymmetry is a global invariance only of the CFT sector, and this partial supersymmetry is responsible for the generation of the tree-level quartic coupling in the Higgs potential without introducing a mass term. Having a non-zero Higgsino mass, however, will require some additional source of supersymmetry breaking. On the other hand, if supersymmetry is broken on the TeV brane in the 5D theory, the holographic theory is a locally supersymmetric theory with supersymmetry broken at the TeV scale by the CFT dynamics. The scale of supersymmetry breaking in this case should not be

⁷Large brane kinetic terms on the TeV brane may also help to reduce constraints from precision electroweak measurements [18].

very high, as the tree-level Higgs quartic coupling becomes zero if the second Higgs boson, which is not a PGB, obtains a large supersymmetry breaking mass.

The quartic coupling in non-supersymmetric theories remains as a difficult issue. However, we can at least adopt the mechanism considered in the previous section with a little modification. For example, in the case of an $SU(3)_L$ theory, we can introduce a bulk scalar field Φ , transforming as a $\mathbf{6}$ under $SU(3)_L$. By introducing a tadpole on the TeV brane for the $SU(2)_L$ singlet component of Φ , we can generate the Higgs quartic coupling in essentially the same way as discussed in section 3.2. This possibility seems to indicate that we can have realistic theories in the non-supersymmetric case as well.

5 Phenomenological Scales and Comparison with Pions in Large N QCD

To better understand the present theory of PGBs, it is instructive to look at the different physical scales of the model from the 4D perspective. This will elucidate the PGB nature of the Higgs and its similarities with pions in QCD.

Let us consider the case in which the PGB is A_5 with the symmetry breaking pattern of Fig. 5. This is equivalent to the model of section 3 if $v \gg \Lambda_{\text{IR}}$, since in this limit the $SU(3)_L$ breaking by Σ reproduces the breaking by boundary conditions. The original 5D scales of the model are Λ_{IR} , $1/g_5^2$, $k = L_0^{-1}$ and L_1^{-1} . They can be related to 4D physical quantities in the following way:

$$g_\rho \equiv \sqrt{g_5^2 k}, \quad m_\rho \equiv \frac{\pi}{L_1}, \quad g^2 = \frac{g_\rho^2}{\ln(L_1/L_0)}, \quad (45)$$

where g_ρ measures the strength of the KK coupling, m_ρ is the mass splitting of the KK towers (approximately this is the first-KK mass), and g is the 4D gauge coupling for the gauge bosons of H . Let us see how the different scales are related. Using naive dimensional analysis we can estimate Λ_{IR} (the scale at which the 5D gauge theory becomes strongly coupled for an observer on the TeV brane):

$$\Lambda_{\text{IR}} \approx \frac{24\pi^3}{g_5^2} \frac{L_0}{L_1} \approx \frac{24\pi^2 m_\rho}{g_\rho^2}. \quad (46)$$

We can define a decay constant f_π for the PGBs in our theory. We follow the usual definition: $m_W^2 = g^2 f_\pi^2$, where m_W is the mass that the gauge bosons $A_\mu^{\bar{a}}$ obtain from the strong dynamics. In the 5D picture this is the mass of the zero-mode gauge bosons with $(+, -)$ boundary conditions. One finds $m_W^2 = 2/(L_1^2 \ln(L_1/L_0))$, and

therefore

$$f_\pi = \frac{\sqrt{2}m_\rho}{\pi g_\rho}. \quad (47)$$

Using the AdS/CFT relation $g_\rho \sim 1/\sqrt{N}$ [5], we obtain $f_\pi \sim \sqrt{N}$ as expected in strongly coupled large N theories [20]. Notice that Λ_{IR} can be larger than the naive value of $4\pi f_\pi$. This is because the PGBs in our theory arise from higher dimensional gauge bosons, and the 5D gauge invariance improves the high energy behavior of the theory. The smallest scale in the model is the mass of the PGBs. It appears at loop level, as that of charged pions in the massless quark limit. We obtained in section 4 that this is of order

$$m_\pi^2 = m_{A_5}^2 \approx \frac{g^2}{16\pi^2} \frac{m_\rho^2}{\pi^2}. \quad (48)$$

The value of the PGB mass from 5D shows the same dependence on m_ρ as that of pions in QCD [21].

We then find that, in general, these theories have the following pattern of scales

$$\Lambda_{\text{IR}} > f_\pi > m_\rho > m_\pi. \quad (49)$$

In real QCD this pattern is not completely followed, since the pion decay constant is smaller than the ρ mass. This could be due to the fact that in QCD we have $N = 3$, which is not really a large number. In spite of this, other predictions of large N QCD agree surprisingly well with the experimental data. Similarly, when the above 5D AdS model is used for the standard model, one realizes that the pattern of scales in Eq. (49) is not really fulfilled. This is because $g_\rho \sim 4$ in order to reproduce the 4D gauge coupling values from $g^2 = g_\rho^2/\ln(L_1/L_0)$. Therefore, the theory of KK states (resonances) is very close to the non-perturbative limit (in the large N expansion). The pattern of scales that we obtain is similar to real QCD:

$$\Lambda_{\text{IR}} \gtrsim m_\rho > f_\pi > m_\pi. \quad (50)$$

We must emphasize, however, that the relation $g^2 = g_\rho^2/\ln(L_1/L_0)$ is subject to large logarithmic corrections and large dependence on brane-kinetic terms, so it is possible that g_ρ takes smaller values than what are naively obtained from the 4D gauge coupling values, making the KK theory more perturbative. This is in fact also needed if we do not want to have large corrections to electroweak observables coming from virtual KK states. These states couple to the Higgs (also a composite object in these models) with a strength g_ρ , and for the value $g_\rho \sim 4$ and $L_1 \sim 1/\text{TeV}$, they gives a too large deviation from the standard-model predictions.

6 Conclusions

We have presented a class of models where the standard-model Higgs appears as a composite PGB from a strongly coupled theory, similar to pions in QCD. We have used the AdS/CFT correspondence to describe the models in terms of the weakly coupled dual theory. The dual theory corresponds to a gauge theory in a slice of 5D AdS in which the bulk gauge symmetry is broken to the standard-model gauge group on both boundaries. This automatically delivers massless scalars (at tree level) on the TeV brane that we associate to the standard-model Higgs field. A remnant global symmetry, under which the Higgs transforms non-linearly, protects the Higgs mass from large radiative corrections. The Higgs mass is generated at one-loop level through the explicitly breaking of the global symmetry due to the standard-model gauge interactions. We have shown that this one-loop contribution is not quadratically divergent. In the 5D AdS picture, this is because of the locality. The Higgs lives on the TeV brane away from the other scalar that breaks the bulk gauge symmetry (which is located on the Planck brane). Therefore, the Higgs can learn this breaking only by bulk fields that propagate from one brane to the other. This is a non-local effect and thus is finite. In the 4D CFT picture, the cancellation of quadratic divergences is understood in a different way. The Higgs is a composite state of CFT which decouples at high energies from the standard-model fields that are elementary states. We have calculated the effective potential of the Higgs from gauge loops and have shown that the Higgs squared-mass is finite and a loop factor smaller than the first resonance mass. This is a very appealing property, since it gives a rationale for the electroweak scale smaller than the new physics scale, as experiments seem to indicate.

If the breaking of the bulk gauge symmetry is due to boundary conditions, the massless scalar corresponds to the fifth component of the bulk gauge boson. Therefore, we find that Higgs-gauge unification in warped space is equivalent to a Higgs as a composite PGB. We have also discussed the similarities and differences of our PGBs with pions in QCD.

The models with the PGB Higgs generically suffer from the absence of a tree-level Higgs quartic coupling, needed to generate a physical Higgs mass larger than the experimental bound. We have presented a mechanism that can generate a quartic coupling without inducing a large quadratic term. This requires a specific assumption about the breaking of the global symmetry at high energies.

The models constructed here have an important phenomenological difference from little Higgs models. The global symmetry that protects the Higgs mass is a symmetry of the strong CFT sector of the theory, but not a symmetry of the standard model. Therefore, there is no partner of the standard-model fields to form a complete multiplet of the global symmetry. New electroweak-scale states appear as resonances

that are in complete multiplets of the global group, similar to the situation in QCD. Detecting these resonances in future colliders will allow us to find the symmetries of the strong CFT, and tell us about the symmetry that protects the electroweak scale from potentially large radiative corrections.

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Appendix

In this appendix we give the propagators for a bulk scalar, fermion and gauge fields in a slice of 5D AdS (see also Ref. [22]). We start by giving the scalar field propagator in the presence of general brane kinetic and mass terms. The free action for a scalar field ϕ is given by

$$\begin{aligned}
S = \int d^4x \int_{L_0}^{L_1} dz \left\{ \sqrt{g} \left[g^{MN} \partial_M \phi^\dagger \partial_N \phi - M^2 \phi^\dagger \phi \right] \right. \\
+ \delta(z - L_0) \sqrt{-g_{\text{ind}}} \left[z_0 g_{\text{ind}}^{\mu\nu} \partial_\mu \phi^\dagger \partial_\nu \phi - m_0 \phi^\dagger \phi \right] \\
\left. + \delta(z - L_1) \sqrt{-g_{\text{ind}}} \left[z_1 g_{\text{ind}}^{\mu\nu} \partial_\mu \phi^\dagger \partial_\nu \phi - m_1 \phi^\dagger \phi \right] \right\}.
\end{aligned} \tag{51}$$

The propagator \hat{G} is given as a solution of

$$[z^2 \partial_z^2 + z \partial_z - (-p^2 z^2 + \alpha^2)] \hat{G}(z, z'; p) = -\frac{z}{k} \delta(z - z'). \tag{52}$$

Here, $\alpha = \sqrt{4 + M^2/k^2}$ and \hat{G} represents the propagator for the rescaled field $\hat{\phi} \equiv (kz)^{-2} \phi$, which is related to the propagator G for the unrescaled field, ϕ , as $G = (kz)^2 (kz')^2 \hat{G}$.

If ϕ has boundary conditions $(+, +)$, the scalar propagator (for the rescaled field)

is given by

$$\hat{G}(z, z'; p) = \frac{-L_0}{(X_I/X_K - Z_I/Z_K)} \times \left(I_\alpha(|p|z_<) - \frac{Z_I}{Z_K} K_\alpha(|p|z_<) \right) \left(I_\alpha(|p|z_>) - \frac{X_I}{X_K} K_\alpha(|p|z_>) \right), \quad (53)$$

where $|p| \equiv \sqrt{-p^2}$ and $z_<$ ($z_>$) is the lesser (greater) of z and z' ; $I_\alpha(x)$ and $K_\alpha(x)$ are the modified Bessel functions. The coefficients X_I , X_K , Z_I and Z_K are given by

$$\begin{aligned} X_I &= |p|L_1 I_{\alpha-1}(|p|L_1) - (\alpha - s/2 - z_1|p|^2 L_1^2 L_0^{-1} - m_1 L_0) I_\alpha(|p|L_1), \\ X_K &= -|p|L_1 K_{\alpha-1}(|p|L_1) - (\alpha - s/2 - z_1|p|^2 L_1^2 L_0^{-1} - m_1 L_0) K_\alpha(|p|L_1), \end{aligned} \quad (54)$$

$$\begin{aligned} Z_I &= |p|L_0 I_{\alpha-1}(|p|L_0) - (\alpha - s/2 + z_0|p|^2 L_0 + m_0 L_0) I_\alpha(|p|L_0), \\ Z_K &= -|p|L_0 K_{\alpha-1}(|p|L_0) - (\alpha - s/2 + z_0|p|^2 L_0 + m_0 L_0) K_\alpha(|p|L_0), \end{aligned} \quad (55)$$

where $s = 4$. The propagator for a field having the odd boundary condition at the Planck brane (TeV brane) is obtained by taking the limit $m_0 \rightarrow \infty$ ($m_1 \rightarrow \infty$).

Restricting the end points to the TeV brane, $z = z' = L_1$, and taking the zero-momentum limit, $p \rightarrow 0$, the scalar propagator of Eq. (53) becomes

$$\begin{aligned} \hat{G}(z = z' = L_1; p \rightarrow 0) &= \frac{L_0}{2\alpha} \left\{ \frac{\alpha + 2 + m_1 L_0}{\alpha - 2 - m_1 L_0} - \frac{\alpha + 2 - m_0 L_0}{\alpha - 2 + m_0 L_0} \left(\frac{L_0}{L_1} \right)^{2\alpha} \right\}^{-1} \\ &\times \left\{ 1 + \frac{\alpha + 2 - m_0 L_0}{\alpha - 2 + m_0 L_0} \left(\frac{L_0}{L_1} \right)^{2\alpha} \right\} \left\{ 1 + \frac{\alpha + 2 + m_1 L_0}{\alpha - 2 - m_1 L_0} \right\}. \end{aligned} \quad (56)$$

For $\alpha = 0$ ($M^2 = -4k^2$), this is further simplified as

$$\hat{G}(z = z' = L_1; p \rightarrow 0) = \frac{(1 + (m_0 L_0 - 2) \ln(L_1/L_0))}{m_0 + m_1 + L_0(m_0 - 2L_0^{-1})(m_1 + 2L_0^{-1}) \ln(L_1/L_0)}, \quad (57)$$

giving the propagator used in section 3.2 (Eq. (24)).

The fermion propagators used in section 4.1 are given by

$$S^{(\pm, \pm)}(z, z'; p) = -(k^2 z z')^{5/2} \left[-\not{p} + \gamma^5 \left(\partial_z + \frac{1}{2z} \right) + \frac{M_\Psi}{kz} \right] \left(P_R \hat{G}_R^{(\pm, \pm)} + P_L \hat{G}_L^{(\mp, \mp)} \right), \quad (58)$$

where $P_{L,R} = (1 \mp \gamma^5)/2$ and M_Ψ is the bulk mass of the fermion. The quantity $\hat{G}_R^{(+, +)}$ is given by Eq. (53) for $\alpha = |M_\Psi/k + 1/2|$, $s = 1$, $m_0 = -M_\Psi$, $m_1 = M_\Psi$ and $z_0 = z_1 = 0$. The case of \hat{G}_R with the odd boundary condition at the Planck brane (TeV brane) is reproduced by taking the limit $m_0 \rightarrow \infty$ ($m_1 \rightarrow \infty$). The expression for \hat{G}_L is obtained from that of \hat{G}_R by simply making the replacement $M_\Psi \rightarrow -M_\Psi$.

Finally, the rescaled gauge boson propagator is given by taking $\alpha = 1$ and $s = 2$ in Eq. (53). The parameters m_0 and m_1 then represent brane masses for the gauge boson induced by spontaneous symmetry breaking caused by brane Higgs fields. The case of boundary condition breaking at the Planck brane (TeV brane) is reproduced by taking the limit $m_0 \rightarrow \infty$ ($m_1 \rightarrow \infty$).

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