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Spectrum of TeV Particles in Warped Supersymmetric Grand Unification

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Abstract

In warped supersymmetric grand unification, XY gauge particles appear near the TeV scale along with Kaluza-Klein towers of the standard model gauge fields. In spite of this exotic low-energy physics, MSSM gauge coupling unification is preserved and proton decay is naturally suppressed. In this paper we study in detail the low-lying mass spectrum of superparticles and GUT particles in this theory, taking supersymmetry breaking to be localized to the TeV brane. The masses of the MSSM particles, Kaluza-Klein modes, and XY states are all determined by two parameters, one which fixes the strength of the supersymmetry breaking and the other which sets the scale of the infrared brane. A particularly interesting result is that for relatively strong supersymmetry breaking, the XY gauginos and the lowest Kaluza-Klein excitations of the MSSM gauginos may both lie within reach of the LHC, providing the possibility that the underlying unified gauge symmetry and the enhanced $N = 2$ supersymmetry of the theory will both be revealed.

1 Introduction

One of the most striking results of any extension of the standard model is the unification of gauge couplings in theories with a supersymmetric desert above the TeV scale. Certain mysteries of the standard model, such as the stability of the Higgs potential and hypercharge quantization, can be elegantly addressed by a combination of low-energy supersymmetry and grand unification at high energies [1], making the supersymmetric desert seem even more compelling. An immediate consequence of this framework is the presence of superparticles at the electroweak scale. In the standard paradigm these superparticles are assumed to be in the smallest possible supersymmetric representations: the standard model particles and their superpartners form multiplets of $N = 1$ supersymmetry, and there are essentially no other fields at the TeV scale charged under the standard model gauge group. This is the basis of the minimal supersymmetric standard model (MSSM), which has been the main focus of phenomenological studies in physics beyond the standard model.

In a previous paper [2] we studied an alternative to this framework with Goldberger. We constructed a realistic theory of grand unification in warped space, in which the unified gauge symmetry is broken by boundary conditions and the electroweak scale is generated by the warp factor. The theory predicts a rich spectrum of new particles at the TeV scale; in addition to the usual superpartners, there are Kaluza-Klein (KK) towers for the standard model fields as well as their supersymmetric and $SU(5)$ partners. The appearance of these particles allows the theory to be "higher dimensional" at the TeV scale. In particular, radiative corrections to the Higgs potential are extremely soft, namely exponentially shut off above the TeV scale, and we can have a complete understanding of the MSSM Higgs sector through the $U(1)_R$ symmetric structure of the theory. We showed that the theory preserves the successful MSSM prediction for gauge coupling unification, despite the drastic departure of the matter content from that of the MSSM at the TeV scale. The theory also preserves a number of the successes of conventional high-scale unification: proton decay is sufficiently suppressed and small neutrino masses are naturally obtained through the seesaw mechanism. This theory thus naturally synthesizes two dominant approaches to physics beyond the standard model: Planck-cutoff and TeV-cutoff paradigms. Some of these features were suggested earlier in [3], and an understanding of logarithmic gauge coupling evolution in warped space was developed in [3 – 9].

In this paper we study the phenomenology of the warped supersymmetric grand unified theory (GUT) described above. Due to the rich spectrum near the TeV scale, the experimental implications of the theory can be quite different from the conventional MSSM. In the dual 4D picture of our theory supersymmetry is broken at the TeV scale by strong dynamics. In the 5D picture, the effects of this supersymmetry breaking are parameterized by operators localized on the TeV brane. The phenomenology of the theory then crucially depends on the form and size of these operators and the location of matter and Higgs fields in the fifth dimension. In this paper we consider the case where matter fields are localized to the Planck brane, which is consistent with the requirements from proton decay suppression and gauge coupling unification. The Higgs fields can either be localized to the Planck brane or propagate in the bulk, though we focus on the localized case in the latter part of our analysis. The supersymmetry breaking operator on the TeV brane is taken to be a linear term for a singlet superfield in the superpotential, which was introduced in [10] and considered in the unified theory context in [2, 11]. This gives gaugino masses at tree level through an operator localized on the TeV brane. At one-loop level, squarks and sleptons obtain masses that are insensitive to physics above the TeV scale. These masses are flavor universal, so that the supersymmetric flavor problem is naturally solved in this setup.

An important feature of the present framework is that the masses for the electroweak-scale particles are determined in terms of only a few free parameters. In [2] we have shown that the simplest theory of warped GUTs is obtained with the TeV brane respecting the full $SU(5)$ symmetry and the Planck brane respecting only the standard model gauge symmetry. This implies that the operators on the TeV brane, including the one that generates the gaugino masses, must respect $SU(5)$. The coefficient of this gaugino mass operator determines the masses for all the superparticles, GUT particles and KK towers, up to the overall mass scale and small effects from electroweak symmetry breaking. This situation is quite different from that in non-unified theories [10], where we can have arbitrary values for $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ gaugino masses and thus have less predictive power. An especially interesting parameter region for our theory is where the coefficient of the gaugino mass operator becomes large. In this parameter region, the spectrum appears quite different from that of the MSSM: the MSSM gauginos become pseudo-Dirac states and one of the XY gauginos becomes quite light, even lighter than some of the superparticles [2]. This makes future experimental searches for these particles quite exciting and promising.

The organization of the paper is as follows. In section 2 we review the theory and summarize the framework for our computation. In section 3 we calculate the masses of the superparticles and GUT particles, including one-loop radiative effects. We find that even with the present experimental bounds on the superparticle masses, the lightest XY gaugino is well within the reach of the LHC for moderately large supersymmetry breaking. In section 4 we discuss the physical Higgs-boson mass and the naturalness of electroweak symmetry breaking. Conclusions are given in section 5.

2 Theory and Framework

2.1 Warped supersymmetric GUTs

We begin by reviewing the warped supersymmetric grand unified theory of Ref. [2]. The theory is formulated in a warped 5D spacetime with the extra dimension compactified on an S^1/Z_2 orbifold: $0 \leq y \leq \pi R$, where y represents the coordinate of the extra dimension. The metric of this space is given by

$$ds^2 \equiv G_{MN} dx^M dx^N = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2. \tag{1}$$

Here, k is the AdS curvature, which is taken to be somewhat (typically a factor of a few) smaller than the 5D Planck scale M_5; the 4D Planck scale, $M_{\rm Pl}$, is given by $M_{\rm Pl}^2 \simeq M_5^3/k$ and we take $k \sim M_5 \sim M_{\rm Pl}$. We choose $kR \sim 10$ so that the TeV scale is naturally generated by the AdS warp factor: $T \equiv k e^{-\pi k R} \sim$ TeV [12].

We consider a supersymmetric $SU(5)$ gauge theory on the above gravitational background. The bulk $SU(5)$ symmetry is broken by boundary conditions imposed at the boundary at $y = 0$. Specifically, the 5D gauge multiplet can be decomposed into a 4D $N = 1$ vector superfield $V(A_\mu, \lambda)$ and a 4D $N = 1$ chiral superfield $\Sigma(\sigma + iA_5, \lambda')$, where both V and Σ are in the adjoint representation of $SU(5)$. The boundary conditions for these fields are given by

$$\begin{pmatrix} V \\ \Sigma \end{pmatrix} (x^\mu, -y) = \begin{pmatrix} PVP^{-1} \\ -P\Sigma P^{-1} \end{pmatrix} (x^\mu, y), \qquad \begin{pmatrix} V \\ \Sigma \end{pmatrix} (x^\mu, -y') = \begin{pmatrix} V \\ -\Sigma \end{pmatrix} (x^\mu, y'), \tag{2}$$

where $y' = y - \pi R$, and P is a 5×5 matrix acting on gauge space: $P = {\rm diag}(+,+,+,-,-)$. This reduces the gauge symmetry at the $y = 0$ brane (Planck brane) to $SU(3)_C \times SU(2)_L \times U(1)_Y$ (321), while the 5D bulk and the $y = \pi R$ brane (TeV brane) respect full $SU(5)$. After KK decomposition, the above boundary conditions ensure that only the 321 components of the 4D vector superfield, V, have massless modes. The typical mass scale for the KK towers is $T \sim$ TeV, so that the lowest KK excitations of the standard model gauge fields and the lightest XY gauge bosons both have masses of order TeV. In fact, the KK towers for these gauge fields turn out to be approximately $SU(5)$ symmetric.

The Higgs fields are introduced in the bulk as two hypermultiplets transforming as the fundamental representation of $SU(5)$. Using the notation where a hypermultiplet is represented by two 4D $N = 1$ chiral superfields $\Phi(\phi, \psi)$ and $\Phi^c(\phi^c, \psi^c)$ with the opposite gauge transformation properties, our two Higgs hypermultiplets can be written as $\{H, H^c\}$ and $\{\bar{H}, \bar{H}^c\}$, where H and $\bar{H}^c$ transform as $\mathbf{5}$ and $\bar{H}$ and H^c transform as $\bar{\mathbf{5}}$ under $SU(5)$. The boundary conditions are given by

$$\begin{pmatrix} H \\ H^c \end{pmatrix} (x^\mu, -y) = \begin{pmatrix} -PH \\ PH^c \end{pmatrix} (x^\mu, y), \qquad \begin{pmatrix} H \\ H^c \end{pmatrix} (x^\mu, -y') = \begin{pmatrix} H \\ -H^c \end{pmatrix} (x^\mu, y'), \tag{3}$$

(p,p')	gauge and Higgs fields	bulk matter fields
$(+,+)$	$V_{321},\ H_D,\ \bar{H}_D$	$T_{U,E},\ T'_Q,\ F_D,\ F'_L$
$(-,-)$	$\Sigma_{321},\ H^c_D,\ \bar{H}^c_D$	$T^c_{U,E},\ T'^c_Q,\ F^c_D,\ F'^c_L$
$(-,+)$	$V_X,\ H_T,\ \bar{H}_T$	$T_Q,\ T'_{U,E},\ F_L,\ F'_D$
$(+,-)$	$\Sigma_X,\ H^c_T,\ \bar{H}^c_T$	$T^c_Q,\ T'^c_{U,E},\ F^c_L,\ F'^c_D$

Table 1: Boundary conditions for the bulk fields under the orbifold reflections. Here, $T^{(\prime)}_{Q,U,E}$ ($F^{(\prime)}_{D,L}$) are the components of $T^{(\prime)}$ ($F^{(\prime)}$) decomposed into irreducible representations of the standard model gauge group. The fields written in the (p,p') row, φ, obey the boundary conditions $\varphi(-y) = p\,\varphi(y)$ and $\varphi(-y') = p'\,\varphi(y')$.

for $\{H, H^c\}$, and similarly for $\{\bar{H}, \bar{H}^c\}$. After KK decomposition, only the two Higgs doublets from H and $\bar{H}$ have massless modes. All the other KK modes, including those of colored Higgs fields, are massive with characteristic mass scale given by $T \sim$ TeV. The masses for these modes are approximately $SU(5)$ symmetric as in the case of the gauge fields. Therefore, at this stage, the mass spectrum of the theory is given as follows: we have a 321 vector multiplet, V_{321}, and the two Higgs doublets, H_D and $\bar{H}_D$, at the massless level together with $SU(5)$ symmetric (and $N = 2$ supersymmetric) KK towers for the gauge and Higgs fields with characteristic mass scale $T \sim$ TeV.

Despite the fact that XY gauge and colored Higgs fields have masses of order TeV, proton decay can be adequately suppressed. One way to achieve this is to impose baryon number, which is possible even if matter propagates in the bulk, and this approach allows matter fields to have wavefunctions spread over the extra dimension. Another, probably more satisfactory way is to localize matter toward the Planck brane – either strictly localized as brane fields, or approximately localized using bulk mass parameters. In particular, the latter possibility appropriately quantizes matter hypercharges while avoiding rapid proton decay caused by exchanges of TeV-scale GUT particles.

To be more explicit, the bulk matter model has the following four hypermultiplets for each generation: $\{T, T^c\}(\mathbf{10}), \{T', T'^c\}(\mathbf{10}), \{F, F^c\}(\mathbf{5}^*)$ and $\{F', F'^c\}(\mathbf{5}^*)$, where the numbers in parentheses represent the transformation properties of the non-conjugated fields under $SU(5)$. The boundary conditions for the matter fields are given similarly to the Higgs fields, Eq. (3), but for $\{T, T^c\}$ and $\{T', T'^c\}$ the matrix P acts on both $SU(5)$ fundamental indices and the overall parities under $y \to -y$ are taken to be opposite between T and T' multiplets and between F and F' multiplets (these boundary conditions are summarized in Table 1). With these boundary conditions, a complete generation, Q, U, D, L and E, arises at the massless level as $T(U,E), T'(Q), F(D), F'(L)$. The wavefunction profiles for these modes depend on the bulk hypermultiplet masses

$$S = \int d^4x \int_0^{\pi R} dy \sqrt{-G} \left[\int d^2\theta\, c_\Phi k \Phi \Phi^c + \text{h.c.} \right], \tag{4}$$

parameterized by dimensionless quantities c_Φ, where Φ runs for T, T', F and F'. For $c_\Phi > 1/2$, we find that the wavefunctions for the zero modes are strongly localized to the Planck brane as $e^{-(c_\Phi - 1/2)k|y|}$ and that proton decay rates are sufficiently suppressed for $c_\Phi \gtrsim 1$.

From the 5D viewpoint, there are three local operators that can contribute to the low-energy 4D gauge couplings:

$$S = -\frac{1}{4} \int d^4x \int_0^{\pi R} dy \sqrt{-G} \Big[\frac{1}{g_B^2} F_{MN} F^{MN} + 2\delta(y) \frac{1}{\tilde{g}_{0,a}^2} F^a{}_{\mu\nu} F^{a\mu\nu} + 2\delta(y - \pi R) \frac{1}{\tilde{g}_\pi^2} F_{\mu\nu} F^{\mu\nu} \Big], \tag{5}$$

where the index a runs over $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ ($a = 3, 2, 1$, respectively). The structure of these terms are determined by the restricted 5D gauge symmetry, which reduces to 321 on the $y = 0$ brane but is $SU(5)$ at all the other points in the extra dimension. At the fundamental scale $M_* \sim M_5$, the coefficients of these operators are incalculable parameters of the effective field theory. Therefore, one might worry that one cannot obtain any prediction for the low-energy gauge couplings, which in general depend on these unknown parameters. This difficulty, however, is avoided by requiring that the theory is strongly coupled at the scale M_*. In this case the sizes of these coefficients are estimated as $1/g_B^2 \simeq M_*/16\pi^3$ and $1/\tilde{g}_{0,a}^2 \simeq 1/\tilde{g}_\pi^2 \simeq 1/16\pi^2$, and one finds that the low-energy prediction is insensitive to the parameters $\tilde{g}_{0,a}$ and $\tilde{g}_\pi$ evaluated at M_*, which encode unknown physics above the cutoff scale of the theory. The prediction for the low-energy 4D gauge couplings, g_a, is then written in the form

$$\frac{1}{g_a^2(T)} \simeq (SU(5) \text{ symmetric}) + \frac{1}{8\pi^2} \Delta^a(T, k), \tag{6}$$

where $\Delta^a(T, k)$ is the quantity whose non-universal part can be unambiguously computed in the effective theory. In our theory, this quantity is given by

$$\begin{pmatrix} \Delta^1 \\ \Delta^2 \\ \Delta^3 \end{pmatrix} (T, k) \simeq \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix} \ln \left(\frac{k}{T} \right), \tag{7}$$

at one-loop leading-log level, regardless of the values of the bulk mass parameters as long as they are larger than or equal to 1/2: $c_H, c_{\bar{H}}, c_T, c_{T'}, c_F, c_{F'} \geq 1/2$ (we have absorbed a possible $SU(5)$ symmetric piece into the first term of Eq. (6)). This is exactly the relation obtained in conventional 4D supersymmetric unification with the parameter k identified with the unification scale. Therefore, we find that our theory preserves the successful 4D MSSM prediction for gauge coupling unification, despite the drastic departure of the matter content from the MSSM at the TeV scale [2].

Given that MSSM gauge coupling unification is naturally preserved when matter and Higgs fields have $c \geq 1/2$ and that proton stability is ensured when matter fields have $c \gtrsim 1$, it is natural to focus on the case with all matter fields strongly localized to the Planck brane. The c parameters for the Higgs multiplets are less constrained, but we can certainly consider the case where the Higgs fields are also effectively localized to the Planck brane. In this case the physics is well approximated by simply regarding matter and Higgs as brane fields, as will be done in our calculations in the subsequent sections. (The analysis of section 3 assumes matter to be localized to the Planck brane but is independent of the Higgs profiles; parts of section 4 assume that the Higgs fields are also localized to the Planck brane.) However, one should keep in mind that our analyses also apply for bulk matter provided the lowest KK modes are localized toward the Planck brane by bulk hypermultiplet masses.

Finally, we discuss the Yukawa couplings. The Yukawa couplings are written on the Planck brane as

$$S = \int d^4x \int_0^{\pi R} dy \, \sqrt{-G} \, 2\delta(y) \left[\int d^2\theta \left(y_u Q U H_D + y_d Q D \bar{H}_D + y_e L E \bar{H}_D \right) + \text{h.c.} \right]. \qquad (8)$$

Since the gauge symmetry on the $y = 0$ brane is only 321, we do not have unwanted $SU(5)$ mass relations such as $m_s/m_d = m_\mu/m_e$. The above Yukawa couplings respect a $U(1)_R$ symmetry, under which the 4D superfields V, Σ, H and $\bar{H}$ are neutral, $T, T^c, F, F^c, T', T'^c, F'$ and F'^c have unit charge, and H^c and $\bar{H}^c$ have charge $+2$. This $U(1)_R$ forbids dangerous dimension four and five proton decay operators together with a potentially large supersymmetric mass term for the Higgs fields, thus providing a complete solution to the doublet-triplet splitting and proton decay problems (the $U(1)_R$ symmetry is broken to its Z_2 subgroup through supersymmetry breaking discussed in the next subsection, but without reintroducing phenomenological problems). Small neutrino masses can be naturally generated by introducing right-handed neutrino fields with the Majorana mass terms and neutrino Yukawa couplings on the Planck brane, through the conventional seesaw mechanism.

2.2 Framework for the analyses

To calculate physical quantities such as superparticle and GUT particle masses, we must specify how supersymmetry is broken. We also have to specify a calculational scheme for computing radiative effects, which are quite important for the phenomenology of the theory.

Since any mass parameter on the $y = \pi R$ brane of order the fundamental scale appears as a TeV scale parameter in the 4D picture, it is quite natural to consider supersymmetry breaking on the TeV brane. Specifically, we introduce a supersymmetry breaking potential

$$S = \int d^4x \int_0^{\pi R} dy\, \sqrt{-G}\, 2\delta(y - \pi R) \left[\int d^2\theta d^2\bar{\theta} Z^\dagger Z + \left(\int d^2\theta \Lambda_Z^2 Z + \text{h.c.} \right) \right], \tag{9}$$

on the TeV brane [10]. Here, Z is a gauge singlet chiral superfield, and Λ_Z is a mass parameter of order the fundamental scale $M_* \sim M_5$. The resulting supersymmetry breaking is transmitted to the $SU(5)$ sector through the following operator:

$$S = \int d^4x \int_0^{\pi R} dy\, \sqrt{-G}\, 2\delta(y - \pi R) \left[\int d^2\theta \frac{\lambda}{2M_*} Z \mathcal{W}^\alpha \mathcal{W}_\alpha + \text{h.c.} \right], \tag{10}$$

where $\mathcal{W}^\alpha$ represents the gauge field strength superfield for the bulk $SU(5)$ gauge multiplet. (The presence of both operators in Eqs. (9, 10) breaks the $U(1)_R$ symmetry discussed in the previous subsection, but this breaking does not reintroduce phenomenological problems such as rapid proton decay [2].) When expanded into component fields, this gives the gaugino masses localized on the TeV brane

$$S = \int d^4x \int_0^{\pi R} dy\, \sqrt{-G}\, 2\delta(y - \pi R) \left[-\frac{M_\lambda}{2} \lambda^\alpha \lambda_\alpha + \text{h.c.} \right], \tag{11}$$

where $M_\lambda \equiv \lambda \Lambda_Z^2 / M_*$, and λ^α is the $SU(5)$ gaugino. After KK decomposition, this term gives TeV scale masses for the 321 gauginos. Since the full $SU(5)$ symmetry is respected on the TeV brane, we find that all the gaugino masses are fixed by the single parameter M_λ at tree level. The splitting among the three gaugino masses then arises through radiative effects.

Squarks and sleptons, which are localized to the Planck brane, obtain masses at one-loop level. Because of the geometrical separation between supersymmetry breaking and the place where squarks and sleptons are located, the resulting masses are finite and calculable in the effective field theory. In fact, the loop integrals are cut off at the scale T and are insensitive to unknown UV physics. However, to find the detailed structure of the mass spectrum, for example that coming from the splitting of the gaugino masses, we have to include higher order effects.

What calculational scheme should we use to compute superparticle masses including radiative effects? One way of computing radiative corrections in truncated AdS_5 is to calculate them directly in perturbation theory, using the KK decomposed 4D theory and retaining all KK modes in loops. This procedure is justified as long as the external momenta, p, are sufficiently smaller than the threshold for the KK towers, T [5, 8], because then the effects from unknown physics above the cutoff scale are suppressed by powers of p/M'_*. Here M'_* is the cutoff scale on the IR brane, $M'_* \equiv M_* e^{-\pi k R}$. However, this procedure is not quite suitable for computing radiative corrections to superparticle masses, as we expect to get powers of large logarithms at higher loop orders, $(\alpha/4\pi)^n (\ln(k/T))^n$ for the n-th loop, which invalidate the perturbative expansions.[1]

[1]For radiative corrections to the gauge couplings, the form of the one-loop renormalization group equations ensures that there are no such terms beyond one loop, if we compute corrections to $1/g^2$.

Although general renormalization theory relates the coefficients of these leading logarithms, in principle allowing their summation, this procedure requires computation of at least the lowest loop diagrams containing the large logarithms: one loop diagrams for the gauginos and two loops for the sfermions. These calculations are somewhat involved, so we do not adopt this scheme for computing the superparticle masses.

As in 4D theories, leading-log effects can be taken into account using the renormalization group method. In truncated AdS_5, the direct analog of integrating out higher momentum modes is to integrate out the space closest to the Planck brane. The specific procedure [13] is summarized as follows. Starting from the theory where the two branes are located at $y = 0$ and $y = \pi R$ with the AdS curvature given by k, we can construct a theory in which the two branes are located at $y = \epsilon R$ and $y = \pi R$ by integrating out the region $0 \leq y \leq \epsilon R$. Then, rescaling all the mass scales of the theory as $m \to m' \equiv e^{-\pi\epsilon k R} m$, we obtain a theory with the AdS curvature given by k' ($< k$), which has the same IR scale, $k' e^{-(\pi-\epsilon)kR} = k e^{-\pi k R} = T$, and gives the same low-energy predictions as the original theory. By repeating this procedure, we can obtain the theory where k' is sufficiently close to T that there is no large logarithm in loop calculations. However, the theory obtained in this way contains a series of higher dimensional operators on the UV brane suppressed only by powers of $k' \ll k$, whose effects on predictions are $O((T/k')^n)$ for some power n. Without knowing the coefficients of these higher dimensional operators, the only way of obtaining reliable predictions is to choose k' to be somewhat larger than T, but then we cannot really sum logarithms down to the scale T. Due to this limitation, we do not choose this "floating cutoff" scheme either, although one can compute superparticle masses in this scheme if one is satisfied with the precision in which one does not distinguish the values for running couplings at T and $k' \simeq T/\varepsilon$, where ε sets the typical size of errors in the predictions.

Instead of changing the cutoff, one can also sum up the large logarithms by defining the couplings of the theory using the sliding renormalization scale μ. Suppose we compute radiative corrections to the mass of a particle that is localized on the Planck brane. If we use the couplings appearing in the bare Lagrangian, the resulting expression contains large logarithms $\ln(k/T)$. However, these large logarithms can be successfully resummed if we use the couplings defined at the scale $\mu \sim T$, say through momentum subtraction, measured in terms of the 4D metric $\eta_{\mu\nu}$. This procedure effectively corresponds to integrating out physics above the scale μ and encoding it into the couplings defined at μ. For this procedure to work, the effective theory obviously has to be valid up to k, the scale below which the large logarithms are generated. This is indeed the case when we compute radiative corrections to Planck-brane localized quantities [5], because on the Planck brane physics is essentially four dimensional up to the scale k. This implies that large logarithms that could invalidate perturbative expansions, *i.e.* $\ln k/T$'s appearing in the superparticle mass calculation, are effectively resummed into the coefficients of operators located on the Planck brane. Since we did not lower the cutoff scale of the theory, the mass scale on this brane is given by k, and the coefficients of these operators still scale by powers of k. Therefore, (UV insensitive) low-energy quantities, such as squark and slepton masses, can be reliably computed using the lowest dimension operators on the Planck brane with coefficients evaluated at the scale $\mu \sim T$.

Calculating superparticle masses in our setup requires only the values of the brane localized gauge couplings, $\tilde{g}_{0,a}$ and $\tilde{g}_{\pi}$, at the 4D scale T. Recall that our strong coupling assumption sets these couplings to be $\simeq 4\pi$ at the scale M_* measured in terms of the 5D metric (see the sentences above Eq. (6)). This corresponds to $1/\tilde{g}_{0,a}^2(\mu = k) \simeq 1/16\pi^2$ and $1/\tilde{g}_{\pi}^2(\mu = T) \simeq 1/16\pi^2$ in terms of the 4D metric, so all $\ln(k/T)$ effects can be resummed by running down the couplings $\tilde{g}_{0,a}$ from the scale k down to T. Since the solution to the one-loop renormalization group equation (RGE) takes the form

$$\frac{1}{\tilde{g}_{0,a}^2(T)} = \frac{1}{\tilde{g}_{0,a}^2(k)} + \frac{\tilde{b}_a}{8\pi^2} \ln\left(\frac{k}{T}\right), \tag{12}$$

the required couplings $\tilde{g}_{0,a}(T)$ can be obtained from $\tilde{b}_a$ without knowing the precise values for the initial couplings, $\tilde{g}_{0,a}(k)$.

Because the non-universal part of the low-energy 4D gauge couplings (*i.e.* the differences of the three gauge couplings) comes only from the Planck-brane localized couplings $\tilde{g}_{0,a}(T)$, we know that the non-universal part of $\tilde{b}_a$ must be the same as that of the MSSM: $\tilde{b}_a - \tilde{b}_b = b_a^{\rm MSSM} - b_b^{\rm MSSM}$. This allows us to write $\tilde{b}_a = b_a^{\rm MSSM} + \tilde{b}$, where $\tilde{b}$ takes a universal value for all the 321 gauge groups. To determine the value of $\tilde{b}$, we focus on the $U(1)_Y$ component ($a = 1$). For a $U(1)$ theory, the computation of Ref. [5] explicitly shows that contributions to $\tilde{b}_a$ are saturated by zero-mode fields for bulk scalars and fermions (for non-Abelian gauge fields this saturation was shown only for the non-universal part). We also notice that the XY gauge bosons and gauginos are all strongly localized to the TeV brane and thus do not contribute to $\tilde{b}_1$. This is sufficient to conclude that $\tilde{b} = 0$, because the contribution to $\tilde{b}_1$ comes entirely from the scalars and fermions in the bulk and on the Planck brane, giving $\tilde{b}_1 = b_1^{\rm MSSM}$.

Our procedure for computing superparticle masses can now be summarized as follows. We first integrate out physics above the scale T (in the 4D metric) by defining the couplings at the sliding renormalization scale $\mu \sim T$. Using these couplings, we compute lowest order contributions to the superparticle masses: tree level for the gauginos and one loop for the sfermions. Higher loop effects are expected not to contain large logarithms because they are already included in the renormalized couplings, so that perturbation theory must work well. The remaining uncertainty arises from possible TeV-brane operators suppressed by powers of $M'_* = M_* e^{-\pi k R}$. These operators, which are intrinsically incalculable in the effective theory, bring uncertainties of $O((T/M'_*)^n)$ in the predictions, where n is some power depending on the dimension of the operator. However, 4D Lorentz invariance implies that these corrections are at most of order $(k/M_*)^2 \lesssim 10\%$, and we can trust our leading-order computations up to uncertainties of $O(10\%)$.[2] Keeping this remark in mind, in the next section we compute the mass spectrum of the theory.

3 Spectra for Superparticles and GUT Particles

In the supersymmetric limit, the massless sector of the theory consists of the fields of the MSSM. The gauge multiplets propagate in the bulk, so the MSSM gauge bosons and gauginos are accompanied by KK towers of massive gauge multiplets. These massive KK levels are $N = 2$ supersymmetric and approximately $SU(5)$ symmetric, with masses given at tree level by $m_n \simeq (n - 1/4)\pi\, T$ for $n = 1, 2, \cdots$. The lightest gaugino and gauge boson KK modes thus have masses $m_1 \simeq 2.4\, T$ in this limit. If they propagate in the bulk, Higgs and matter fields will also have KK excitations, but the analysis of this section will treat matter as localized to the Planck brane (or approximately localized with a large bulk hypermultiplet mass term). Our results will apply regardless of whether or not the Higgs fields propagate in the bulk, however.

When supersymmetry is broken as described in section 2.2, the MSSM gauginos acquire masses at tree level and the squarks and sleptons obtain masses at one loop. Supersymmetry breaking also feeds into the spectrum of the KK excitations in potentially crucial fashion. In particular, the masses of the lightest XY gauginos and of the first KK excitations of the 321 gauginos can be pushed well below the $2.4\, T$ value that applies in the supersymmetric limit, improving the prospects for their discovery at colliders [2]. Our aim in this section is to compute the masses of the MSSM superparticles along with those of the lightest 321 KK states, XY gauge bosons, and XY gauginos, including one-loop radiative effects.

Before getting into the details of these computations, we refer the reader to Fig. 1, where a schematic depiction of the effects of supersymmetry breaking on the particle spectrum is given. It is useful to introduce the supersymmetry-breaking parameter

$$x \equiv M_\lambda / k, \tag{13}$$

which we will take to be $.01 - 10$ corresponding to relatively unsuppressed supersymmetry breaking on the TeV brane. Once this single parameter is specified, the masses of all the particles in the theory are fixed up to an overall scale and small electroweak symmetry breaking effects. For $x = 0$ (Fig. 1a), the massless particles are the 321 gauge bosons A_μ^{321}, the 321 gauginos

[2]Corrections to the scalar masses induced by a supersymmetry-breaking gaugino kinetic operator on the TeV brane can be calculated using the gaugino propagators derived in Appendix A, by choosing $1/\tilde{g}_\pi^2$ appropriately. We find that these corrections do not significantly affect our results below.

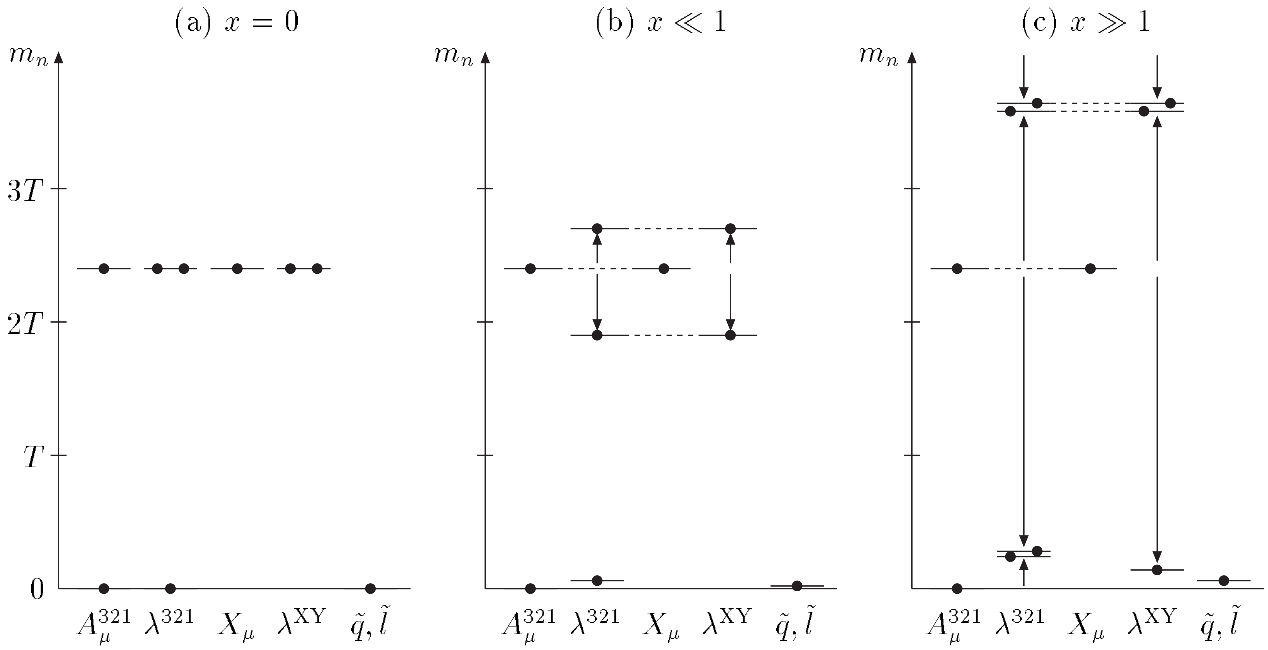

Figure 1: The lowest-lying masses for the 321 gauge bosons (A_μ^{321}), 321 gauginos (λ^{321}), XY gauge bosons (X_μ), XY gauginos ($\lambda^{\rm XY}$), and MSSM scalars ($\tilde{q}$, $\tilde{l}$), in the presence of no supersymmetry breaking (a), weak supersymmetry breaking (b), and strong supersymmetry breaking (c). Each bullet for λ^{321} and $\lambda^{\rm XY}$ represents a Majorana and Dirac degree of freedom, respectively. Dotted lines connect nearly degenerate $SU(5)$ partners, and arrows indicate displacements of the mass levels relative to their supersymmetric positions. The states not shown here include the standard-model quarks and leptons, the two Higgs hypermultiplets, and the real scalar fields arising from the 5D gauge multiplet, which have the 321 and XY quantum numbers and are degenerate with the excited states of A_μ^{321} and X_μ.

λ^{321}, and the MSSM scalars $\tilde{q}$, $\tilde{l}$ (and the standard-model quarks and leptons, of course). The first KK level, at $m_1 \simeq (3\pi/4)\,T$, includes the first A_μ^{321} KK mode and a degenerate pair of Majorana fermions for λ^{321}, which form a Dirac state. Nearly degenerate with these are their $SU(5)$ partners, the lightest XY gauge bosons X_μ and a degenerate pair of Dirac XY gauginos $\lambda^{\rm XY}$. When supersymmetry is broken by a small amount ($x \ll 1$; Fig. 1b), the λ^{321} zero modes pick up small ($\ll T$) tree-level masses and the MSSM scalars pick up even smaller masses at loop level. Meanwhile, the degeneracy between the λ^{321} KK excitations is spoiled: one Majorana fermion's mass increases, while the other's decreases. The XY gaugino masses move in essentially the same way: one Dirac state becomes heavier while the other becomes lighter. Thus, as long as x is sufficiently small, the gaugino states are still $SU(5)$ symmetric. In the limit of very strong supersymmetry breaking ($x \gg 1$; Fig. 1c), the 321 gauginos once again form near-degenerate pairs, but now the formerly massless gaugino is approximately degenerate with one of the light KK excitations, with a mass $\simeq T/4$. Meanwhile the other light gaugino KK excitation becomes nearly degenerate with one of the 321 gauginos from the second KK level, with mass $\simeq (5\pi/4)\,T$. On the other hand, the lightest XY gaugino has no nearly degenerate partner in this regime. Moreover, its mass approaches zero, rather than a finite limiting value, as $x \to \infty$. Therefore, it is possible that this XY gaugino is quite light.

Let us now compute the particle masses. To calculate supersymmetry-breaking effects on the spectrum, we adopt the framework described in the previous section. As discussed there, we assume that the effects of physics above the energy scale T are encoded in local operators that reside on the Planck brane. The operators of interest for our purposes are the Planck-brane localized gauge kinetic terms. We compute tree-level gaugino and gauge boson masses and one-loop scalar masses under the presence of these (radiatively induced) Planck-brane operators. This requires knowing the numerical values of the Planck-brane, TeV-brane, and bulk gauge couplings at the scale T. The TeV-brane coefficient $1/\tilde{g}_\pi^2$ is assumed to be very small at that scale, $\sim 1/16\pi^2$, and we neglect it entirely. The Planck-brane coefficients $1/\tilde{g}_{0,a}^2$ are similarly assumed to be negligibly small at the scale k, and their values at T are obtained using the MSSM RGEs (see Eq. (12) and the discussion below it). Finally, the value of the bulk gauge coupling g_B is determined from

$$\frac{1}{g_a^2(T)} = \frac{\pi R}{g_B^2} + \frac{1}{\tilde{g}_{0,a}^2(T)} + \frac{1}{\tilde{g}_\pi^2(T)}, \tag{14}$$

where $g_a(T)$ are the 4D gauge couplings evaluated at the scale T. These are approximated by running the experimentally measured values at m_Z up to $\simeq 1$ TeV using standard model RGEs and then from $\simeq 1$ TeV to T using MSSM RGEs. Fixing in this way the numerical values of the various gauge couplings, we can now give results.

(i) 321 gauginos

The equation determining the 321 gaugino masses in the presence of Planck-brane and TeV-brane gauge kinetic terms is presented in Appendix B. In the limit where the TeV-brane coefficient vanishes, the equation becomes

$$\frac{J_0\left(\frac{m_n}{k}\right)+\frac{g_B^2}{\tilde{g}_0^2}m_nJ_1\left(\frac{m_n}{k}\right)}{Y_0\left(\frac{m_n}{k}\right)+\frac{g_B^2}{\tilde{g}_0^2}m_nY_1\left(\frac{m_n}{k}\right)}=\frac{J_0\left(\frac{m_n}{T}\right)+g_B^2M_\lambda J_1\left(\frac{m_n}{T}\right)}{Y_0\left(\frac{m_n}{T}\right)+g_B^2M_\lambda Y_1\left(\frac{m_n}{T}\right)}, \tag{15}$$

where we have suppressed the 321 index. Using the values for the gauge couplings at the scale T, the lowest mass solutions give the lightest gaugino masses at T, which we run down from T to the gaugino masses themselves using MSSM RGEs. We will be interested primarily in the masses of the two lightest sets of gauginos.

(ii) MSSM scalars

The MSSM scalars acquire masses at one-loop level due to their gauge interactions. We obtain these contributions to the scalar masses by computing the gaugino loop in the presence of the TeV-brane gaugino mass M_λ, and then subtracting the value of the same diagram in the supersymmetric limit. The results are

$$m_{\tilde{q}}^2 = \frac{1}{2\pi^2}\left(\frac{4}{3}\mathcal{I}_3+\frac{3}{4}\mathcal{I}_2+\frac{1}{60}\mathcal{I}_1\right), \tag{16}$$

$$m_{\tilde{u}}^2 = \frac{1}{2\pi^2}\left(\frac{4}{3}\mathcal{I}_3+\frac{4}{15}\mathcal{I}_1\right), \tag{17}$$

$$m_{\tilde{d}}^2 = \frac{1}{2\pi^2}\left(\frac{4}{3}\mathcal{I}_3+\frac{1}{15}\mathcal{I}_1\right), \tag{18}$$

$$m_{\tilde{l}}^2 = \frac{1}{2\pi^2}\left(\frac{3}{4}\mathcal{I}_2+\frac{3}{20}\mathcal{I}_1\right), \tag{19}$$

$$m_{\tilde{e}}^2 = \frac{1}{2\pi^2}\left(\frac{3}{5}\mathcal{I}_1\right), \tag{20}$$

and, if the Higgs doublets are effectively localized to the Planck brane, $m_{h_u}^2 = m_{h_d}^2 = m_{\tilde{l}}^2$. Here $\mathcal{I}_3$, $\mathcal{I}_2$ and $\mathcal{I}_1$ are loop integrals defined as

$$\mathcal{I}_a = \int_0^\infty dq\, q^3\left[f_{z,a}\Big(z=z'=\frac{1}{k};q\Big)-f_{z,a}\Big(z=z'=\frac{1}{k};q\Big)\Big|_{M_\lambda=0}\right], \tag{21}$$

where $a=1,2,3$ labels the gauge groups of the standard model. The functions $f_{z,a}(z,z';q)$ are defined in Appendix A (Eq. (57)). As shown there, the integrand in Eq. (21) has an exponential suppression $\sim e^{-2q/T}$ above T. This suppression at high momentum comes from the spatial separation between the matter fields on the Planck brane and the supersymmetry breaking on the TeV brane: to communicate the supersymmetry breaking to the MSSM scalars, the gaugino in the loop must propagate from the Planck brane to the TeV brane, where its Majorana mass term is localized, and back.

(iii) 321 KK states, XY gauge bosons, and XY gauginos

We also calculate the tree-level masses of the 321 gauge-boson KK excitations, XY gauge bosons, and XY gauginos. For the 321 gauge bosons the equation determining the masses is

$$\frac{J_0\left(\frac{m_n}{k}\right)+\frac{g_B^2}{\tilde{g}_0^2}m_n J_1\left(\frac{m_n}{k}\right)}{Y_0\left(\frac{m_n}{k}\right)+\frac{g_B^2}{\tilde{g}_0^2}m_n Y_1\left(\frac{m_n}{k}\right)}=\frac{J_0\left(\frac{m_n}{T}\right)}{Y_0\left(\frac{m_n}{T}\right)}, \tag{22}$$

while for the XY gauge bosons we have

$$\frac{J_1\left(\frac{m_n}{k}\right)}{Y_1\left(\frac{m_n}{k}\right)}=\frac{J_0\left(\frac{m_n}{T}\right)}{Y_0\left(\frac{m_n}{T}\right)}, \tag{23}$$

and for the XY gauginos,

$$\frac{J_1\left(\frac{m_n}{k}\right)}{Y_1\left(\frac{m_n}{k}\right)}=\frac{J_0\left(\frac{m_n}{T}\right)-g_B^2 M_\lambda J_1\left(\frac{m_n}{T}\right)}{Y_0\left(\frac{m_n}{T}\right)-g_B^2 M_\lambda Y_1\left(\frac{m_n}{T}\right)}. \tag{24}$$

In each case we will be interested only in the lightest massive mode.

The results for the various spectra are shown in Figs. 2 – 4 for x ranging from .01 to 10. In these figures, we have chosen to normalize the particle masses, m, in units of $10\, m_{\tilde{e}}$ (*i.e.* the vertical axes represent $0.1\, m/m_{\tilde{e}}$), so that if the right-handed slepton masses are near their current experimental lower bound of around 100 GeV, the numbers labeling the vertical axes correspond roughly to masses in TeV units. In Fig. 2, the masses for the MSSM particles, XY gauge bosons and gauginos, and KK excitations of the 321 gauge bosons and gauginos are all shown. The value of T for a given x can be deduced from the fact that the XY and 321 KK gauge bosons have masses $\simeq 2.4\,T$; T ranges from $\simeq 100$ TeV on the left hand side of the plot to $\simeq 5$ TeV on the right hand side.[3] From this plot one sees some overall features of the spectrum: the XY and 321 KK gauge bosons are quite heavy, with masses larger than 10 TeV even if one assumes that $m_{\tilde{e}} \simeq 100$ GeV. On the other hand the masses of the 321 gaugino KK excitations decrease much more rapidly as x is increased, and form pseudo-Dirac states with the lightest gauginos at large x, where the values of the masses plateau at $m \simeq T/4$. Finally, the XY gaugino masses are lighter still, and continue to decrease with increasing x even after the 321 gaugino masses level off. In fact, we find that for large x the XY gaugino masses are given by [2]

$$M_{\rm XY} \simeq \frac{2}{g_{\rm 4D}^2 \ln(k/T)\, x} T, \tag{25}$$

[3]For these values of T, constraints from the precision electroweak measurements ($T \gtrsim 250$ GeV [10, 14]) are completely negligible.

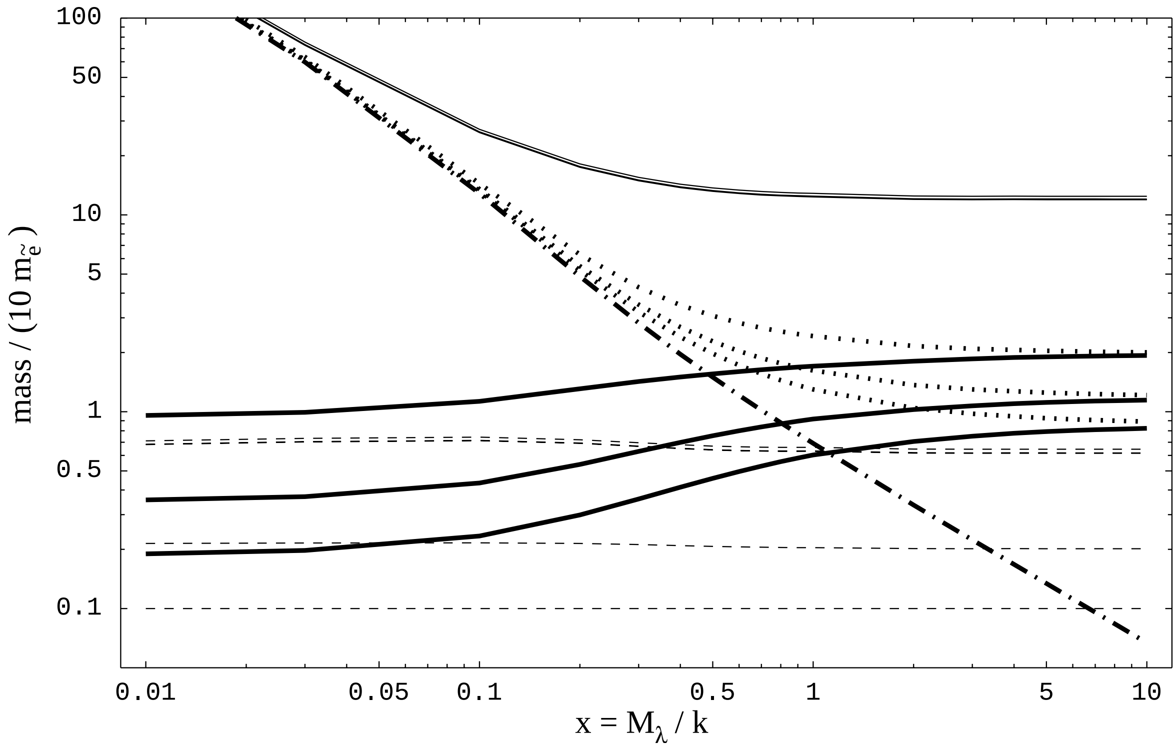

Figure 2: Masses of the MSSM scalars (dashed, with $m_{\tilde{q}}$, $m_{\tilde{u}}$, and $m_{\tilde{d}}$ closely spaced and $m_{\tilde{l}}$ and $m_{\tilde{e}}$ below), MSSM gauginos (thick solid), XY gauginos (dot-dashed), 321 gaugino KK modes (dotted), and XY and 321 KK gauge bosons (thin solid, nearly degenerate and most massive). As explained in the text, we give the masses in units of $10\, m_{\tilde{e}}$.

where $g_{4\mathrm{D}} = g_B/\sqrt{\pi R}$ is the "4D gauge coupling", which takes a value of $O(1)$. This fact significantly improves the discovery potential of the GUT particles at colliders.

Figs. 3 and 4 focus on the low-lying masses. In Fig. 3 the masses of the MSSM scalars and gauginos are plotted. From the figure we immediately see that the ratios of gaugino masses to scalar masses become larger for larger values for x. This is because for large x ($x \gtrsim 1$) the scale of superparticle masses is close to the scale at which they are generated, so that the scalar masses are purely one-loop suppressed compared with the gaugino masses; on the other hand, for $x \ll 1$, the scalar masses are enhanced by a logarithm between the scale of superparticle mass generation and the gaugino masses, $\ln(1/x)$, and become close to the gaugino masses. Another interesting feature is that the ratios among the scalar masses are relatively insensitive to x for the range considered here, while the gaugino mass ratios change significantly as x does (their masses are less hierarchical for larger x, a feature that may actually be easier to see on the log-scale plot of Fig. 2). It turns out that the gaugino masses satisfy $M_a \propto g_a^2$ for $x \ll 1$ and $M_a \propto g_a$ for $x \gg 1$ [11]. For scalar masses, we can see their rough scaling by studying

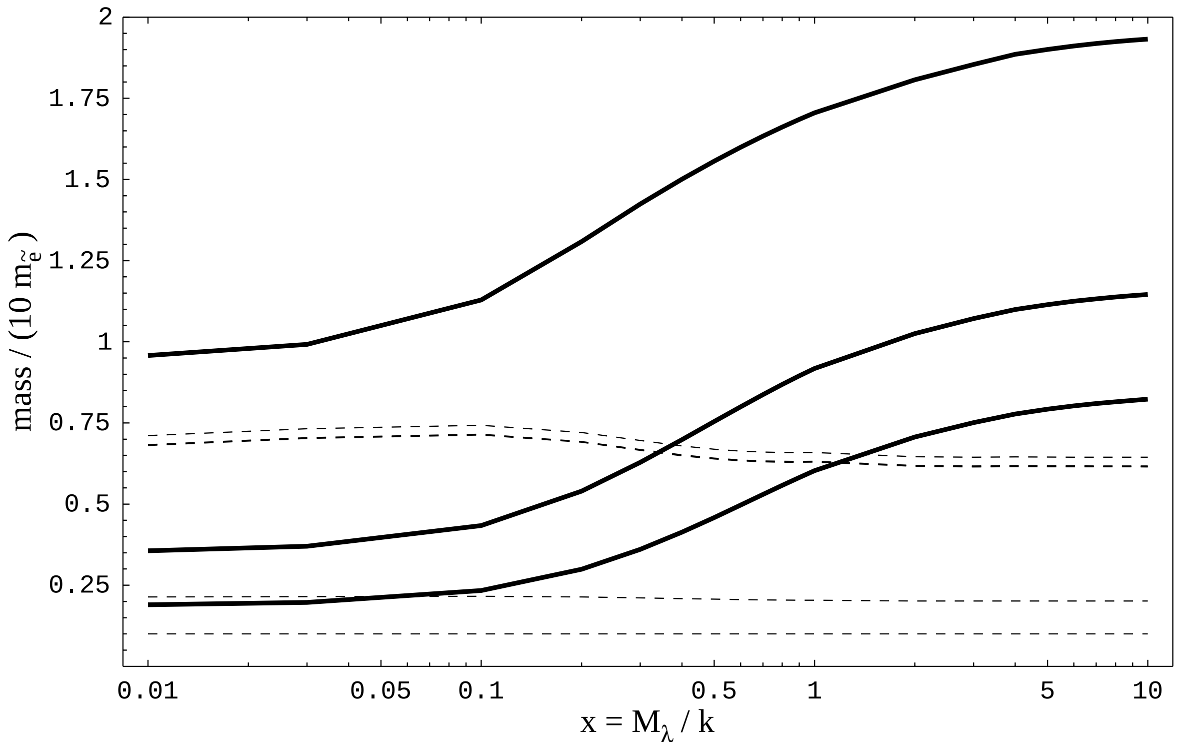

Figure 3: Masses of the MSSM scalars (dashed) and gauginos (solid).

the integrand of Eq. (21) evaluated at $q \sim T$, where the dominant contribution to the integral arises. This suggests that the scalar masses are approximately proportional to the square of the gauge couplings; in fact, we numerically find that the scalar masses scale roughly as the relevant Casimir times the relevant 4D gauge coupling squared (*e.g.* $(4/3)\, g_3^2$ for the squark masses). Thus, the lightest among the superpartners of the standard model fields are the right-handed sleptons for the entire range of x considered. Including effects of the Yukawa couplings, the lightest one will be the right-handed stau. Since the lightest supersymmetric particle (LSP) of the model is the gravitino $G_{3/2}$ with mass of order $T^2/M_{\rm Pl} \sim .01 - .1$ eV, it decays as $\tilde{\tau} \rightarrow \tau + G_{3/2}$ with a lifetime of order $8\pi T^4/m_{\tilde{e}}^5 \sim 10^{-18} - 10^{-14}$ sec.

In Fig. 4 the masses of the XY gauginos and 321 gaugino KK modes are shown with the MSSM gaugino masses. We see that for $x \gtrsim 0.4$, the XY gauginos can be lighter than 2 TeV, well within the reach of the LHC.[4] For these larger values of x, the 321 gaugino KK modes also become light, and for much of the parameter space for which the XY gauginos can be discovered, these KK modes are also experimentally accessible. Therefore, the experimental signatures in

[4]Although the XY particles are strongly localized to the TeV brane, giving exponentially small couplings to matter (and ensuring sufficient proton stability), their production rates are still unsuppressed because they have order one couplings to the standard model gauge fields.

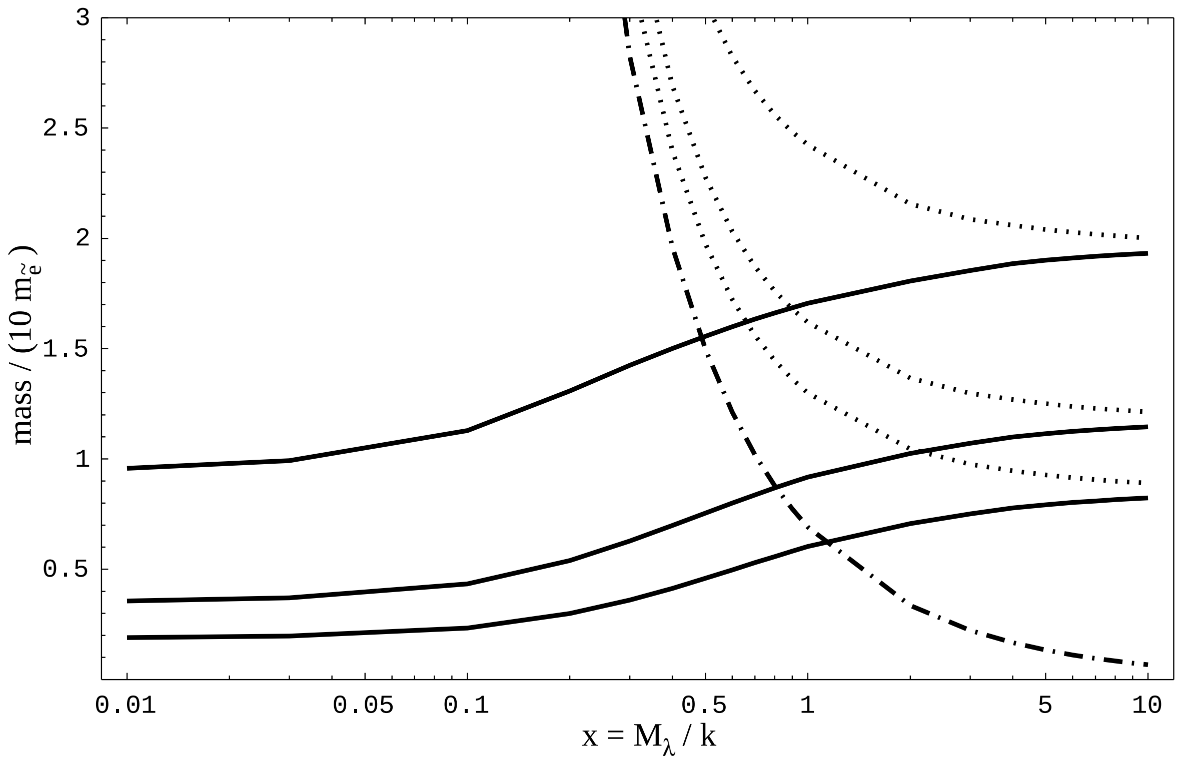

Figure 4: Masses of the MSSM gauginos (solid), XY gauginos (dot-dashed), and 321 gaugino KK modes (dotted).

this parameter region are quite distinct: we would find two gauginos for each 321 group, whose masses are very close for large values for x, and one XY gaugino, which will be stable and seen as highly ionizing tracks at colliders [2].[5] This raises the exciting possibility that both the underlying $SU(5)$ gauge structure and $N = 2$ supersymmetric structure of the model will be revealed by discovering these gauginos at the LHC.

It is interesting to point out that the three MSSM gaugino masses are determined by just two parameters, T and x, so that a single relation can be established among them. In particular, given the mass scale T, all the gaugino mass ratios are determined by a single mass ratio. This situation is depicted in Fig. 5, where we plot M_2/M_1 versus M_3/M_1 for the fixed value of $m_{\tilde{e}} = 100$ GeV. Moreover, the same two free parameters also determine the mass of the XY gaugino, $M_{\rm XY}$, so a measurement of, say, M_3 and M_1 would determine not only M_2, but also $M_{\rm XY}$. The labeled points on the line in Fig. 5 are meant to give a sense of how $M_{\rm XY}/M_1$ varies

[5] In fact, if the theory preserves a certain parity of the bulk Lagrangian called GUT parity, the XY gaugino is absolutely stable. In the present case with matter strongly localized to the Planck brane, the XY gaugino is effectively stable for collider purposes even in the absence of GUT parity.

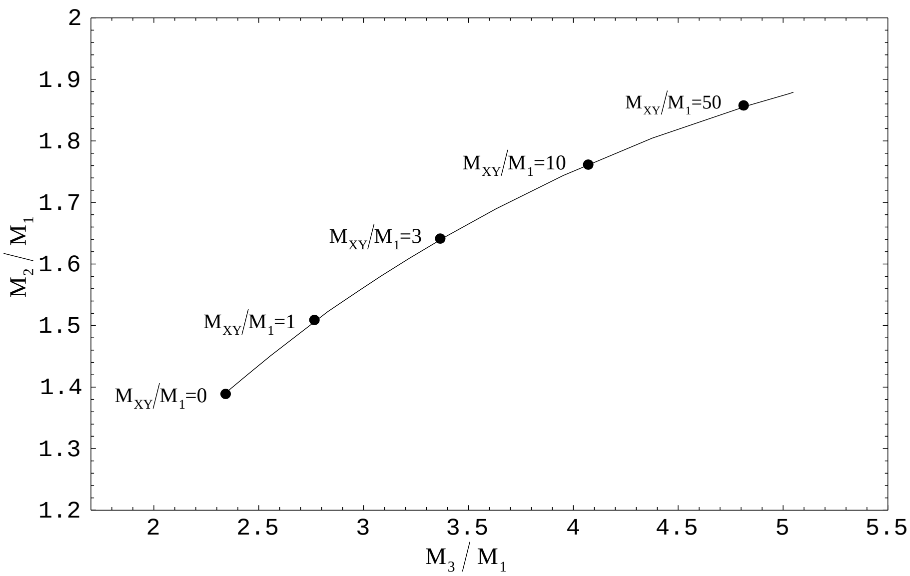

Figure 5: One gaugino mass ratio, M_2/M_1, given as a function of another, M_3/M_1. Each point on the line specifies a different value of $x = M_\lambda/k$, and therefore a different value of $M_{\rm XY}/M_1$. The labeled points correspond to $M_{\rm XY}/M_1 = 0, 1, 3, 10$, and 50.

as the MSSM gaugino mass ratios change. These constraints among the gaugino masses make this part of the spectrum especially interesting from an experimental standpoint. In particular, one could predict the mass of the XY gaugino once the 321 gaugino masses were measured with sufficient accuracy.[6]

We finally discuss uncertainties. As was already mentioned in section 2.2, higher dimensional operators on the TeV brane introduce an uncertainty in the superparticle masses that we expect to be of order $(k/M_*)^2 \lesssim 10\%$. We have used one-loop RGEs to evaluate the Planck-brane gauge kinetic terms at T, but errors coming from higher loops are small, especially because our results are not so sensitive to the relative size of the Planck-brane and bulk gauge couplings. Two-loop contributions to the sfermion masses could be sizable, especially for $x \ll 1$: in this regime the logarithmically enhanced higher-loop effects become larger. For $x \gtrsim .01$, however, we expect that those are still $O(10\%)$ effects. Based on these arguments we expect that our results are robust, at worst, at the $20-30\%$ level.

[6]It is worth mentioning that the gaugino mass relations shown in Fig. 5 (and in Fig. 4 up to an overall mass scale) are insensitive to certain model details. In particular, these results apply even if matter propagates in the bulk, in which case squark and slepton masses can arise at tree level, allowing much lower values for T.

4 Electroweak Symmetry Breaking

The naturalness of the electroweak symmetry breaking sector is an important issue for any extension of the standard model, and the original motivation for low-energy supersymmetry. The fact that we have not seen physics beyond the standard model typically requires some degree of fine-tuning among parameters in this sector. In supersymmetric theories, the experimental results that are most relevant for naturalness considerations are (i) the non-discovery of superparticles, and in particular gauginos, which can easily be lighter than the other superparticles because their masses are protected by an R symmetry, and (ii) the non-discovery of the lightest Higgs boson, which is predicted to be light in supersymmetric extensions of the standard model. In the MSSM, the physical mass of the lightest Higgs boson can be as large as ~ 130 GeV, but for the Higgs boson to have evaded detection at LEP II requires somewhat large top squark masses. These in turn generate a large negative contribution to the soft mass-squared parameter for the Higgs field, introducing some degree of fine-tuning in the Higgs potential.

In our theory all superparticles are heavy enough to evade experimental bounds provided that the right-handed slepton mass is larger than about 100 GeV. This is clear from Figs. 2 and 3: the gaugino masses are significantly heavier than the scalars, especially for larger x, because they acquire tree-level masses. The right-handed stau will be somewhat lighter than the other right-handed sleptons due to Yukawa-induced radiative corrections (not included in our analysis), but the experimental bound on the stau mass is also somewhat less stringent, so we keep 100 GeV as a representative value for the lower bound.

Now we can ask the following question: for a given value of x, if we take T large enough to evade the bound on the right-handed slepton mass, how heavy is the lightest Higgs boson? Keeping all other parameters fixed, the Higgs mass decreases as x increases because the top squark masses decrease (see Fig. 3). The left-right mixing in the top squark mass matrices is dominated for moderately large $\tan\beta$ by a loop-generated A_t term,

$$A_t = \frac{2y_t}{3\pi^2} \int_0^\infty dq\, q\, h_{z,3}\Big(z = z' = \frac{1}{k}; q\Big). \tag{26}$$

Here we have included only the $SU(3)_C$ loop, y_t is the top Yukawa coupling, and $h_{z,3}$ is the function defined in Appendix A (for $SU(3)_C$). We find that the mixing induced is quite small. A no-mixing scenario requires a somewhat larger overall mass scale for superparticles, especially for $x \gtrsim 1$, to obtain a sufficiently large Higgs boson mass: we estimate $m_{\tilde{e}} \gtrsim 200$ GeV is required for $x \gg 1$. Even with these large values of superparticle masses, however, the fine-tuning in our theory is not as severe as one might naively imagine based on the squark masses alone. This is because the scale of the superpartner masses is close to the the scale where they are generated, especially for large values of x, and thus even if the top squark is somewhat heavy, it will not give too large a negative contribution to the Higgs mass-squared parameter. This is an interesting point: the correction to the physical Higgs-boson mass arises below the stop mass, and goes like $\ln(m_{\tilde{t}}/m_t)$, while the correction to the Higgs mass-squared parameter arises above the stop mass, and goes like $\ln(M_{\rm mess}/m_{\tilde{t}})$, where $M_{\rm mess}$ is the scale where soft masses are generated.

An alternative way of obtaining a large Higgs boson mass is to introduce a singlet superfield S on the Planck brane with the superpotential interactions $2\delta(y)(\lambda S H_D \bar{H}_D + \kappa S^3)$. This setup is also motivated as a way to naturally induce the supersymmetric mass term (μ term) for the Higgs doublets through the vacuum expectation value for the S field.[7] In this case the Higgs boson mass receives additional contribution at tree level thorough the coupling λ. The size of this contribution depends on the value of λ, whose upper bound is set by Landau pole considerations. An interesting point is that in our theory the 321 gauge couplings become strong at high energies so that the bound on λ is significantly weaker. (This fact can be understood more easily in terms of the 4D dual picture of the theory [2].) This allows us to have the weak scale value of λ as large as $\lambda \simeq 0.8$, and we obtain a large enough Higgs boson mass for $\tan\beta \lesssim 5$ even with $m_{\tilde{e}} \simeq 100$ GeV.

How finely tuned is electroweak symmetry breaking in our theory? A precise discussion of naturalness requires the values of the Higgs mass-squared parameters, and these depend on the details of the Higgs sector. Here we simply estimate their sizes for Planck-brane localized Higgs doublets, in which case the soft masses vanish at tree level. The dominant radiative corrections come from the one-loop $SU(2)_L$ gauge contribution and the two-loop top-Yukawa $SU(3)_C$-gauge contribution. The former is calculated in the previous section (given by Eq. (19) with $m_{h_u}^2|_{\alpha_2} = m_{h_d}^2|_{\alpha_2} = m_{\tilde{l}}^2$), which gives $m_{h_u}^2|_{\alpha_2} \simeq 4\, m_{\tilde{e}}^2$ for large x. The latter we expect is similar to the flat space case for $x \gg 1$, with the gluino masses in the two theories identified. Referring to [15], we obtain $m_{h_u}^2|_{\alpha_3\alpha_t} \simeq -5\, m_{\tilde{e}}^2$, although the precise value is quite sensitive to the choice of the renormalization scale *etc.*, and the exact number we quote is not really trustworthy. Nevertheless, it is reasonable to expect $m_{h_u}^2 = m_{h_u}^2|_{\alpha_2} + m_{h_u}^2|_{\alpha_3\alpha_t} \simeq -c\, m_{\tilde{e}}^2$, where c is a factor of a few. The amount of fine-tuning is given roughly by $|m_Z/m_{h_u}|^2/2$ for moderately large $\tan\beta$, and this is of order 10% if we take $m_{\tilde{e}}$ at its present experimental bound. This is relatively mild. The bound on the physical Higgs boson mass may push up the superparticle mass scale higher, and for smaller x, the fine-tuning might change due to $\ln(1/x)$ enhancements in the loop-induced scalar masses. However, we still expect that the fine-tuning is not very severe. Although this estimate is based on top-derived radiative electroweak symmetry breaking, it illustrates the required fine-tuning in our theory even for more general Higgs sectors.

[7] With these superpotential interactions, the $U(1)_R$ symmetry discussed in section 2.1 is explicitly broken to the $Z_{4,R}$ subgroup, under which S carries a charge of $+2$. This $Z_{4,R}$ symmetry, however, is still sufficient to forbid unwanted dimension four and five proton decay operators and a large supersymmetric mass term for the Higgs doublets.

We finally comment on the possibility of a very simple and constrained scenario. Suppose that both matter and Higgs fields are localized to the Planck brane (either exactly or approximately, by bulk hypermultiplet masses), and a μ term of order the weak scale is generated on the Planck brane. Then, assuming top-Yukawa driven radiative electroweak symmetry breaking, the entire superpartner spectrum and the parameters of the Higgs potential can all be calculated in terms of only three parameters, x, T and μ, one of which is fixed by the observed value of the Higgs expectation value v. In this setup, not only supersymmetric flavor problem, but also the supersymmetric CP problem is solved, because the A terms and the B term are all generated radiatively through gaugino loops, and are thus all real in the basis where the gaugino masses are real. The sign of μ is determined to be negative in the standard phase convention. A detailed study of this scenario will be interesting.

5 Conclusions

Warped supersymmetric grand unification has some remarkable features. Most notably, it preserves MSSM-like gauge coupling unification even though it predicts that the exotic particles associated with grand unification – XY gauge bosons, for example – appear near the TeV scale. These light exotics would normally induce baryon-number violating processes at disastrous rates, but here proton decay is naturally suppressed simply by localizing matter to the Planck brane.

This framework also accommodates simple, constrained possibilities for supersymmetry breaking. In this paper we considered a setup with unsuppressed, or only mildly suppressed, supersymmetry breaking localized to the $SU(5)$-preserving TeV brane. In such a scenario, the superpartners of the standard model fields naturally acquire TeV-scale masses. Taking the quark and lepton superfields to be localized to the Planck brane, advantageous for suppressing proton decay and enforcing flavor universality in the squark and slepton masses, one is led to a highly predictive model. In this case, the masses of the MSSM squarks, sleptons, and gauginos, along with those of the XY gauge bosons and gauginos and KK excitations of MSSM gauge particles, are all calculable in terms of two parameters: T, the scale of the infrared brane, and $x \equiv M_\lambda/k$, the ratio of the supersymmetry-breaking gaugino mass on the TeV brane to the curvature scale. Calculating these masses was our main purpose.

In this setup the gauginos acquire masses at tree level and are heavier than the squarks and sleptons, which only acquire masses at loop level, and the lightest MSSM superparticles are the right-handed sleptons (including Yukawa effects the lightest one is the right-handed stau, which decays into the LSP gravitino). Requiring that the right-handed sleptons are heavy enough to evade detection at colliders sets a lower bound on T for a given value of x. From this lower bound we infer that the XY gauge bosons and KK excitations of the standard model gauge bosons are too heavy to be detected at the LHC, with masses larger than 10 TeV regardless of how large x becomes. On the other hand, supersymmetry breaking has a dramatic impact on the masses of the supersymmetric partners of these gauge bosons, pushing some of the gaugino states to be considerably lighter. In fact, for $x \gtrsim 0.4$, we find that the XY gauginos can have masses below 2 TeV, and for these larger values of x, the KK excitations of the MSSM gauginos are also relatively light, and approach degeneracy with the MSSM gauginos themselves as x is increased. The fact that the masses of these particles can be light enough to be discovered at the LHC – which would reveal both the underlying unified gauge symmetry and the enhanced $N = 2$ supersymmetry of the theory – is the most important result of this paper.

Our results for the spectra of GUT particles and superparticles are independent of the Higgs sector, but we also briefly considered electroweak symmetry breaking in the particularly constrained setup where the Higgs doublets are localized to the Planck brane. In the minimal scenario, the experimental bound on the Higgs boson mass may require the right-handed slepton mass to be somewhat larger than its experimental lower bound of 100 GeV. On the other hand, a superpotential coupling between the Higgs doublets and a Planck-brane localized singlet S, motivated independently as a means for generating a weak-scale μ term, allows the Higgs mass to be raised above its lower bound quite easily. The effect on the Higgs mass can be stronger than in the conventional NMSSM because our theory has relatively strong gauge interactions at high energies, which drive the $SH_D\bar{H}_D$ coupling away from its Landau pole. It will be interesting to explore electroweak symmetry breaking in this model in greater detail.

Acknowledgments

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Appendix A

A.1 Gaugino propagators

In this appendix we derive the propagators for the 321 gauginos in the presence of general brane kinetic terms and a TeV-brane localized Majorana mass. The free action for the gauginos is given by

$$\begin{aligned} S &= \int d^4x \int_0^{\pi R} dy \left\{ \frac{e^{-4ky}}{g_B^2} \Big[e^{ky}(\lambda^\dagger i\bar\sigma^\mu \partial_\mu \lambda + \lambda' i\sigma^\mu \partial_\mu \lambda'^\dagger) + \lambda'(\partial_y - \frac{3k}{2})\lambda + \lambda^\dagger(-\partial_y - \frac{3k}{2})\lambda'^\dagger \Big] \right. \\ & \left. +2\delta(y)\Big[\frac{1}{\tilde g_0^2}\lambda^\dagger i\bar\sigma^\mu\partial_\mu\lambda\Big] + 2\delta(y-\pi R)e^{-4\pi kR}\Big[\frac{e^{\pi kR}}{\tilde g_\pi^2}\lambda^\dagger i\bar\sigma^\mu\partial_\mu\lambda - \frac{M_\lambda}{2}\lambda\lambda - \frac{M_\lambda}{2}\lambda^\dagger\lambda^\dagger\Big] \right\}, \end{aligned} \quad (27)$$

where λ and λ' are the two-component gaugino fields that are contained in 4D superfields V and Σ, respectively. Here, we have suppressed the index a running over $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ for the simplicity of the notation, but it should be understood that the Planck-brane localized kinetic term, $\tilde g_0$, takes different values for $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$: $\tilde g_0 \to \tilde g_{0,a}$. The other parameters, g_B, $\tilde g_\pi$ and M_λ, are $SU(5)$ symmetric and have universal values for $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$.

It is useful to define the rescaled gaugino fields $\hat\lambda \equiv e^{-2ky}\lambda$ and $\hat\lambda' \equiv e^{-2ky}\lambda'$. Then, in terms of the Fourier transformed fields $\tilde\lambda(p,y) = \int d^4x \hat\lambda(x,y)e^{ipx}$ and $\tilde\lambda'(p,y) = \int d^4x \hat\lambda'(x,y)e^{ipx}$, the above action is written as

$$\begin{aligned} S &= \int \frac{d^4p}{(2\pi)^4} \int_0^{\pi R} dy \left\{ \Big[\frac{e^{ky}}{g_B^2}\big(\tilde\lambda^\dagger(p)\bar\sigma^\mu p_\mu \tilde\lambda(p) + \tilde\lambda'(-p)\sigma^\mu p_\mu \tilde\lambda'^\dagger(-p)\big) \right. \\ & + \frac{1}{g_B^2}\big(\tilde\lambda'(-p)(\partial_y + \frac{k}{2})\tilde\lambda(p) + \tilde\lambda^\dagger(p)(-\partial_y + \frac{k}{2})\tilde\lambda'^\dagger(-p)\big)\Big] + 2\delta(y)\Big[\frac{1}{\tilde g_0^2}\tilde\lambda^\dagger(p)\bar\sigma^\mu p_\mu\tilde\lambda(p)\Big] \\ & \left. + 2\delta(y-\pi R)\Big[\frac{e^{\pi kR}}{\tilde g_\pi^2}\tilde\lambda^\dagger(p)\bar\sigma^\mu p_\mu \tilde\lambda(p) - \frac{M_\lambda}{2}\tilde\lambda(p)\tilde\lambda(-p) - \frac{M_\lambda}{2}\tilde\lambda^\dagger(p)\tilde\lambda^\dagger(-p)\Big]\right\}. \end{aligned} \quad (28)$$

The content of this action can be divided into two parts. First, by examining the region near $y=0$ and πR in the equations of motion derived from Eq. (28), we find the following conditions imposed on the fields:

$$-\frac{1}{g_B^2}\tilde\lambda'^\dagger(-p)\Big|_{y=\epsilon} + \frac{1}{\tilde g_0^2}\bar\sigma^\mu p_\mu\tilde\lambda(p)\Big|_{y=0} = 0, \quad (29)$$

$$\frac{1}{g_B^2}\sigma^\mu p_\mu\tilde\lambda'^\dagger(-p)\Big|_{y=\epsilon} + \frac{1}{g_B^2}(\partial_y + \frac{k}{2})\tilde\lambda(p)\Big|_{y=\epsilon} = 0, \quad (30)$$

$$\frac{1}{g_B^2}\tilde\lambda'^\dagger(-p)\Big|_{y=\pi R-\epsilon} + \frac{e^{\pi kR}}{\tilde g_\pi^2}\bar\sigma^\mu p_\mu\tilde\lambda(p)\Big|_{y=\pi R} - M_\lambda\tilde\lambda^\dagger(-p)\Big|_{y=\pi R} = 0, \quad (31)$$

$$\frac{e^{\pi kR}}{g_B^2}\sigma^\mu p_\mu\tilde\lambda'^\dagger(-p)\Big|_{y=\pi R-\epsilon} + \frac{1}{g_B^2}(\partial_y + \frac{k}{2})\tilde\lambda(p)\Big|_{y=\pi R-\epsilon} = 0, \quad (32)$$

together with the equations obtained by interchanging $\tilde{\lambda}(p) \leftrightarrow \tilde{\lambda}^\dagger(-p)$, $\tilde{\lambda}'(p) \leftrightarrow \tilde{\lambda}'^\dagger(-p)$ and $\sigma^\mu \leftrightarrow \bar{\sigma}^\mu$ in the above equations; here $\epsilon \to 0$. These conditions provide boundary conditions for the corresponding propagators because the propagators are given by the time-ordered products of the two fields evaluated on the vacuum: $G_{\varphi\varphi'} = \langle 0|T\{\varphi\varphi'\}|0\rangle$. Second, the bulk piece (the terms that do not involve delta functions) of Eq. (28) is written as

$$\begin{aligned} S_{\text{bulk}} &= \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\int_0^{\pi R} dy \left(\begin{array}{cc|cc} \tilde{\lambda}^\dagger(p) & \tilde{\lambda}'(-p) & \tilde{\lambda}(-p) & \tilde{\lambda}'^\dagger(p) \end{array} \right) \\ &\times \left(\begin{array}{cc|cc} \frac{e^{ky}}{g_B^2}\bar{\sigma}^\mu p_\mu & \frac{1}{g_B^2}(-\partial_y + \frac{k}{2}) & 0 & 0 \\ \frac{1}{g_B^2}(\partial_y + \frac{k}{2}) & \frac{e^{ky}}{g_B^2}\sigma^\mu p_\mu & 0 & 0 \\ \hline 0 & 0 & \frac{e^{ky}}{g_B^2}\sigma^\mu p_\mu & \frac{1}{g_B^2}(-\partial_y + \frac{k}{2}) \\ 0 & 0 & \frac{1}{g_B^2}(\partial_y + \frac{k}{2}) & \frac{e^{ky}}{g_B^2}\bar{\sigma}^\mu p_\mu \end{array} \right) \left(\begin{array}{c} \tilde{\lambda}(p) \\ \tilde{\lambda}'^\dagger(-p) \\ \hline \tilde{\lambda}^\dagger(-p) \\ \tilde{\lambda}'(p) \end{array} \right). \end{aligned} \tag{33}$$

This piece dictates the form of the propagators in the bulk. Defining the 2×2 matrix appearing in Eq. (33) as M_{bulk}, the propagators

$$\hat{G} \equiv \left(\begin{array}{cc|cc} \hat{G}_{\lambda\lambda^\dagger}(y,y';p) & \hat{G}_{\lambda\lambda'}(y,y';p) & \hat{G}_{\lambda\lambda}(y,y';p) & \hat{G}_{\lambda\lambda'^\dagger}(y,y';p) \\ \hat{G}_{\lambda'^\dagger\lambda^\dagger}(y,y';p) & \hat{G}_{\lambda'^\dagger\lambda'}(y,y';p) & \hat{G}_{\lambda'^\dagger\lambda}(y,y';p) & \hat{G}_{\lambda'^\dagger\lambda'^\dagger}(y,y';p) \\ \hline \hat{G}_{\lambda^\dagger\lambda^\dagger}(y,y';p) & \hat{G}_{\lambda^\dagger\lambda'}(y,y';p) & \hat{G}_{\lambda^\dagger\lambda}(y,y';p) & \hat{G}_{\lambda^\dagger\lambda'^\dagger}(y,y';p) \\ \hat{G}_{\lambda'\lambda^\dagger}(y,y';p) & \hat{G}_{\lambda'\lambda'}(y,y';p) & \hat{G}_{\lambda'\lambda}(y,y';p) & \hat{G}_{\lambda'\lambda'^\dagger}(y,y';p) \end{array} \right), \tag{34}$$

are given as a solution of

$$M_{\text{bulk}} \cdot \hat{G} = i\delta(y - y')\,\mathbf{1}, \tag{35}$$

in the bulk, where $\mathbf{1}$ is the 4×4 unit matrix. Note that $\hat{G}$ represents propagators for the rescaled fields, $\hat{\lambda}$ and $\hat{\lambda}'$, which are related to propagators G for the unrescaled fields, λ and λ', as $G = e^{2k(y+y')}\hat{G}$.

Now, let us derive the bulk propagator by solving Eq. (35). Parameterizing the matrix $\hat{G}$ of Eq. (34) as

$$\hat{G} = \left(\begin{array}{cc|cc} i\sigma^\mu p_\mu\, f & ie^{-ky}(\partial_y - \frac{k}{2})\, f' & i\, h & \frac{i\sigma^\mu p_\mu}{p^2} e^{-ky}(\partial_y - \frac{k}{2})\, h' \\ ie^{-ky}(-\partial_y - \frac{k}{2})\, f & i\bar{\sigma}^\mu p_\mu\, f' & \frac{i\bar{\sigma}^\mu p_\mu}{p^2} e^{-ky}(-\partial_y - \frac{k}{2})\, h & i\, h' \\ \hline i\, h & \frac{i\bar{\sigma}^\mu p_\mu}{p^2} e^{-ky}(\partial_y - \frac{k}{2})\, h' & i\bar{\sigma}^\mu p_\mu\, f & ie^{-ky}(\partial_y - \frac{k}{2})\, f' \\ \frac{i\sigma^\mu p_\mu}{p^2} e^{-ky}(-\partial_y - \frac{k}{2})\, h & i\, h' & ie^{-ky}(-\partial_y - \frac{k}{2})\, f & i\sigma^\mu p_\mu\, f' \end{array} \right), \tag{36}$$

where f, f', h and h' are the functions of y, y' and p, we find that the functions f, f', h and h' obey the following equations:

$$\frac{1}{g_B^2}\Big[p^2 e^{ky} + e^{-ky}(\partial_y^2 - k\partial_y - \frac{3}{4}k^2)\Big] f(y,y';p) = \delta(y - y'), \tag{37}$$

$$\frac{1}{g_B^2}\Big[p^2 e^{ky} + e^{-ky}(\partial_y^2 - k\partial_y + \frac{1}{4}k^2)\Big] f'(y,y';p) = \delta(y-y'), \tag{38}$$

$$\Big[p^2 e^{ky} + e^{-ky}(\partial_y^2 - k\partial_y - \frac{3}{4}k^2)\Big] h(y,y';p) = 0, \tag{39}$$

$$\Big[p^2 e^{ky} + e^{-ky}(\partial_y^2 - k\partial_y + \frac{1}{4}k^2)\Big] h'(y,y';p) = 0. \tag{40}$$

Changing the variable from y to $z = e^{ky}/k$, the equations become

$$\frac{1}{g_B^2}\Big[p^2 + \partial_z^2 - \frac{3}{4}\frac{1}{z^2}\Big] f_z(z,z';p) = \delta(z-z'), \tag{41}$$

$$\frac{1}{g_B^2}\Big[p^2 + \partial_z^2 + \frac{1}{4}\frac{1}{z^2}\Big] f'_z(z,z';p) = \delta(z-z'), \tag{42}$$

$$\Big[p^2 + \partial_z^2 - \frac{3}{4}\frac{1}{z^2}\Big] h_z(z,z';p) = 0, \tag{43}$$

$$\Big[p^2 + \partial_z^2 + \frac{1}{4}\frac{1}{z^2}\Big] h'_z(z,z';p) = 0, \tag{44}$$

where the functions with the subscript z represent the functions obtained by changing variables from y and y' to z and z': $f_z(z,z';p) \equiv f(\ln(kz)/k, \ln(kz')/k;p)$ and so on. Considering the region $z \neq z'$, we find that the solutions to Eqs. (41 – 44) are given as

$$f_z(z,z';p) = \sqrt{z}\Big(a_>(z')\, I_1(|p|z) + b_>(z')\, K_1(|p|z)\Big), \tag{45}$$

$$f'_z(z,z';p) = \sqrt{z}\Big(a'_>(z')\, I_0(|p|z) + b'_>(z')\, K_0(|p|z)\Big), \tag{46}$$

$$h_z(z,z';p) = \sqrt{z}\Big(\alpha_>(z')\, I_1(|p|z) + \beta_>(z')\, K_1(|p|z)\Big), \tag{47}$$

$$h'_z(z,z';p) = \sqrt{z}\Big(\alpha'_>(z')\, I_0(|p|z) + \beta'_>(z')\, K_0(|p|z)\Big), \tag{48}$$

for $z > z'$, where $I_n(x)$ and $K_n(x)$ are the modified Bessel functions of order n and $|p| \equiv \sqrt{-p^2}$. For $z < z'$ the functions $a_>$, $b_>$, $a'_>$, $b'_>$, $\alpha_>$, $\beta_>$, $\alpha'_>$ and $\beta'_>$ take different forms, which we denote as $a_<$, $b_<$, $a'_<$, $b'_<$, $\alpha_<$, $\beta_<$, $\alpha'_<$ and $\beta'_<$, respectively.

The propagators must obey the conditions of Eqs. (29 – 32). This allows us to solve the functions $b_>$, $b'_>$, $\beta_>$ and $\beta'_>$ in terms of $a_>$, $a'_>$, $\alpha_>$ and $\alpha'_>$; and similarly, $b_<$, $b'_<$, $\beta_<$ and $\beta'_<$ in terms of $a_<$, $a'_<$, $\alpha_<$ and $\alpha'_<$. Further constraints on the remaining functions come from the continuity of the propagators at $z = z'$:

$$f_z(z,z';p)|_{z=z'+\epsilon} = f_z(z,z';p)|_{z=z'-\epsilon}, \tag{49}$$

$$f'_z(z,z';p)|_{z=z'+\epsilon} = f'_z(z,z';p)|_{z=z'-\epsilon}, \tag{50}$$

$$h_z(z,z';p)|_{z=z'+\epsilon} = h_z(z,z';p)|_{z=z'-\epsilon}, \tag{51}$$

$$h'_z(z,z';p)|_{z=z'+\epsilon} = h'_z(z,z';p)|_{z=z'-\epsilon}, \tag{52}$$

and the junction conditions following from Eqs. (41 – 44) at $z = z'$:

$$\begin{aligned}
\partial_z f_z(z,z';p)|_{z=z'+\epsilon} - \partial_z f_z(z,z';p)|_{z=z'-\epsilon} &= g_B^2, & (53)\\
\partial_z f'_z(z,z';p)|_{z=z'+\epsilon} - \partial_z f'_z(z,z';p)|_{z=z'-\epsilon} &= g_B^2, & (54)\\
\partial_z h_z(z,z';p)|_{z=z'+\epsilon} - \partial_z h_z(z,z';p)|_{z=z'-\epsilon} &= 0, & (55)\\
\partial_z h'_z(z,z';p)|_{z=z'+\epsilon} - \partial_z h'_z(z,z';p)|_{z=z'-\epsilon} &= 0. & (56)
\end{aligned}$$

These equations completely determine the functions $a_>$, $a'_>$, $\alpha_>$, $\alpha'_>$, $a_<$, $a'_<$, $\alpha_<$ and $\alpha'_<$.

After some algebra, we finally find that the gaugino propagators defined in Eq. (34) are given by Eq. (36) with

$$\begin{aligned}
f_z(z,z';p) &= \frac{g_B^2\sqrt{z_<z_>}}{(C-A)^2+B^2}\Big(I_1(|p|z_<)+CK_1(|p|z_<)\Big)\\
&\quad\times\Big((C-A)\Big\{I_1(|p|z_>)+AK_1(|p|z_>)\Big\}-B^2K_1(|p|z_>)\Big), & (57)\\
f'_z(z,z';p) &= -\frac{g_B^2\sqrt{z_<z_>}}{(C-A)^2+B^2}\Big(I_0(|p|z_<)-CK_0(|p|z_<)\Big)\\
&\quad\times\Big((C-A)\Big\{I_0(|p|z_>)-AK_0(|p|z_>)\Big\}+B^2K_0(|p|z_>)\Big), & (58)\\
h_z(z,z';p) &= -\frac{g_B^2|p|\sqrt{z_<z_>}}{(C-A)^2+B^2}\Big(I_1(|p|z_<)+CK_1(|p|z_<)\Big)B\Big(I_1(|p|z_>)+CK_1(|p|z_>)\Big), & (59)\\
h'_z(z,z';p) &= \frac{g_B^2|p|\sqrt{z_<z_>}}{(C-A)^2+B^2}\Big(I_0(|p|z_<)-CK_0(|p|z_<)\Big)B\Big(I_0(|p|z_>)-CK_0(|p|z_>)\Big), & (60)
\end{aligned}$$

where $|p| = \sqrt{-p^2}$ and $z_<$ ($z_>$) is the lesser (greater) of z and z'; the functions f_z, f'_z, h_z and h'_z are related to f, f', h and h' by $f_z(z,z';p) = f(y,y';p)$ *etc.* with $z = e^{ky}/k$. The coefficients A, B and C are given by

$$A = \frac{X_IX_K - Y_IY_K}{X_K^2+Y_K^2}, \qquad B = \frac{X_IY_K + X_KY_I}{X_K^2+Y_K^2}, \qquad C = \frac{Z_I}{Z_K}. \tag{61}$$

Here,

$$\begin{cases} X_I = \frac{1}{g_B^2}I_0(\frac{|p|}{T}) + \frac{|p|}{\tilde{g}_\pi^2}\frac{k}{T}I_1(\frac{|p|}{T}), \\ X_K = \frac{1}{g_B^2}K_0(\frac{|p|}{T}) - \frac{|p|}{\tilde{g}_\pi^2}\frac{k}{T}K_1(\frac{|p|}{T}), \end{cases} \quad \begin{cases} Y_I = M_\lambda I_1(\frac{|p|}{T}), \\ Y_K = M_\lambda K_1(\frac{|p|}{T}), \end{cases} \quad \begin{cases} Z_I = \frac{1}{g_B^2}I_0(\frac{|p|}{k}) - \frac{|p|}{\tilde{g}_0^2}I_1(\frac{|p|}{k}), \\ Z_K = \frac{1}{g_B^2}K_0(\frac{|p|}{k}) + \frac{|p|}{\tilde{g}_0^2}K_1(\frac{|p|}{k}), \end{cases} \tag{62}$$

where $T \equiv ke^{-\pi kR}$.

A.2 Limiting behaviors

We here consider various limits of the gaugino propagators. We first consider the "4D limit", in which the compactification scale is sent to infinity and the theory reduces to the 4D MSSM.

This limit is obtained by taking $|p|/T \to 0$ keeping T/k and $|p|/M_\lambda$ fixed. The function f_z then becomes

$$f_z(z, z'; p) \to -\frac{1}{k\sqrt{z_< z_>}} \frac{g_{4\mathrm{D}}^2}{|p|^2 + (g_{4\mathrm{D}}^2 M_\lambda \frac{T}{k})^2}, \tag{63}$$

where $g_{4\mathrm{D}}$ is the 4D gauge coupling given by $1/g_{4\mathrm{D}}^2 = \pi R/g_B^2 + 1/\tilde{g}_0^2 + 1/\tilde{g}_\pi^2$. The MSSM gaugino propagator, $G_{\lambda\lambda^\dagger}^{4\mathrm{D}}(p)$, is obtained by multiplying the propagator in this "4D limit", $\hat{G}_{\lambda\lambda^\dagger}(y, y'; p) = i\sigma^\mu p_\mu f(y, y'; p)$, by the MSSM gaugino wavefunction in $\hat{\lambda}(x, y)$, $e^{3ky/2}$, and setting $y = y' = 0$. Considering the 4D gaugino mass is given by $M_{\lambda,4\mathrm{D}} = g_{4\mathrm{D}}^2 M_\lambda T/k$ (see Appendix B), this is given by

$$G_{\lambda\lambda^\dagger}^{4\mathrm{D}}(p) = g_{4\mathrm{D}}^2 \frac{i\sigma^\mu p_\mu}{p^2 - M_{\lambda,4\mathrm{D}}^2}, \tag{64}$$

reproducing the 4D gaugino propagator.

We next consider the propagator with the external points both on the Planck brane in the limit where the momentum scale $|p|$ is much larger than the IR scale T: $|p| \gg T$. We are interested in the difference between the gaugino propagator in the presence and absence of supersymmetry breaking. For $|p| \gg T$, the relevant quantity $\bar{f}(p) \equiv f_z(z = z' = 1/k; p) - f_z(z = z' = 1/k; p)|_{M_\lambda = 0}$ is given by

$$\bar{f}(p) = \frac{2\pi M_\lambda^2}{k g_B^4} \frac{\left(\frac{I_1(\frac{|p|}{k}) K_0(\frac{|p|}{k}) + I_0(\frac{|p|}{k}) K_1(\frac{|p|}{k})}{\frac{1}{g_B^2} K_0(\frac{|p|}{k}) + \frac{|p|}{\tilde{g}_0^2} K_1(\frac{|p|}{k})} \right)^2}{\left(\frac{1}{g_B^2} + \frac{|p|}{\tilde{g}_\pi^2}\frac{k}{T} \right) \left(\left(\frac{1}{g_B^2} + \frac{|p|}{\tilde{g}_\pi^2}\frac{k}{T} \right)^2 + M_\lambda^2 \right)} e^{-\frac{2|p|}{T}}, \tag{65}$$

which shows that $\bar{f}(p)$ is exponentially suppressed for $p \gg T$. This ensures the UV insensitivity for the scalar masses computed in section 3 (see Eqs. (16 – 21)).

Appendix B

In this appendix we present a formula for the 321 gaugino masses, applicable for any value of $x \equiv M_\lambda/k$, in the presence of general brane-localized kinetic terms. We start with the action given in Eq. (27). This action gives the following equations of motion in the bulk:

$$\frac{e^{ky}}{g_B^2} i\bar{\sigma}^\mu \partial_\mu \hat{\lambda} + \frac{1}{g_B^2}(-\partial_y + \frac{k}{2})\hat{\lambda}'^\dagger = 0, \tag{66}$$

$$\frac{e^{ky}}{g_B^2} i\sigma^\mu \partial_\mu \hat{\lambda}'^\dagger + \frac{1}{g_B^2}(\partial_y + \frac{k}{2})\hat{\lambda} = 0, \tag{67}$$

where we have presented the equations in terms of the rescaled gaugino fields, $\hat{\lambda} \equiv e^{-2ky}\lambda$ and $\hat{\lambda}' \equiv e^{-2ky}\lambda'$. Looking for solutions of the form

$$\hat{\lambda}(x,y) = \sum_n \lambda_n(x) f_n^{\lambda}(y), \qquad \hat{\lambda}'(x,y) = \sum_n \lambda_n(x) f_n^{\lambda'}(y), \tag{68}$$

the bulk equations of motion, Eqs. (66, 67), lead to the following differential equation for f_n^{λ}:

$$\partial_y^2 f_n^{\lambda} - k\partial_y f_n^{\lambda} - \frac{3k^2}{4} f_n^{\lambda} + m_n^2 e^{2ky} f_n^{\lambda} = 0. \tag{69}$$

Here, m_n is the 4D masses and we have used the 4D relation $i\bar{\sigma}^{\mu}\partial_{\mu}\lambda_n = m_n \lambda_n^{\dagger}$. The solution of this equation is given by

$$f_n^{\lambda}(y) = \frac{e^{\frac{k}{2}y}}{N_n}\left[J_1\left(\frac{m_n}{k}e^{ky}\right) + b_{\lambda} Y_1\left(\frac{m_n}{k}e^{ky}\right)\right], \tag{70}$$

where N_n and b_{λ} are coefficients that do not depend on y.

The boundary conditions for f_n^{λ} and $f_n^{\lambda'}$ at $y=0$ and πR are given by examining the equations of motion, Eqs. (66, 67):

$$-f_n^{\lambda'}\Big|_{y=\epsilon} + \frac{g_B^2}{\tilde{g}_0^2} m_n f_n^{\lambda}\Big|_{y=0} = 0, \tag{71}$$

$$m_n f_n^{\lambda'}\Big|_{y=\epsilon} + (\partial_y + \frac{k}{2}) f_n^{\lambda}\Big|_{y=\epsilon} = 0, \tag{72}$$

$$f_n^{\lambda'}\Big|_{y=\pi R-\epsilon} + \frac{g_B^2 e^{\pi kR}}{\tilde{g}_{\pi}^2} m_n f_n^{\lambda}\Big|_{y=\pi R} - g_B^2 M_{\lambda} f_n^{\lambda}\Big|_{y=\pi R} = 0, \tag{73}$$

$$m_n e^{\pi kR} f_n^{\lambda'}\Big|_{y=\pi R-\epsilon} + (\partial_y + \frac{k}{2}) f_n^{\lambda}\Big|_{y=\pi R-\epsilon} = 0, \tag{74}$$

where $\epsilon \to 0$. Eliminating $f_n^{\lambda'}$ gives boundary conditions for f_n^{λ} at $y=0$ and πR, each of which determines the coefficient b_{λ}. Equating b_{λ} obtained from the boundary conditions at $y=0$ with that from $y=\pi R$, we obtain the equation that determines the 321 gaugino masses:

$$\frac{J_0\left(\frac{m_n}{k}\right) + \frac{g_B^2}{\tilde{g}_0^2} m_n J_1\left(\frac{m_n}{k}\right)}{Y_0\left(\frac{m_n}{k}\right) + \frac{g_B^2}{\tilde{g}_0^2} m_n Y_1\left(\frac{m_n}{k}\right)} = \frac{J_0\left(\frac{m_n}{T}\right) - \frac{g_B^2}{\tilde{g}_{\pi}^2}\frac{k}{T} m_n J_1\left(\frac{m_n}{T}\right) + g_B^2 M_{\lambda} J_1\left(\frac{m_n}{T}\right)}{Y_0\left(\frac{m_n}{T}\right) - \frac{g_B^2}{\tilde{g}_{\pi}^2}\frac{k}{T} m_n Y_1\left(\frac{m_n}{T}\right) + g_B^2 M_{\lambda} Y_1\left(\frac{m_n}{T}\right)}, \tag{75}$$

where $T = ke^{-\pi kR}$.

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