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Comment on Quark Masses in SCET

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Abstract

Quark masses are included in the SCET Lagrangian. Treating the strange quark mass as order $\Lambda_{\rm QCD}$, we find that strange quark mass terms are suppressed in $\mathrm{SCET_I}$, but are leading order in $\mathrm{SCET_{II}}$. This is relevant for B decays to K^* and K. Strange quark mass effects in semileptonic and weak radiative form factors are studied. They give corrections to the form factors that are not suppressed by powers of the bottom quark mass, or, equivalently, by the large recoil energy of the final state meson, and preserve the heavy to light form factor relations that follow from using the leading order current.

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Our understanding of processes involving a collinear jet of particles has improved significantly recently due to the construction of the Soft-Collinear Effective Theory (SCET) [1, 2, 3, 4]. The effective field theory couples soft physics to highly energetic quarks and gluons moving in a collinear jet. The symmetries of the SCET simplify proofs of factorization [3, 4, 5] and calculations of Sudakov logarithms. SCET has been applied to many processes including B meson [1, 2, 4, 6, 7] and quarkonium [8] decays, jet physics [9], and the pion form factor [10]. Light quark mass terms in SCET were first considered in Ref. [10], and our work elaborates on the discussion there.

The lightcone components of the collinear particles scale as $p = (p^+, p^-, p_\perp) \approx Q(\lambda^2, 1, \lambda)$, where Q is the large energy scale, and the expansion parameter $\lambda \ll 1$ depends on the particular process. For example, $\lambda = \sqrt{1 - 2E_\gamma/M}$ for inclusive meson decays (i.e., B or Υ) to a photon. In B decays, there are two appropriate choices, $\lambda = \sqrt{\Lambda_{\rm QCD}/m_b}$ or $\lambda = \Lambda_{\rm QCD}/m_b$. To distinguish between the two cases, the effective theories are called $\rm SCET_I$ and $\rm SCET_{II}$, respectively [11].

Factorization is proven in SCET by using a field redefinition which decouples soft quanta from collinear particles. For example, in the proof of factorization for $B \to D\pi$, after the soft physics is decoupled, there is a matrix element of a pair of collinear quark fields which becomes the pion decay constant [4]. At leading order in λ, no soft physics couples to the pion in the rest frame of the B meson, and so the nonperturbative strong interaction physics that gives rise to confinement and chiral symmetry breaking must exist in the collinear sector of the theory if this approach to factorization is correct.

Intuitively, the light quark mass should be suppressed in the collinear Lagrangian, since at very large energies, the quark behaves as if it were massless. However this leads to a problem. Apart from a factor of the Cabibbo angle, the only difference between $\bar{B}^0 \to D^+\pi^-$ and $\bar{B}^0 \to D^+K^-$ is the flavor of one of the final state collinear quarks. If light quark mass terms are suppressed in the collinear Lagrangian then differences in the flavor of light quarks can only enter in the soft physics, which at leading order in λ does not couple to the final state meson. However, the difference between the π and K decay constants is not small (i.e., around 30%), and furthermore this difference enters the decay rates in a way that is not suppressed by the available energy $1/(m_B - m_D)$. So if m_s is treated as order $\Lambda_{\rm QCD}$, the strange quark mass terms must be order one in the collinear Lagrangian.

Apart from isospin violation, the up and down quark masses will always be negligible, however the strange quark mass, $m_s \sim \Lambda_{\rm QCD}$, could be important. Below we investigate how quark masses enter into SCET and under what circumstances the strange quark mass is important.

Begin by defining two lightcone vectors, n^μ and $\bar{n}^\mu$, such that $n^2 = \bar{n}^2 = 0$ and $n \cdot \bar{n} = 2$. In SCET there are fundamental fields and Wilson lines, which are built out of the fields. Furthermore, there are two separate sectors to the theory: collinear and ultrasoft (usoft). In the collinear sector there is a collinear fermion field $\xi_{n,p}$, which is obtained by decomposing the full QCD quark field as

$$\psi(x) = \sum_{\tilde{p}} e^{-i\tilde{p}\cdot x} \left(\frac{\not{n}\,\bar{\not{n}}}{4} + \frac{\bar{\not{n}}\,\not{n}}{4} \right) \psi_{n,p} \equiv \sum_{\tilde{p}} e^{-i\tilde{p}\cdot x} (\xi_{n,p} + \xi_{\bar{n},p}) \,, \tag{1}$$

a collinear gluon field $A^\mu_{n,q}$, and a collinear Wilson line

$$W_n(x) = \Big[\sum_{\rm perms} \exp\left(-g \, \frac{1}{\bar{\mathcal{P}}} \, \bar{n} \cdot A_{n,p}(x) \right) \Big] \,. \tag{2}$$

The subscripts on the collinear fields are the lightcone direction n^μ, and the large components of the lightcone momenta, $\tilde{p} = n\,(\bar{n}\cdot p)/2 + p_\perp$. Derivatives acting on collinear fields are order λ^2, since the p^- and $p_\perp$ components have been removed. The operator $\mathcal{P}^\mu$ projects out the momentum label [3]. For example $\bar{n}\cdot\mathcal{P}\,\xi_{n,p} \equiv \bar{\mathcal{P}}\xi_{n,p} = \bar{n}\cdot p\,\xi_{n,p}$. Functions of the operator $\bar{\mathcal{P}}$ have the property

$$f(\bar{\mathcal{P}} + i\bar{n}\cdot D) = W_n f(\bar{\mathcal{P}}) W_n^\dagger \,. \tag{3}$$

Likewise in the usoft sector there is a usoft fermion field q_s, a usoft gluon field A^μ_s, and a usoft Wilson line Y.

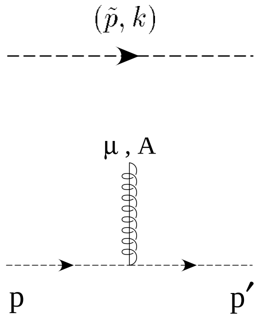
Operators in SCET are constructed from these objects such that they are gauge invariant. For example, under collinear-gauge transformations $\xi_{n,p} \to U_n\,\xi_{n,p}$ and $W_n \to U_n\,W_n$, so

$$\chi_n \equiv W_n^\dagger\,\xi_{n,p} \tag{4}$$

is collinear-gauge invariant. This combination, χ_n, however, still transforms under a usoft-gauge transformation $\chi_n \to V(x)\chi_n$.

The SCET collinear Lagrangian is obtained by substituting Eq. (1) into the QCD Lagrangian, $\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi$, which gives

$$\begin{aligned} \mathcal{L} \;=\; & \bar{\xi}_{n,p'}\, in\cdot D\, \frac{\bar{\not{n}}}{2}\, \xi_{n,p} + \bar{\xi}_{\bar{n},p'}\, (\not{\mathcal{P}}_\perp + i\not{D}_\perp - m)\, \xi_{n,p} \\ +\; & \bar{\xi}_{n,p'}\, (\not{\mathcal{P}}_\perp + i\not{D}_\perp - m)\, \xi_{\bar{n},p} + \bar{\xi}_{\bar{n},p'}\, (\bar{\mathcal{P}} + i\bar{n}\cdot D)\, \frac{\not{n}}{2}\, \xi_{\bar{n},p} \,, \end{aligned} \tag{5}$$



$$= i\,\frac{\not{n}}{2}\,\frac{\bar{n}\cdot p}{n\cdot k\;\bar{n}\cdot p + p_\perp^2 - m^2 + i\epsilon}$$

$$= ig\,T^A\left\{ n^\mu + \frac{\gamma_\perp^\mu \not{p}_\perp}{\bar{n}\cdot p} + \frac{\not{p}'_\perp \gamma_\perp^\mu}{\bar{n}\cdot p'} - \frac{\not{p}'_\perp \not{p}_\perp}{\bar{n}\cdot p\;\bar{n}\cdot p'}\,\bar{n}^\mu \right.$$
$$\left. + \frac{m}{\bar{n}\cdot p\;\bar{n}\cdot p'}\Big[\gamma_\perp^\mu(\bar{n}\cdot p' - \bar{n}\cdot p) + \bar{n}^\mu(\not{p}_\perp - \not{p}'_\perp + m)\Big]\right\}\frac{\not{\bar{n}}}{2}$$

FIG. 1: Order λ^0 Feynman rules: collinear quark propagator with label $\tilde{p}$ and residual momentum k, and collinear quark interactions with one collinear gluon, respectively.

where we have used the convention where momentum labels are implicitly summed over [3].

We can use the equations of motion to remove $\xi_{\bar{n},p}$,

$$(\bar{\mathcal{P}} + i\bar{n}\cdot D)\xi_{\bar{n},p} = (\not{\mathcal{P}}_\perp + i\not{D}_\perp + m)\frac{\not{\bar{n}}}{2}\,\xi_{n,p}\,, \tag{6}$$

and make a field redefinition to remove the couplings to the (u)soft degrees of freedom. This gives the usual leading order collinear Lagrange density

$$\mathcal{L}_0 = \bar{\xi}_{n,p'}\left\{ in\cdot\partial + (\not{\mathcal{P}}_\perp + g\not{A}^\perp_{n,q})W_n\frac{1}{\bar{\mathcal{P}}}W_n^\dagger(\not{\mathcal{P}}_\perp + g\not{A}^\perp_{n,q'})\right\}\frac{\not{\bar{n}}}{2}\,\xi_{n,p}\,, \tag{7}$$

and the following mass terms

$$\mathcal{L}_m = m\,\bar{\xi}_{n,p'}\left[(\not{\mathcal{P}}_\perp + g\not{A}^\perp_{n,q}), W_n\frac{1}{\bar{\mathcal{P}}}W_n^\dagger\right]\frac{\not{\bar{n}}}{2}\,\xi_{n,p} - m^2\,\bar{\xi}_{n,p'}W_n\frac{1}{\bar{\mathcal{P}}}W_n^\dagger\,\frac{\not{\bar{n}}}{2}\,\xi_{n,p}\,. \tag{8}$$

The Feynman rules for the collinear quark propagator and the interaction of a collinear quark with a single collinear gluon are show in Fig. 1.

The next question is how m scales. If $m \sim Q\lambda^2$, the terms in Eq. (8) are suppressed and can be dropped from the collinear Lagrangian at leading order. Thus we would be left with only $\mathcal{L}_0$, which agrees with [2]. The masses would still be important in the usoft Lagrangian. If, however, $m \sim Q\lambda$, all terms in Eq. (8) are equally important compared with Eq. (7), and must be kept. Note that when $m \sim Q\lambda \sim \Lambda_{\rm QCD}$, i.e., in SCET$_{\rm II}$, the perpendicular label should be interpreted as a derivative. Thus Eq. (8) becomes

$$\mathcal{L}_m = m\,\bar{\xi}_{n,p'}\left[(i\not{\partial}_\perp + g\not{A}^\perp_{n,q}), W_n\frac{1}{\bar{\mathcal{P}}}W_n^\dagger\right]\frac{\not{\bar{n}}}{2}\,\xi_{n,p} - m^2\,\bar{\xi}_{n,p'}W_n\frac{1}{\bar{\mathcal{P}}}W_n^\dagger\,\frac{\not{\bar{n}}}{2}\,\xi_{n,p}\,. \tag{9}$$

The leading order collinear Lagrangian in Eq. (7) has a $U(1)$ helicity symmetry which is generated by γ_5 acting on the collinear quark field. However, this symmetry is explicitly broken by the terms linear in the quark mass in Eq. (9). It is also broken by nonperturbative strong interaction physics which causes the vacuum expectation value,

$$\langle\Omega|\,\bar{\xi}_{n,p'}\left[(i\partial\!\!\!/_\perp + gA\!\!\!/^{\,\perp}_{n,q}), W_n\frac{1}{\bar{\mathcal{P}}}W_n^\dagger\right]\frac{\bar{n}\!\!\!/}{2}\,\xi_{n,p}|\Omega\rangle\,, \tag{10}$$

to differ from zero.

For inclusive $B \to X_s\gamma$ or $B \to X_u\ell\bar{\nu}$ decays, the interesting region of phase space is $E \simeq m_b/2 - \Lambda_{\rm QCD}$, giving $\lambda = \sqrt{\Lambda_{\rm QCD}/m_b}$, where we have taken $Q = \mathcal{O}(m_b)$. The appropriate effective theory is SCET$_{\rm I}$, and therefore the dependence on the light quark flavor is power suppressed in these decays, implying the usual relationship between the shape functions [12].

For B decays to a light meson M, however, the invariant mass square of the outgoing meson is of order $m_M^2 \sim m_b^2\lambda^2 = \mathcal{O}(\Lambda_{\rm QCD}^2)$, where again we have taken $Q = \mathcal{O}(m_b)$. Therefore, $\lambda = \Lambda_{\rm QCD}/m_b$ and SCET$_{\rm II}$ is the proper theory. Then, if $m_s \sim \Lambda_{\rm QCD}$, we must include the strange quark mass effects in the leading order collinear Lagrangian. In practice it may be appropriate to treat the strange quark mass as somewhat smaller than the strong interaction scale in which case it can be treated as a perturbation. Then the term linear in m_s in Eq. (9) gives rise to corrections suppressed by $m_s/\Lambda_{\rm QCD}$ but not by powers of λ. Our results show explicitly that $SU(3)$ breaking in the relations between the $B \to K^*$ and $B \to \rho$ (or the $B \to K$ and $B \to \pi$) form factors that describe semileptonic and weak radiative decays is not suppressed by $\Lambda_{\rm QCD}/m_b$ in any region of phase space.

In Ref. [11] the form factors for heavy to light weak transitions are considered using SCET. They adopt a two step process where one first considers contributions in SCET$_{\rm I}$ and then matches onto SCET$_{\rm II}$. The terms that give a leading contribution to the form factors are suppressed by $\lambda^2 = \Lambda_{\rm QCD}/m_b$ in SCET$_{\rm I}$. For $B_{u,d}$ decays, when the spectator quark is not a strange quark, the terms in the mixed collinear-usoft Lagrangian that cause the usoft spectator quark to transition to a collinear quark are the same as in [11]. These are suppressed by at least one power of λ. As we have remarked earlier, in SCET$_{\rm I}$ the strange quark mass terms are also suppressed by at least one power of λ. Therefore, to get a leading form factor contribution involving the strange quark mass term, the time ordered products must contain the leading order current,

$$J^{(0)} = \bar{\xi}_n W_n \Gamma h_v, \tag{11}$$

and these terms preserve the usual heavy to light form factor relations [13]. On transitioning to $\mathrm{SCET_{II}}$, the strange quark mass terms become leading order. However, the non-factorizable pieces which involve $J^{(0)}$ will automatically preserve the form factor relations in $\mathrm{SCET_{II}}$, even with the strange quark mass term included in the leading order $\mathrm{SCET_{II}}$ Lagrangian. In fact there is a simple physical argument that this should be the case. Constituent quark masses induced by chiral symmetry breaking often act, in low energy phenomenology, much like explicit light quark masses. Given this it would be puzzling if explicit light quark masses violated the heavy to light form factor relations that follow from only including the leading order current, $J^{(0)}$, but spontaneous chiral symmetry breaking did not cause such violations. It is possible that the factorizable terms that violate the leading order form factor relations are suppressed by a factor of $\alpha_s(\sqrt{m_b \Lambda_{\mathrm{QCD}}})$ compared to those that preserve them [14]. If this is the case then there will be no corrections to the form factor relations to any order in $m_s/\Lambda_{\mathrm{QCD}}$.

Finally we note that for the case of B_s decays the strange quark mass terms in $\mathrm{SCET_I}$ that cause the usoft spectator strange quark to transition to a collinear one could be important. However, using the equations of motion, it is not difficult to show that no terms of this type appear at order λ or λ^2 [15]. Note that this does not mean that the differences between form factors for B_s and B decays are suppressed by powers of λ.

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