Fermilab FERMILAB-Pub-03/033-T March 2003

UCB-PTH-03/04
LBNL-52212
FERMILAB-Pub-03/033-T

Explicit Supersymmetry Breaking on Boundaries of Warped Extra Dimensions

Lawrence J. Hall*, Yasunori Nomura†, Takemichi Okui*, and Steven J. Oliver*

* *Department of Physics, University of California, Berkeley,*
and
Theoretical Physics Group, Lawrence Berkeley National Laboratory,
Berkeley, CA 94720, USA
† *Theoretical Physics Department, Fermi National Accelerator Laboratory,*
Batavia, IL 60510, USA

Abstract

Explicit supersymmetry breaking is studied in higher dimensional theories by having boundaries respect only a subgroup of the bulk symmetry. If the boundary symmetry is the maximal subgroup allowed by the boundary conditions imposed on the fields, then the symmetry can be consistently gauged; otherwise gauging leads to an inconsistent theory. In a warped fifth dimension, an explicit breaking of all bulk supersymmetries by the boundaries is found to be inconsistent with gauging; unlike the case of flat 5D, complete supersymmetry breaking by boundary conditions is not consistent with supergravity. Despite this result, the low energy effective theory resulting from boundary supersymmetry breaking becomes consistent in the limit where gravity decouples, and such models are explored in the hope that some way of successfully incorporating gravity can be found. A warped constrained standard model leads to a theory with one Higgs boson with mass expected close to the experimental limit. A unified theory in a warped fifth dimension is studied with boundary breaking of both $SU(5)$ gauge symmetry and supersymmetry. The usual supersymmetric prediction for gauge coupling unification holds even though the TeV spectrum is quite unlike the MSSM. Such a theory may unify matter and Higgs in the same $SU(5)$ hypermultiplet.

1 Introduction

A light Higgs boson, suggested by precision electroweak data, together with a heavy top quark, has direct and consequential implications. Virtual top quarks necessarily induce a large quadratic divergence to the Higgs mass parameter, hence a light Higgs boson is expected only if this is canceled by additional radiative contributions from new physics at energy scales not far above the top quark mass. The most obvious origin for this cancellation is weak scale supersymmetry, and the case for this is greatly strengthened by the successful prediction from gauge coupling unification. The experimental implications for such theories have focused almost exclusively on theories which are four dimensional at the TeV scale — especially the minimal supersymmetric standard model (MSSM). However, higher dimensional supersymmetric theories can also tame the divergences of scalar mass parameters, with cancellations from Kaluza-Klein (KK) modes playing as important a role as cancellations from superpartners [1]. Furthermore, beneath the mass scale of the lightest KK modes, the 4D effective theory need not be supersymmetric — there is no MSSM limit of the theory.

In the MSSM the weak scale is understood as a byproduct of the more fundamental supersymmetry breaking scale. When KK modes play a crucial role in canceling the Higgs mass divergence, the more fundamental scale is the effective compactification scale, M_c, which is the mass threshold for the KK modes. This mass scale, which should not be far above the top quark mass, should trigger the breaking of both supersymmetry and electroweak symmetry. In the constrained standard model of Ref. [2] $M_c \simeq 350$ GeV, and it is obvious that there is no MSSM limit: there is only one Higgs doublet, and it couples to both up and down type quarks. Two Higgs theories can also be constructed [3], as can theories with M_c considerably higher, in the 30 TeV region [4]. These latter theories may mimic the MSSM at future collider experiments.

A common feature of these theories is that supersymmetry breaking arises because the boundary conditions in the fifth dimension are taken to differ for fermions and bosons. The non-locality of this breaking implies supersymmetry breaking counterterms cannot be induced by radiative corrections. The cancellations in the Higgs mass parameters are more precise than in 4D theories, so that the Higgs mass parameters are finite and calculable. In the constrained standard model there is only a single Higgs field, with the potential calculated in terms of a single free parameter M_c, making it possible to determine M_c by the Z mass and predict the physical Higgs boson mass: 127 ± 8 GeV.

Despite these successes, one must admit a significant drawback. In all these theories the gauge and Yukawa couplings become strongly coupled in the multi-TeV domain and the UV cutoff of the effective 5D field theory is reached long before the unification scale, so that the successful prediction from conventional logarithmic unification is lost. Furthermore, since the cutoff of the theory is in the multi-TeV domain, one must address the question of why gravity is so weak. This apparently requires further extra dimensions, either in the sub mm domain [5] or with a warp factor [6].

There is a very simple way to maintain gauge coupling unification even when supersymmetry is broken by boundary conditions in a fifth dimension. It is possible that the difference in the boundary conditions between fermions and bosons is described by a very small angle α [7], so that the scale of the superpartners, αM_c, can become decoupled from the compactification scale. Gauge coupling unification is recovered if M_c is taken at or above the unification scale. However, in this case, since the KK modes are at or above the unification scale, the cancellation of the top divergence in the Higgs mass parameter reduces precisely to the usual 4D supersymmetric case. In this paper we want to ask a different question: is it possible for the KK modes to take part in the cancellation of the Higgs mass divergence, while allowing conventional logarithmic gauge coupling unification? Furthermore, how would the weakness of gravity be understood in such a theory with TeV scale KK modes? One possibility is to also have sub mm scale extra dimensions for the propagation of gravity, but then it is not clear how to recover gauge coupling unification. A second possibility is that there is a warped extra dimension, in which case the running of gauge couplings is logarithmic above the mass threshold for the KK towers [8, 9 – 14]. In general $SU(3)_C \times SU(2)_L \times U(1)_Y$ theories the low energy gauge couplings cannot be predicted, because they depend on the tree-level 5D gauge couplings, which are free parameters of the theory. However, if the bulk of this warped dimension has unified gauge symmetry such as $SU(5)$, one can show that the successful prediction of the MSSM for gauge coupling unification can be obtained [15]. In fact, such theories can be constructed by breaking the unified gauge symmetry either by the vacuum expectation value of a Planck-brane localized field [8] or by boundary conditions imposed at the Planck brane [15]. This offers the possibility of exceptional economy: the warped dimension that generates the TeV scale and the dimension which contains supersymmetry breaking boundary conditions could be one and the same.

With the above motivation, in this paper we study boundary condition breaking of supersymmetry in warped space, in particular in a slice of AdS_5. Our aim is to construct a theory of electroweak symmetry breaking where a crucial role is played by the TeV mass KK modes of this warped extra dimension, while simultaneously solving the gauge hierarchy problem and addressing logarithmic gauge coupling unification. However, before attempting to construct a model, we must study whether it is consistent to impose supersymmetry breaking boundary conditions in a supersymmetric theory in a warped 5D spacetime.

We note that it is straightforward to construct 5D warped, supersymmetric theories with supersymmetry broken spontaneously by a vacuum expectation value (VEV) located on the TeV brane [16]. With gauge interactions in the bulk, but matter and Higgs fields on the Planck brane, supersymmetry breaking is mediated to matter and Higgs via gaugino mass terms. By introducing the bulk $SU(5)$, one can also recover the MSSM prediction for gauge coupling unification [15].

These theories are rather interesting, since the 5D nature for the Higgs mass cancellation is obtained by taking the VEV to be large. However, in these theories the scale of supersymmetry breaking is in principle a free parameter and is not strictly related to the KK mass scale. It is also difficult to construct one Higgs theories with TeV brane localized supersymmetry breaking. In this paper we explore theories where the two scales are tightly related through compactifications.

The boundary condition supersymmetry breaking in warped space has been considered before in Ref. [17], but without addressing the issue of the consistency of the theory. Potential difficulties of the theory with supersymmetry broken by boundary conditions in a warped fifth dimension has been noted in Ref. [18]. In this paper we study the consistency of the theory in detail, and during that course we develop the concept of symmetry breaking defects in higher dimensional spacetime. In general, higher dimensional theories compactified on a spacetime with boundaries can possess symmetry breaking defects at the boundaries. When do such defects lead to consistent theories, and when do difficulties arise? In section 2 we study the local breaking of global internal symmetries in flat space, and introduce a distinction between two types of defect: type I (type II) defects which are (are not) consistent with a gauging of the global symmetry. For example, we find that the boundary condition breaking of a $U(1)$ gauge symmetry, or of the electroweak gauge symmetry $SU(2)_L \times U(1)_Y \to U(1)_{EM}$, leads to type II defects and thus is not consistent in a flat fifth dimension. In section 3 we show that the defects arising when supersymmetry is broken by a boundary condition in a warped fifth dimension are of type II, preventing a consistent construction of the corresponding supergravity theory. Despite this difficulty, in section 4 we construct an $SU(3)_C \times SU(2)_L \times U(1)_Y$ model in a warped 5D background with supersymmetry broken by boundary conditions. Such an effective theory may follow from some consistent fundamental theory. We explore electroweak symmetry breaking in this theory when there is a single Higgs hypermultiplet. We also construct an $SU(5)$ theory in warped 5D spacetime where supersymmetry is broken by boundary conditions in the fifth dimension, and show that consistent phenomenology is obtained in the theory.

2 Symmetry Breaking Defects in Higher Dimensions

In this section we carefully study the notion of symmetry breaking defects in higher dimensional effective field theories. These defects arise on a boundary of the bulk when the Lagrangian at that boundary is invariant under a smaller internal symmetry than that of the bulk Lagrangian. We find that there are two types of defect: type I defects arise when the reduction in symmetry from bulk to boundary is entirely forced by the boundary conditions imposed on the fields of the theory. On the other hand, for type II defects not all of the symmetry reduction is required by the boundary conditions. For the first kind of defect, the internal symmetry can be gauged, and such defects were considered in Ref. [19] in the context of higher dimensional grand unified theories.

We discuss these defects in sub-section 2.1 using the example of $SU(5)$ symmetry in 5D. In sub-section 2.2, we introduce the second type of defect and find that the internal symmetry cannot be consistently gauged. In section 3 we extend this analysis of defects to the case of supersymmetry, and find that it has important consequences for supersymmetry breaking in truncated AdS_5 space.

2.1 Type I symmetry breaking defect

In this sub-section we discuss the first kind of symmetry breaking defect. This type of defect allows the whole symmetry structure to be gauged, and we call them type I symmetry breaking defects. To illustrate the point, in this sub-section we consider 5D theories compactified on a flat S^1/Z_2 orbifold: a line segment parameterized by $y : [0, \pi R]$ with the metric of the spacetime given by $-ds^2 = g_{MN} dx^M dx^N = \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$.

Let us first consider the theory in which the bulk Lagrangian possesses a global $SU(5)$ symmetry: for example, the bulk Lagrangian is invariant under the transformation

$$\phi \to \exp(i T^A \xi^A) \phi, \tag{1}$$

where the field ϕ is in the **5** representation, T^A are the generators of $SU(5)$ and ξ^A are arbitrary constants. If the spacetime we consider were non-compact, this would be the end of the story. However, since we are considering the theory on a compact space (S^1/Z_2 orbifold), we have to specify the boundary conditions on the fields to define the theory. Suppose we require that all fields in a single irreducible representation of $SU(5)$ obey the same boundary conditions. In this case the full theory can possess the global $SU(5)$ symmetry of Eq. (1), and the resulting space does not have any symmetry breaking defect. What happens if we impose different boundary conditions on fields in the same irreducible representation of $SU(5)$? This is the situation we want to consider in this sub-section.

The boundary conditions on S^1/Z_2 are completely specified if we specify the conditions which the fields must satisfy at $y = 0$ and $y = \pi R$. In general these conditions are written as

$$\varphi(y) = \mathbf{Z}\, \varphi(-y), \qquad \varphi(y') = \mathbf{Z}'\, \varphi(-y'), \tag{2}$$

where $y' \equiv y - \pi R$; φ is a column vector collecting all the fields in the theory, while $\mathbf{Z}$ and $\mathbf{Z}'$ are matrices acting on this vector. The precise meaning of these conditions is the following. Although our space is only for $0 \leq y \leq \pi R$, we can fictitiously extend it to the domain $y < 0$ or $y > \pi R$ using the above equations. Then the dynamics of the fields (wavefunctions of the fields) are obtained by solving the equations of motion in the whole covering space, including the terms arising from brane-localized operators. (The importance of thinking in this way becomes clearer in the next section because, unlike the flat space case, in AdS space we cannot construct the theory on S^1/Z_2

by simple identification procedures from the corresponding theory on the non-compactified AdS space.)

Now, we consider the matrices $\mathbf{Z}$ and/or $\mathbf{Z}'$ which do not give the same boundary conditions for all the fields in a single irreducible representation of $SU(5)$. For illustrative purposes, we choose these matrices to be $\mathbf{Z} = \mathrm{diag}(1,1,1,1,1)$ and $\mathbf{Z}' = \mathrm{diag}(-1,-1,-1,1,1)$ when acting on an $SU(5)$ fundamental index. For instance, the triplet and doublet components, ϕ_T and ϕ_D, of the $\mathbf{5}$ representation obey the boundary conditions $\phi_T(+,-)$ and $\phi_D(+,+)$, where the first and second signs represent the eigenvalues of $\mathbf{Z}$ and $\mathbf{Z}'$.

What are the consequences of imposing the above boundary conditions? First of all, the whole theory obviously does not have a global $SU(5)$ symmetry, since we have imposed different boundary conditions on, say, ϕ_T and ϕ_D and they have different wavefunctions. The transformation of Eq. (1) is inconsistent with the boundary conditions at $y = \pi R$, again demonstrating the absence of the global $SU(5)$ symmetry. However, physically we suspect that the physics at any local neighborhood of the bulk must still reflect the original global $SU(5)$ symmetry. This is because the effect of the boundary conditions at $y = \pi R$, which is $SU(5)$ violating, is suppressed by locality in any point in the bulk. On the other hand, at the $y = \pi R$ brane, the effect of $SU(5)$-violating boundary conditions is maximal, and we suspect that physics will not reflect the original global $SU(5)$ symmetry. For example, the wavefunction values for ϕ_D can be non-zero at $y = \pi R$, while those for ϕ_T must always be zero. This implies that it does not make sense to impose the $SU(5)$ symmetry on the operators on the $y = \pi R$ brane. Hence we are led to ask: what is the most general form of the action, and is there a symmetry transformation which guarantees this form?

We find the most general form for the action to be

$$S = \int d^4x \int dy \Big[\mathcal{L}_{\rm 5D}^{SU(5)} + \delta(y) \mathcal{L}_{\rm 4D}^{SU(5)} + \delta(y - \pi R) \mathcal{L}_{\rm 4D}^{3-2-1} \Big]. \tag{3}$$

Here, $\mathcal{L}_{\rm 5D}^{SU(5)}$ and $\mathcal{L}_{\rm 4D}^{SU(5)}$ respect the full $SU(5)$ symmetry, while $\mathcal{L}_{\rm 4D}^{3-2-1}$ respects only the $SU(3) \times SU(2) \times U(1)$ subgroup of $SU(5)$. The different pieces of the Lagrangian are invariant under global transformations of different size:

$$\mathcal{L}_{\rm 5D}^{SU(5)}, \mathcal{L}_{\rm 4D}^{SU(5)}: \quad \phi \;\rightarrow\; \exp(i T^A \xi^A) \phi, \tag{4}$$

$$\mathcal{L}_{\rm 4D}^{3-2-1}: \quad \phi \;\rightarrow\; \exp(i T^a \xi^a) \phi, \tag{5}$$

where A runs over all $SU(5)$ generators while a runs over the subset of those forming the $SU(3) \times SU(2) \times U(1)$ subgroup, and ξ^A are constant and do not depend on the coordinates. This is an unusual situation — while the theory does possess a global $SU(3) \times SU(2) \times U(1)$ symmetry, the other transformations of $SU(5)$ are not symmetries, since not all pieces of the Lagrangian are invariant under them. In general in higher dimensional theories, it is useful to consider an action where the bulk Lagrangian and the boundary Lagrangian possess different invariances. We will say that such theories possess restricted symmetries. Where the invariance at a boundary is less than in the bulk we will say that there is a symmetry breaking defect at the boundary. In our $SU(5)$ example, we therefore find that the boundary conditions have forced a reduction of the original global symmetry to a restricted global symmetry, with a symmetry breaking defect appearing at the $y = \pi R$ brane. The question is whether this new concept of a restricted global symmetry, such as Eqs. (4, 5), is really useful: does it lead to relations amongst counterterms, for example sufficient to yield Eq. (3) as the most general action? Locality suggests that this is so: at short distances (*i.e.* with large momentum) in the bulk, the effect from the $y = \pi R$ boundary is exponentially suppressed due to Yukawa suppression (the 4D momentum appears as a mass in the direction of the fifth dimension). The same argument applies to the Lagrangian at the $y = 0$ brane. Therefore, we expect that all divergences are absorbed into the counterterms preserving the form of Eq. (3).

This expectation is confirmed because the theory defined by Eq. (3) possesses a conserved $SU(5)$ current in the bulk, and a conserved $SU(3) \times SU(2) \times U(1)$ current at $y = \pi R$, at the quantum level. The notion of a restricted global symmetry, which takes a different form at different locations, makes sense because current conservation occurs locally. We can demonstrate that these currents are conserved, for instance, by the Noether procedure in the path integral formalism. We consider varying the fields with position dependent ξ's. The position dependence of ξ's must be consistent with the boundary conditions of the fields and with the form of the restricted global symmetry. Specifically, we have to restrict the y dependence of ξ's as $\xi^a(+,+)$ and $\xi^{\hat{a}}(+,-)$ where a and $\hat{a}$ run for $SU(3) \times SU(2) \times U(1)$ and $SU(5)/(SU(3) \times SU(2) \times U(1))$, respectively. When expanded in the complete set in the fifth dimension, they are written as

$$\xi^a(x^\mu, y) = \sum_{n=0} \xi^a_n(x^\mu) \cos\left(\frac{ny}{R}\right), \tag{6}$$

$$\xi^{\hat{a}}(x^\mu, y) = \sum_{n=0} \xi^{\hat{a}}_n(x^\mu) \cos\left(\frac{(n+1/2)y}{R}\right). \tag{7}$$

Note that with $\xi^{\hat{a}}$ having boundary conditions $(+,-)$ we automatically have $\xi^{\hat{a}}(x^\mu, y = \pi R) = 0$, ensuring that we restrict transformations to be in $SU(3)\times SU(2)\times U(1)$ at $y = \pi R$. The rest of the procedure is the usual one. Although the action, Eq. (3), is not invariant under the transformation by Eqs. (6, 7), the variation is proportional to the derivatives of ξ's since Eq. (3) is invariant under transformations with constant ξ's. This leads to a conservation law, which tells us that there is a conserved current for $SU(5)$ in the bulk and on $y = 0$, but only the $SU(3) \times SU(2) \times U(1)$ part of it is conserved at $y = \pi R$.

We are now in a position to consider gauging the restricted global symmetry of the system. It is the gauging which distinguishes between the two types of defects discussed in this and the next sub-sections. The gauging of the restricted global symmetry is accomplished by requiring the theory to be *invariant* under the transformations of Eqs. (6, 7) with arbitrary functions of $\xi_n^a(x^\mu)$ and $\xi_n^{\hat{a}}(x^\mu)$.[1] Since the kinetic terms of the original Lagrangian with restricted global symmetry are not invariant under these transformations, we have to introduce the connection fields $A_M^A(x^\mu, y)$, which are in the adjoint representation of $SU(5)$. The boundary conditions for these fields are determined to be $A_\mu^a(+,+)$ and $A_\mu^{\hat{a}}(+,-)$ ($A_5^a(-,-)$ and $A_5^{\hat{a}}(-,+)$) from the transformation properties of these fields, $A_M^A \to A_M^A + \partial_M \xi^A + \cdots$. The expansion then goes as in Eqs. (6, 7) with ξ^a and $\xi^{\hat{a}}$ replaced by A_μ^a and $A_\mu^{\hat{a}}$ (for A_5's, replace the cosine by sine and start the sum for $A_{5,n}^a$ from $n = 1$). Therefore, we find that there is a one-to-one correspondence between the modes $A_{\mu,n}^A$ and ξ_n^A. This is crucial for the consistency of the gauge theory: each gauge field requires a corresponding gauge symmetry. This one-to-one correspondence characterizes what we call type I symmetry breaking defects. In our $SU(5)$ example, gauging produces a restricted gauge symmetry (the transformations of Eqs. (4, 5) with all ξ now local) yielding a consistent effective higher dimensional field theory below the cutoff, as discussed in detail in Ref. [21]. Restricted gauge symmetries play an important role for constructing realistic higher dimensional grand unified theories [19], which have automatic doublet-triplet splitting [22], proton decay suppression [19], and an interesting new prediction for gauge coupling unification [23]. In the next sub-section, we consider a different kind of defect, which violates the above one-to-one correspondence, and consequently does not allow the consistent gauging of the symmetry.

2.2 Type II symmetry breaking defect

As in the previous sub-section, we consider a theory on the flat S^1/Z_2 orbifold. We consider a restricted symmetry where the bulk and the $y = 0$ brane possess a global $U(1)$ invariance but the $y = \pi R$ brane does not. The action of this system takes the form:

$$S = \int d^4x \int dy \Big[\mathcal{L}_{\rm 5D}^{U(1)} + \delta(y) \mathcal{L}_{\rm 4D}^{U(1)} + \delta(y - \pi R) \mathcal{L}_{\rm 4D}^{\times} \Big]. \tag{8}$$

Here $\mathcal{L}_{\rm 5D}^{U(1)}$ and $\mathcal{L}_{\rm 4D}^{U(1)}$ are invariant under the field rotation $\phi \to \exp(iQ_\phi \xi)\phi$, but $\mathcal{L}_{\rm 4D}^{\times}$ is not, where ϕ is a field carrying the $U(1)$ charge of Q_ϕ and ξ is an arbitrary constant. The boundary conditions for ϕ are arbitrary: it can either be $(+,+)$, $(+,-)$, $(-,+)$ or $(-,-)$.

Does the above action make sense? To answer this question, we have to study radiative corrections. As in the previous example, we find that all divergences are absorbed in the counterterms preserving the form of the Lagrangian. Here we prove this using the Noether procedure in the path integral formalism. We consider the $U(1)$ transformation parameter ξ to be a function of the spacetime. The boundary conditions for ξ are determined to be $\xi(+,+)$ so that this $U(1)$

[1] The gauging is possible only when the theory is anomaly free. If the low energy 4D theory does not have anomalies, we can in general make the full higher dimensional theory to be anomaly free by introducing an appropriate Chern-Simons term in the bulk [20].

transformation preserves the boundary conditions for ϕ's. A mode expansion gives

$$\xi(x^\mu, y) = \sum_{n=0} \xi_n(x^\mu) \cos\left(\frac{ny}{R}\right). \tag{9}$$

However, we now have an extra constraint. Because the above expansion does not ensure the vanishing of $\xi(x^\mu, y)$ at $y = \pi R$, where $U(1)$ symmetry is supposed to be absent, we have to impose a further condition on the $\xi^a_n(x^\mu)$'s:

$$\sum_{n=0} \xi_n(x^\mu) \cos(\pi n) = 0. \tag{10}$$

We now vary the action with arbitrary $\xi^a_n(x^\mu)$'s under the constraint Eq. (10). Then we find that the variation is proportional to the derivative of $\xi(x^\mu, y)$, giving a current associated with $U(1)$ which is conserved everywhere except $y = \pi R$. Therefore, we find the system with a restricted global $U(1)$ symmetry with a symmetry breaking defect at $y = \pi R$ is meaningful, in the sense that its structure is preserved by radiative corrections. This situation is quite similar to the restricted global $SU(5)$ symmetry with the $SU(3) \times SU(2) \times U(1)$ defect.

Now, we consider gauging this restricted $U(1)$ global symmetry, *i.e.* we require the theory to be *invariant* under position dependent ξ. To make the kinetic term of the original Lagrangian invariant, we must introduce the connection fields, $A_M(x^\mu, y)$. The boundary conditions for these fields are determined to be $A_\mu(+,+)$ and $A_5(-,-)$ from their transformation properties $A_M \to A_M + \partial_M \xi$. Therefore, the mode expansions for these fields are given by Eq. (9) with ξ replaced by A_μ (for A_5, replace the cosine by sine and start the sum from $n = 1$). Unlike the gauge parameter ξ, however, these gauge fields A_M are dynamical fields, so that we cannot simply impose the constraint like Eq. (10). In particular, all $A_{\mu,n}(x^\mu)$ are independent fields. This means that the number of gauge transformation parameters, $\xi_n(x^\mu)$, is smaller than the number of gauge fields, $A_{\mu,n}(x^\mu)$, due to the constraint imposed on the $\xi_n(x^\mu)$'s, Eq. (10). This leads to an inconsistency of the theory, because, from the 4D viewpoint, there is one gauge field which is not accompanied by a corresponding gauge symmetry. As is well known, such a gauge field gives a ghost which can be produced as an external particle, leading to negative probabilities for certain processes.

Therefore, we find that the defect in this $U(1)$ theory has a different character from the one discussed in the previous sub-section, and we call it a defect of type II. When the restricted symmetry is global the two types of defect have similar properties, but when the restricted symmetry is gauged quite different features are revealed: one allows consistent gauging but the other does not. The criterion for distinguishing the two is whether the number of gauge transformation parameters is the same as or smaller than the number of the gauge fields (counting modes in the 4D picture). Type I defects arise naturally when the restricted symmetry is taken to be the largest possible consistent with the boundary conditions imposed on the fields. In the $U(1)$ example, the boundary conditions are consistent with a $(+,+)$ parity assignment to ξ, so that both branes would naturally be expected to have Lagrangians which respect the $U(1)$ symmetry. Type II defects arise when the invariance on the boundary is taken to be less than the maximal consistent with the boundary conditions. Clearly there are much more general possibilities than we have discussed, even on S^1/Z_2. The restricted global symmetry may correspond to invariances of the three pieces of the Lagrangian under transformations of different sizes. Type I defects arise if these transformations on a boundary Lagrangian are the largest consistent with the boundary conditions that have been imposed on the fields. If the boundary Lagrangian is invariant under a smaller set of transformations, then the defect is type II.

A case of potential phenomenological interest has the bulk Lagrangian invariant under the electroweak group $SU(2)_L \times U(1)_Y$, with a boundary Lagrangian invariant under the smaller electromagnetic symmetry group $U(1)_{EM}$. Apparently this provides an alternative electroweak symmetry breaking mechanism — no Higgs bosons or new strong dynamics are needed since the defect explicitly breaks weak interactions. However, this defect is of type II: there is no way of imposing boundary conditions on the fields such that the reduction of symmetry on the boundary is required for consistency with the boundary conditions. Incidentally, in the next section when we consider boundary condition breaking of supersymmetry in warped space, we will similarly discover that the defects are of type II.

Although we find that type II defects do not allow gauging of the symmetry in a straightforward way, we can obtain a low energy effective field theory which mimics the gauging of type II defects. For instance, to mimic the above theory, we can first consider a 5D $U(1)$ gauge theory compactified on the flat S^1/Z_2 orbifold without any defect. Then, if we break this $U(1)$ by the vacuum expectation value for the Higgs field h localized on the $y = \pi R$ brane, we find that the wavefunctions for the gauge field are given by $\sim \cos((n+1/2)y/R)$ in the limit $\langle h \rangle \to \infty$ [24]. The operators on the $y = \pi R$ brane can now pick up the effect of this large expectation value, so that they effectively do not respect the $U(1)$ symmetry. Thus the action of the resulting effective field theory is given by Eq. (8) with the $U(1)$ symmetry gauged. Although this theory with large brane Higgs expectation value does not completely reproduce the properties of the theory where the restricted global symmetry with type II defects were gauged, it shares many properties with such a (non-existent) theory. Therefore, it may not be so meaningless to consider theories with type II defects and consider the gauging of its symmetry, in the sense that we might find some underlying theory reproducing some of the features possessed by such a theory. This is the attitude we will take in section 4 when we consider theories with boundary condition supersymmetry breaking in warped space.

3 Supersymmetry Breaking in Warped Space

In this section we study the supersymmetry structure of theories on truncated AdS_5 space, *i.e.* AdS_5 with the fifth dimension compactified on the S^1/Z_2 orbifold. The metric for 5D AdS space with 4D Poincare invariance is given by

$$-ds^2 = g_{MN}dx^M dx^N = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{11}$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$ and $\sigma(y) = ky$ with $0 \leq y \leq \pi R$.

Although the physical space of S^1/Z_2 is only $0 \leq y \leq \pi R$, we can extend it to all values for y with the understanding that different points are identified as $y \sim -y$ and $y' \sim -y'$, where $y' = y - \pi R$. This is a useful procedure because we can then figure out the physics at the boundaries, $y = 0, \pi R$, just by considering the equations of motion *etc.* across these points. The extension of the metric to the (fictitious) space $y < 0$ and $y > \pi R$ is given by

$$\begin{cases} \sigma(y) = k|y| \ \text{ in } \ -\pi R \leq y \leq \pi R, \\ \sigma(y + 2\pi R) = \sigma(y), \end{cases} \tag{12}$$

since $g_{\mu\nu}$ must be even under $y \to -y$ and $y' \to -y'$.

In sub-section 3.1 we define global supersymmetry in AdS space and write down the bulk Lagrangian. The effects of the boundaries are considered in sub-section 3.2. We show that, if we impose boundary conditions on the fields that preserve $N = 1$ supersymmetry in 4D, the two boundaries at $y = 0$ and πR are supersymmetry breaking defects of type I in the classification of the previous section. On the other hand, if we impose boundary conditions which break all the supersymmetries in 4D, we find that the resulting defect is type II. This leads to an important result that when we gauge supersymmetry, which is required to incorporate gravity into the theory, then the theory with supersymmetry breaking boundary conditions becomes inconsistent. Therefore, if we want to consider warped theories with supersymmetry breaking boundary conditions, such theories must be viewed, at best, as phenomenological approximations to some consistent theories that mimic the desired properties of the theories with boundary condition supersymmetry breaking.

3.1 Supersymmetry in the bulk of AdS_5

In this sub-section we study supersymmetry in AdS space and write down the off-shell Lagrangian in the bulk of S^1/Z_2. Recall that a commutator of two supersymmetry transformations δ_ξ and δ_η, parameterized by two Dirac spinors ξ and η respectively,[2] acts on the coordinates x^M as

$$\begin{aligned} x^M &\longrightarrow x^M + \epsilon^M, \\ \text{where} \quad [\delta_\eta, \delta_\xi] &= 2(\bar{\eta}\gamma^M \xi - \bar{\xi}\gamma^M \eta)\partial_M \equiv \epsilon^M \partial_M. \end{aligned} \tag{13}$$

Under this coordinate transformation, the metric g_{MN} changes as

$$g_{MN} \longrightarrow g_{MN} + \epsilon^L \partial_L g_{MN} + g_{LN}\partial_M \epsilon^L + g_{ML}\partial_N \epsilon^L. \tag{14}$$

Now, a global supersymmetry transformation is defined as the supersymmetry transformation which leads to ϵ^M that leaves g_{MN} unchanged. Namely, we require ϵ^M to satisfy

$$\epsilon^L \partial_L g_{MN} + g_{LN}\partial_M \epsilon^L + g_{ML}\partial_N \epsilon^L = 0, \tag{15}$$

or more explicitly

$$\partial_5 \epsilon^5 = 0, \tag{16}$$

$$g_{\mu\nu}\partial_5 \epsilon^\nu + \partial_\mu \epsilon^5 = 0, \tag{17}$$

$$-2\sigma' g_{\mu\nu}\epsilon^5 + g_{\rho\nu}\partial_\mu \epsilon^\rho + g_{\mu\rho}\partial_\nu \epsilon^\rho = 0, \tag{18}$$

where $\sigma' \equiv \partial\sigma/\partial y$. The vector ϵ^M is called a Killing vector, and the above equations are called Killing vector equations.

By replacing ϵ^M in Eqs. (16 – 18) by Eq. (13), we find that the Killing vector equations are satisfied if ξ (and η) satisfies certain conditions. Such a spinor is called a Killing spinor. We write these conditions, called Killing spinor equations, using the symplectic Majorana spinor notation: we express the 5D supersymmetry transformation parameter ξ by two Dirac spinors ξ^1 and ξ^2 obeying a single relation.[3] In this notation, Eq. (13) simply becomes $\epsilon^M = \bar{\eta}_i \gamma^M \xi^i$. First, we find

[2] We use the following convention for γ-matrices:

$$\{\gamma^M, \gamma^N\} = 2g^{MN}, \quad \gamma^\mu = -ie^{\sigma(y)} \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \equiv e^{\sigma(y)}\hat{\gamma}^\mu, \quad \gamma^5 = \gamma_5 = -i\hat{\gamma}^0\hat{\gamma}^1\hat{\gamma}^2\hat{\gamma}^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

where $\sigma^\mu = (1, \vec{\sigma})$ and $\bar{\sigma}^\mu = (1, -\vec{\sigma})$. The Dirac conjugate is defined as $\bar{\Psi} \equiv \Psi^\dagger i\hat{\gamma}^0$.

[3] Here, ξ^1 and ξ^2 *together* correspond to a *single* Dirac spinor ξ. They are related as

$$\xi^1 \equiv \xi, \quad \xi^2 \equiv -C\xi^*, \quad \text{so that} \quad \xi^i = \epsilon^{ij} C \xi_j^* \quad \text{and} \quad \xi_i^* = (\xi^i)^*,$$

where $C \equiv -\hat{\gamma}^2\gamma^5$ is the 5D charge conjugation matrix and has properties, $C^2 = -1$ and $C\gamma^M C^{-1} = -\gamma^{M*}$. Thus both ξ^1 and ξ^2 properly transform as 5D Dirac spinors, and simultaneously they form a doublet under the $SU(2)_R$ automorphism group of the 5D supersymmetry. In terms of more familiar two component notation, they are:

$$\xi^1 = \begin{pmatrix} \xi_{L\alpha} \\ \bar{\xi}_R^{\dot{\alpha}} \end{pmatrix}, \quad \xi^2 = \begin{pmatrix} -\xi_{R\alpha} \\ \bar{\xi}_L^{\dot{\alpha}} \end{pmatrix}.$$

There is one convenient identity for these spinors: $\bar{\xi}_i \gamma^M \cdots \gamma^K \eta^j = \bar{\eta}^j \gamma^K \cdots \gamma^M \xi_i$, for any ξ^i and η^i.

that the most general form for the constraint that solves Eq. (16) and is consistent with the 4D Lorentz invariance is given by

$$\partial_5 \xi^i = -\frac{\sigma'}{2} H^i_j \gamma_5 \xi^j - \frac{i\sigma'}{2} K^i_j \xi^j, \tag{19}$$

where ξ^i represents a general Killing spinor, and H^i_j and K^i_j are 2×2 arbitrary Hermitian matrices which can even depend on positions in spacetime. At this stage, the only constraints for these matrices come from differentiating the identity $\xi^i = \epsilon^{ij} C(\xi^j)^*$ by y, which leads to

$$\mathrm{Tr}[H] = \mathrm{Tr}[K] = 0. \tag{20}$$

We next consider Eq. (17) and find that, in order to solve this, we need an equation for $\partial_\mu \xi^i$ as well as Eq. (19). The most general form of this is given by

$$\partial_\mu \xi^i = -\frac{\sigma'}{2} H^i_j \gamma_\mu \xi^j - \frac{\sigma'}{2} \gamma_5 \gamma_\mu \xi^i - \frac{i\sigma'}{2} L^i_j \gamma_\mu \xi^j, \tag{21}$$

where L^i_j is a new arbitrary 2×2 Hermitian matrix. Finally, we consider the last equation, Eq. (18). We find that this equation is satisfied if and only if

$$L^i_j = 0. \tag{22}$$

Therefore, we find that the Killing spinor must satisfy the equations

$$\partial_5 \xi^i = -\frac{\sigma'}{2} H^i_j \gamma_5 \xi^j - \frac{i\sigma'}{2} K^i_j \xi^j, \tag{23}$$

$$\partial_\mu \xi^i = -\frac{\sigma'}{2} H^i_j \gamma_\mu \xi^j - \frac{\sigma'}{2} \gamma_5 \gamma_\mu \xi^i, \tag{24}$$

where H^i_j and K^i_j are arbitrary 2×2 traceless Hermitian matrices.

Let us now examine whether Eqs. (23, 24) have a non-trivial solution or not. If there exists a non-trivial and reasonable ξ, it must satisfy

$$[\partial_M, \partial_N] \xi^i = 0. \tag{25}$$

Evaluating the above commutator for $M = \mu$ and $N = \nu$ gives the following constraints on the matrix H:

$$H^2 = \mathbf{1}, \tag{26}$$

$$\partial_\mu H = \mathbf{0}, \tag{27}$$

where $\mathbf{1}$ and $\mathbf{0}$ are the unit and zero 2×2 matrices. On the other hand, $[\partial_\mu, \partial_5]\xi^i = 0$ gives the following constraints on H and K:

$$\partial_\mu K = \mathbf{0}, \tag{28}$$

$$-i\partial_5 H = \left[-\frac{\sigma'}{2} K, H \right], \tag{29}$$

and the conditions for ξ^i:

$$H^i_j(y)\xi^j = \gamma_5 \xi^i \qquad \text{at } y = 0 \text{ and } \pi R. \tag{30}$$

Because the form of Eq. (29) is identical to the Heisenberg equation of motion for the operator H with "time" y and "Hamiltonian" $-(\sigma'/2)K$, we can write the general solution as:

$$H(y) = U(y) H(0) U^\dagger(y), \tag{31}$$

where, having Eqs. (26, 28) in mind, $H(0)$ and $U(y)$ are given by

$$\begin{aligned} H(0) &= n^a \sigma_a, && (32) \\ U(y) &= \hat{\mathbf{Y}} \exp\left[-\frac{i}{2} \int_0^y \sigma' K(y') dy' \right], && (33) \end{aligned}$$

where n^a ($a = 1, 2, 3$) is a constant real vector with unit length $n^a n^a = 1$, σ_a are the Pauli spin matrices, and $\hat{\mathbf{Y}}$ is the "time"-ordering operator. We can check that this solution solves all the constraints on H and K, Eqs. (26 – 29). Note also that $U(y)$ belongs to $SU(2)$ because K is Hermitian and traceless.

The above Killing spinor equation contains important information about the symmetry structure of the theory. We consider the $SU(2)_R$ automorphism group of the 5D supersymmetry, under which ξ^1 and ξ^2 form a doublet. In flat space ($\sigma' = 0$), this $SU(2)_R$ is a symmetry of the algebra and thus respected by the whole theory. In AdS, however, we find that it is broken by the presence of the matrices H and K in Eqs. (23, 24). Now, we consider redefining the fields by a twist inside $SU(2)_R$. This results in the redefinition of ξ^i according to

$$\xi^i(y) \longrightarrow \tilde{U}(y)^i_j \xi^j(y), \tag{34}$$

where $\tilde{U}(y)$ is a y-dependent matrix taking arbitrary values in $SU(2)$. Note that, since we are just redefining the name of the fields, this does not change any physics. Then, substituting Eq. (34) into Eqs. (23, 24) and choosing $\tilde{U}(y) = U(y)$, we find that $H(y)$ is replaced by $H(0)$ and the $K(y)$ term is canceled. We can further make a y-independent $SU(2)_R$ rotation and choose $(n^1, n^2, n^3) = (0, 0, 1)$ for $H(0)$. Therefore, we finally obtain the following simple form for the Killing spinor equations in AdS$_5$:

$$\begin{aligned} \partial_5 \xi^i &= -\frac{\sigma'}{2} (\sigma_3)^i_j \gamma_5 \xi^j, && (35) \\ \partial_\mu \xi^i &= -\frac{\sigma'}{2} (\sigma_3)^i_j \gamma_\mu \xi^j - \frac{\sigma'}{2} \gamma_5 \gamma_\mu \xi^i. && (36) \end{aligned}$$

These equations show that a $U(1)_R$ subgroup of $SU(2)_R$ remains unbroken in the AdS background. The constraint on ξ^i, Eq. (30), now becomes

$$(\sigma_3)^i_j \xi^j = \gamma_5 \xi^i \qquad \text{at } y = 0 \text{ and } \pi R. \tag{37}$$

These three equations define global supersymmetry in the truncated AdS_5 on S^1/Z_2. The form of the bulk Lagrangian is determined by Eqs. (35, 36). The Killing spinor boundary constraint of Eq. (37) is crucially important when we consider the effect of the boundaries in the next sub-section. In particular it requires that $\xi_{R\alpha} = 0$ at both boundaries.

Finally, we write down the off-shell bulk Lagrangians (in the basis where the Killing spinor equations take the form of Eqs. (35, 36)). Effects of boundaries, including Eq. (37), will be considered in the next sub-section. We begin with a hypermultiplet, which consists of two complex scalars, ϕ^1 and ϕ^2, and a Dirac spinor, Ψ, and two complex auxiliary fields, F^1 and F^2. The kinetic part of the action is given by

$$\mathcal{S}_{\text{hyp.kin.}} \equiv \int d^4x \int dy \, \sqrt{-g} \, \mathcal{L}_{\text{hyp.kin.}}, \tag{38}$$

$$\mathcal{L}_{\text{hyp.kin.}} = -g^{MN} \partial_M \phi_i^* \partial_N \phi^i - \frac{1}{2} \bar{\Psi} \gamma^M \partial_M \Psi + \frac{1}{2} \partial_M \bar{\Psi} \gamma^M \Psi + F_i^* F^i + \frac{15}{4} k^2 \phi_i^* \phi^i, \tag{39}$$

where $i = 1, 2$ and both ϕ^i and F^i are doublets under $SU(2)_R$; in particular $\phi_i = \epsilon_{ij} \phi^j$, $\phi_i^* = (\phi^i)^*$, and so on. This action is invariant under the following global supersymmetry transformation:

$$\begin{aligned} \delta \phi^i &= \sqrt{2} \epsilon^{ij} \bar{\xi}_j \Psi, \\ \delta \Psi &= \sqrt{2} \left(\gamma^M \xi^i \partial_M \phi_i - \frac{3}{2} \sigma' \xi^i \phi_j (\sigma_3)^j_i + \xi^i F_i \right), \\ \delta F^i &= \sqrt{2} \epsilon^{ij} \left(\bar{\xi}_j \gamma^M \partial_M \Psi - 2 \sigma' \bar{\xi}_j \gamma_5 \Psi \right), \end{aligned} \tag{40}$$

where the global supersymmetry transformation parameter ξ^i satisfies Eqs. (35, 36). In addition to the above kinetic part, Eq. (39), we can also add a mass to the hypermultiplet:

$$\mathcal{L}_{\text{hyp.mass}} = -c\sigma' \bar{\Psi} \Psi + c\sigma' (F_i^* \phi^i + \phi_i^* F^i) - ck^2 (\sigma_3)^i_j \phi_i^* \phi^j, \tag{41}$$

where c is a dimensionless real constant. This by itself is invariant (up to a total derivative) under the global supersymmetry transformation, Eqs. (40) with Eqs. (35, 36).

The gauge supermultiplet consists of a vector field A_M, a Dirac gaugino Ψ_λ, a real scalar Σ, and three real auxiliary fields X^a ($a = 1, 2, 3$). The Lagrangian is given by

$$\begin{aligned} \mathcal{L}_{\text{gauge}} \;\; = \;\; & \frac{1}{g^2} \Bigg[-\frac{1}{4} g^{ML} g^{NK} F_{MN} F_{KL} - \frac{1}{2} g^{MN} \partial_M \Sigma \, \partial_N \Sigma - \frac{1}{2} \bar{\lambda}_i \gamma^M \partial_M \lambda^i + \frac{1}{2} X^a X^a \\ & + 2k^2 \Sigma^2 - \frac{1}{4} \sigma' (\sigma_3)^i_j \bar{\lambda}_i \lambda^j \Bigg], \end{aligned} \tag{42}$$

where we have chosen the gauge group to be $U(1)$ for simplicity. We have also used the symplectic Majorana notation for the gaugino: Ψ_λ is represented by the two Dirac spinors λ^1 and λ^2. Note that the gaugino and the auxiliary fields form a doublet and a triplet, respectively, under $SU(2)_R$. This Lagrangian is supersymmetric under the following global supersymmetry transformation:

$$\begin{aligned}
&\delta A_M = -\bar{\xi}_i \gamma_M \lambda^i, \\
&\delta \Sigma = i\bar{\xi}_i \lambda^i, \\
&\delta \lambda^i = -i\gamma^M \xi^i \partial_M \Sigma - \frac{1}{2}\gamma^M \gamma^N \xi^i F_{MN} + 2i\sigma' \Sigma (\sigma_3)^i_j \xi^j - i(\sigma_a)^i_j \xi^j X^a, \\
&\delta X^a = i\bar{\xi}_i \gamma^M \partial_M \lambda^j (\sigma^a)^i_j - 2i\sigma' \bar{\xi}_i \gamma_5 \lambda^j (\sigma^a)^i_j,
\end{aligned} \tag{43}$$

where ξ satisfies the condition Eqs. (35, 36). Generalization to a non-Abelian group is fairly straightforward (giving the appropriate gauge structure, adding certain gaugino-gaugino-scalar interactions, changing the derivatives to gauge covariant derivatives, and so on). These bulk Lagrangians, Eqs. (39, 41, 42), reproduce the on-shell bulk Lagrangians given in Refs. [25], after integrating out the auxiliary fields (assuming no boundaries).

3.2 Effects of the boundaries

In this sub-section we consider the effects of the boundaries. We follow the discussion in section 2 and consider the symmetry structure of the theory. A new ingredient compared with the previous case is the constraint coming from the Killing spinor equation, Eq. (37). This additional complication arises from the fact that supersymmetry is a spacetime symmetry. The other parts of the discussion, however, are quite analogous to the previous case.

We begin by considering the boundary conditions on the fields. As explained in the previous section, the boundary conditions are written as Eq. (2), where φ is a column vector collecting all the fields in the theory, including the metric g_{MN}. Since the matrices $\mathbf{Z}$ and $\mathbf{Z}'$ must be representations of the two reflections $\mathcal{Z} : y \to -y$ and $\mathcal{Z}' : y' \to -y'$, respectively, they must obey the relations:

$$\mathbf{Z}^2 = \mathbf{1}, \qquad \mathbf{Z}'^2 = \mathbf{1}. \tag{44}$$

Thus we find that the general boundary conditions are given as follows. Under the reflection $\mathcal{Z}$, the fields obey

$$\begin{aligned}
&\phi^i(y) = P_\Phi U^i_j (\sigma_3)^j_k \phi^k(-y), \\
&\Psi(y) = P_\Phi \gamma_5 \Psi(-y), \\
&F^i(y) = P_\Phi U^i_j (\sigma_3)^j_k F^k(-y),
\end{aligned} \tag{45}$$

and

$$
\begin{aligned}
& A_\mu(y) = A_\mu(-y), \qquad A_5(y) = -A_5(-y), \\
& \lambda^i(y) = U^i_j(\sigma_3)^j_k \gamma_5 \lambda^k(-y), \\
& \Sigma(y) = -\Sigma(-y), \\
& X^a(y) = \frac{1}{2}\mathrm{tr}[\sigma^a U \sigma^3 \sigma^b \sigma^3 U^\dagger] X^b(-y),
\end{aligned}
\tag{46}
$$

where $U = \exp[2\pi i(\alpha_1\sigma_1 + \alpha_2\sigma_2)]$ with $0 \leq \alpha_{1,2} < 1$, and each hypermultiplet can have its own parity $P_\Phi = \pm 1$. The boundary conditions under $\mathcal{Z}'$ is also given similarly, by the replacement $y \to y'$, $U \to U'$ ($\alpha_{1,2} \to \alpha'_{1,2}$) and $P_\Phi \to P'_\Phi$ in Eqs. (45, 46), where $0 \leq \alpha'_{1,2} < 1$ and $P'_\Phi = \pm 1$.[4]

Now, we study the supersymmetry structure of the theory: a conservation law for the supercurrent. Following the discussion in section 2, we consider the Noether procedure in the path integral formalism. What position dependence should we allow for the supersymmetry transformation parameter, and how many supersymmetries are preserved in each point in the extra dimension? First, we can easily see that there are 4D $N = 2$ supersymmetries in the bulk, because in any local neighborhood of the bulk we can solve the Killing spinor equation, Eqs. (35, 36), as $\xi^1(x^\mu, y) = \exp(-\sigma\gamma_5/2)(1 - \sigma' \exp(\sigma)\gamma_\mu x^\mu(1-\gamma_5)/2)\xi_0$, which is parameterized by an arbitrary constant Dirac spinor ξ_0. A non-trivial question is the number of supersymmetries on the boundaries. At the boundaries $y = 0$ and πR, the supersymmetry transformation parameter must obey the condition Eq. (37). On the other hand, the boundary conditions for the fields, Eqs. (45, 46) implies that the supersymmetry transformation parameter must obey

$$
\xi^i(y) = U^i_j(\sigma_3)^j_k \gamma_5 \xi^k(-y), \qquad \xi^i(y') = U'^i_j(\sigma_3)^j_k \gamma_5 \xi^k(-y'),
\tag{47}
$$

to preserve the boundary conditions of the fields. The number of supersymmetries on the boundaries is then determined by these two conditions, Eq. (37, 47).

Let us focus on the $y = 0$ boundary (the discussion for the $y = \pi R$ boundary is identical). We first consider the case $\alpha_1 = \alpha_2 = 0$. In this case, Eq. (37) and Eq. (47) become identical; in other words, the Killing spinor equation does not give an additional constraint on the transformation parameter ξ^i beyond the one arising from the boundary conditions, Eq. (47). This situation is similar to the $SU(5)$ example discussed in sub-section 2.1. In fact, we find that the $y = 0$ brane is supersymmetry breaking defect of type I, on which the 4D $N = 2$ supersymmetry in the bulk is broken to 4D $N = 1$. The number of supersymmetries can easily be understood from Eq. (47): $\xi^i(y) = (\sigma_3)^i_j \gamma_5 \xi^j(-y)$ requires half of ξ^i to vanish at $y = 0$. A defect of type I implies that the symmetries can be consistently gauged, *i.e.* the theory can be embedded into supergravity. In supergravity, the gravitino $\psi_{3/2}$ obeys the boundary conditions analogous to Eq. (47). Thus, when

[4]Here we have assumed the boundary conditions do not break the gauge symmetry, although including such breaking is straightforward. The procedure is exactly identical to that in flat space.

expanded into 4D modes, the number of $\psi_{3/2}$'s and the number of ξ's are the same (there is no need to impose any extra constraint on the gravitino field), ensuring the consistency of the theory with local supersymmetry. In particular, if we choose $\alpha_1 = \alpha_2 = \alpha_1' = \alpha_2' = 0$, the resulting theory possesses unbroken 4D $N=1$ supersymmetry, whose transformation parameter is given by $\xi^1(x^\mu, y) = \exp(-\sigma/2)\xi_L$ where ξ_L is a spinor (dependent on coordinates in supergravity) subject to the condition $\gamma_5 \xi_L = \xi_L$. The explicit realization of this case in the context of supergravity has been extensively studied [26].

We next consider the case where either α_1 or α_2 is non-zero. In this case Eq. (37) and Eq. (47) give different conditions, and we find that the solution to both equations is only the trivial one, $\xi^i = 0$. This implies that we do not have any supersymmetry on the $y=0$ brane. Since the constraint $\xi^i(y=0) = 0$ is an extra condition imposed on ξ, additional to the one arising from the boundary conditions, the situation is similar to the $U(1)$ example discussed in sub-section 2.2 with $\xi^i(y=0) = 0$ corresponding to Eq. (10). The defect is type II and does not allow gauging of the supersymmetry of the theory. The argument is similar to the previous $U(1)$ case. When we gauge supersymmetry, we must introduce the gravitino field and impose boundary conditions like Eqs. (47). Since the gravitino is a dynamical field, we cannot impose any additional constraint by hand. This implies that the number of ξ's is one smaller than that of $\psi_{3/2}$'s (in the 4D picture) due to the extra constraint $\xi^i(y=0) = 0$. Since the consistent treatment of a spin-3/2 field requires a supersymmetry, this leads to an inconsistency; for instance, in such theories ghosts can be physically produced and certain scattering amplitudes lead to negative probabilities (the presence of such ghosts was also noted in Ref. [18]).

Why do we insist on gauging supersymmetry? If supersymmetry were not a spacetime symmetry, we would be able to consider only global supersymmetry. We would be able to use arbitrary values for $\alpha_{1,2}$ and $\alpha_{1,2}'$ to construct models, in which supersymmetry is broken by boundary conditions. However, supersymmetry *is* spacetime symmetry. When we include gravity, we have to consider supergravity, in which supersymmetry is gauged. This means that the boundaries at $y=0$ and πR must be symmetry breaking defects of type I: $\alpha_{1,2}$ and $\alpha_{1,2}'$ must be zero. Therefore, we arrive at the following conclusion. *In AdS_5 the compactification on S^1/Z_2 is unique: we cannot use boundary conditions to break all bulk supersymmetries in a warped extra dimension.*

Nevertheless, in the next section we consider models on the truncated AdS_5 in which supersymmetry is broken by boundary conditions. As mentioned at the end of sub-section 2.2, we do this because some theories can mimic certain properties of the theory with boundary condition supersymmetry breaking. For instance, consider a theory with $\alpha_1 = \alpha_2 = \alpha_1' = \alpha_2' = 0$ and break supersymmetry spontaneously by the expectation value for the F-component of a brane-localized chiral superfield Z at $y = \pi R$. Then, if this expectation value is large (we formally take $F_Z \to \infty$), we find that some properties of the boundary condition breaking, such as strict relations between supersymmetry breaking masses and the KK mass scale, are recovered [3]. Thus, although the models presented in section 4 do not allow consistent inclusion of gravity as they are, we think that it is worthwhile constructing some representative models and exploring their phenomenology.

We note that the case of supersymmetry breaking by boundary conditions in flat space is now very simple to analyze. The Killing spinor equations become trivial, with ξ^i becoming constant for a global transformation; crucially there is no Killing spinor constraint at the boundary, such as Eq. (37). Therefore, the issue we have in AdS space, *i.e.* the incompatibility of Eq. (47) with Eq. (37), does not exist in flat space. Thus, any choices for the matrices U^i_j and U'^i_j in Eq. (47) yield type I defects at both boundaries, where each boundary preserves a single 4D supersymmetry, with the orientation of the supersymmetry in $SU(2)_R$ space depending on the parameter α relevant at that boundary. The entire system preserves a supersymmetry in 4D only if the two boundaries preserve the same supersymmetry, $\alpha = \alpha'$, otherwise supersymmetry is completely broken by the boundary conditions. If either boundary is allowed to have a Lagrangian which is not invariant under any supersymmetry, the resulting $N = 0$ defect is of type II, so that the resulting theories are inconsistent with supergravity.

Finally in this section, we complete the Lagrangian in the case of $\alpha_1 = \alpha_2 = \alpha'_1 = \alpha'_2 = 0$. The bulk Lagrangian of Eqs. (39, 41, 42) is not invariant under the supersymmetry transformation at $y = 0$ and πR. For instance, when we vary the hypermultiplet action Eqs. (38, 39), we find that the terms that spoil invariance appear from ∂_y acting on σ':

$$\sqrt{-g}\,\delta\mathcal{L}_{\text{hyp.kin.}} = \sqrt{-g}\left[\cdots + \frac{3}{2}\sigma''\sqrt{2}\bar{\Psi}\xi^i\phi_i + \text{h.c.}\right], \tag{48}$$

where $\sigma'' = 2k(\cdots + \delta(y) - \delta(y - \pi R) + \cdots)$. However, these terms can be canceled if we add brane mass terms for the scalars:

$$\mathcal{L}_{\text{hyp.kin.}} \to \mathcal{L}_{\text{hyp.kin.}} - \frac{3}{2}\sigma''\phi_i^*\phi^i. \tag{49}$$

This gives the correct supersymmetric Lagrangian on AdS$_5$ compactified on S^1/Z_2. A similar analysis for Eqs. (41, 42) leads to

$$\mathcal{L}_{\text{hyp.mass}} \to \mathcal{L}_{\text{hyp.mass}} + c\sigma''(\sigma_3)^i_j\phi_i^*\phi^j, \tag{50}$$

$$\mathcal{L}_{\text{gauge}} \to \mathcal{L}_{\text{gauge}} - \frac{1}{g^2}\sigma''\Sigma^2. \tag{51}$$

After integrating out the auxiliary fields, these Lagrangians agree with the on-shell Lagrangian given in Ref. [16].

4 Models

4.1 Warped constrained standard model

Consider an $SU(3)_C \times SU(2)_L \times U(1)_Y$ supersymmetric gauge theory on truncated AdS$_5$ space. Each 4D boundary is necessarily a defect in the space of supersymmetries, since the two bulk supersymmetries cannot coexist on a 4D boundary. In the last section we have shown that if both defects are of type I, then the supersymmetries preserved at each boundary must align with each other, so that the entire system preserves a 4D supersymmetry. To break supersymmetry by boundary conditions, we must consider supersymmetry breaking by means of a defect of type II. Furthermore, we assume that the Planck brane located at $y = 0$ is a type I defect preserving one supersymmetry, since, if it were type II, all supersymmetries would be broken at the Planck scale.[5] Therefore the TeV brane at $y = \pi R$ must be of type II, so that couplings on this brane explicitly break all supersymmetries.

The field content and boundary conditions are chosen to be identical to those of the constrained standard model [2], so that gauge, matter, and a single Higgs hypermultiplet all propagate in the bulk. Since there is a single zero-mode Higgs boson, we expect a Higgs sector far more constrained than that of the MSSM. The supersymmetry breaking boundary conditions are those of Eqs. (45, 46) with $\alpha'_2 = 1/2$, $\alpha_{1,2} = \alpha'_1 = 0$ and $P_{\rm matter} = P'_{\rm matter} = +1$, $P_{\rm Higgs} = -P'_{\rm Higgs} = +1$. The mass spectrum for both matter-like and Higgs-like boundary conditions are shown in Fig. 1. While these boundary conditions are identical to those of the flat space constrained standard model, we stress that in that theory both boundary defects were of type I preserving orthogonal supersymmetries, so that the structure of supersymmetry breaking differs greatly in the warped case. We assume that all matter hypermultiplets have a bulk mass $c_M = 1/2$, ensuring that the quark and lepton zero modes are conformally flat. This is analogous to the flat space theory in the absence of bulk masses. We expect that deviations from $c_M = 1/2$ would be analogous to introducing bulk masses in the flat case [28]. To obtain a predictive theory of electroweak symmetry breaking with a single Higgs boson, the Higgs must propagate in the bulk. If we had instead placed the Higgs boson on the Planck brane, then 4D supersymmetry on that brane would have prevented it from generating down-type masses. If we had placed it on the non-supersymmetric TeV brane, the quartic coupling would be arbitrary and there would be no prediction for the physical Higgs boson mass. A bulk Higgs boson, however, is able to generate up-type masses at the $y = 0$ brane and all masses at the $y = \pi R$ brane, and to a large extent radiative corrections are controlled by the unbroken bulk supersymmetry.

The bulk mass for the Higgs hypermultiplet, c, is still free as is the Higgs boson brane mass

[5] We could instead choose the Planck brane to be a type II defect, if we localize the Higgs fields to the TeV brane. Such a construction can lead to theories where there is a little hierarchy between the electroweak and new physics scales [27].

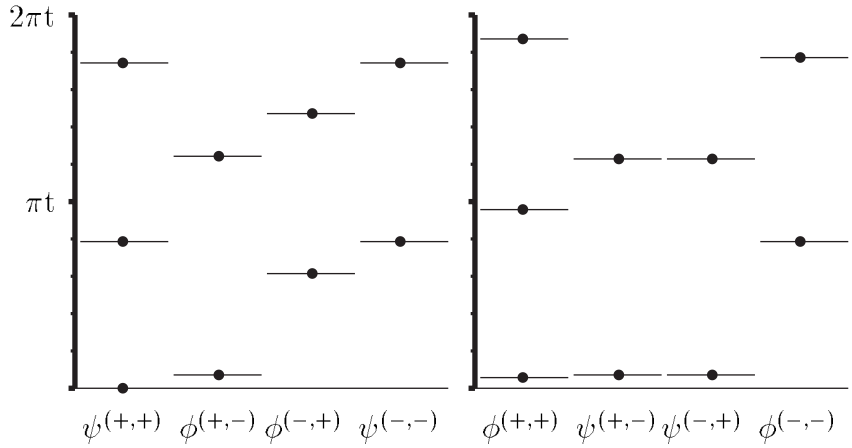

Figure 1: Mass spectrum for matter-like fields (left) with $c_M = 1/2$ and for Higgs-like fields (right) with $c = 1/2$ and $r' = -1$.

term at the TeV brane $\mathcal{L} = r'(\phi^{1*}\phi^1)2k\delta(y - \pi R)$, where ϕ^1 is the first component of a complex $\mathrm{SU}(2)_R$ doublet and gauge indices are contracted. These two parameters in turn determine the profile of the Higgs boson KK modes in the bulk, and the tree-level mass of the Higgs boson KK modes. In particular the lightest mode mass is approximately given by:

$$m_{\text{tree}} \simeq 2\sqrt{(c^2 - 1/4)\frac{3/2 - r' - c}{5/2 - r' + c}}\left(\frac{t}{k}\right)^{c-1/2} t, \tag{52}$$

for $(t/k)^{c-1/2} \ll 1$, where $t = e^{-k\pi R}k$ is the scale of physics at the $y = \pi R$ brane. An important result is that for $c > 1/2$ the tree-level mass is much smaller than the typical KK mass scale t so that the full Higgs mass parameter becomes only weakly sensitive to r'. This is because the lowest level wavefunction is strongly peaked around the $y = 0$ brane. Therefore, while we have no knowledge of the TeV brane parameters (and radiative corrections to some of these parameters are even power divergent), the low energy physics is largely insensitive to their values.

Since the tree level mass of the Higgs boson rapidly becomes small for $c > 1/2$, electroweak symmetry breaking is triggered radiatively via the top Yukawa coupling, which we assume to be located dominantly on the Planck brane. As discussed in section 3.2, the supercurrent is conserved locally in the bulk, so radiative effects must respect supersymmetry there. Therefore, supersymmetry guarantees that the bulk Higgs mass is not renormalized. Thus, as in models of boundary condition supersymmetry breaking on flat extra dimension, we expect corrections to the 4D Higgs boson mass to be finite, except for the contribution from the $y = \pi R$ brane, which we have argued is small. Therefore, by taking r' to be the renormalized brane mass we are able to calculate the physical Higgs mass in terms of c and r'. We have computed radiative corrections from the top quark Yukawa coupling to the Higgs boson effective potential. After minimizing this effective potential, the mass scale of the KK modes is determined from M_Z, and the resulting

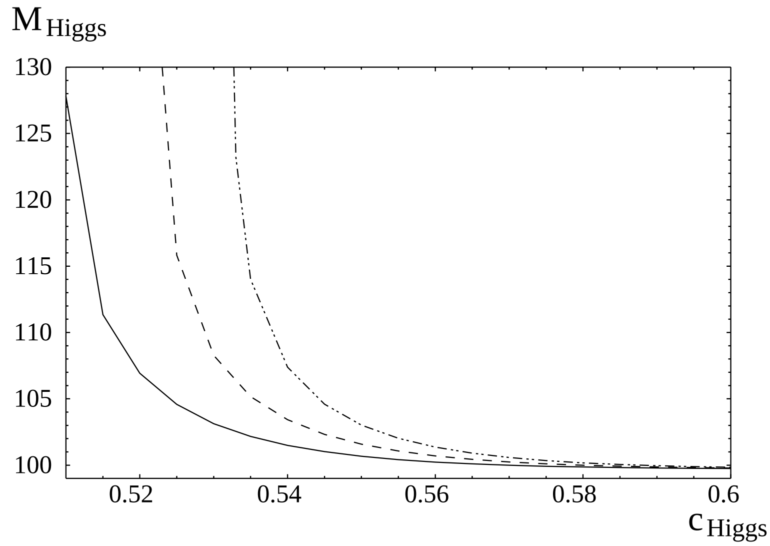

Figure 2: Physical Higgs boson mass in GeV for $r' = -1$ (dash-dot-dot), $r' = 0$ (dashed) and $r' = 0.5$ (solid).

prediction for the Higgs boson mass is shown in Fig. 2, for a range of c and for three values of r'. The insensitivity to r' as c increases above $1/2$ is striking, but not unexpected as in this region the tree-level mass effectively vanishes. For the same reason, the physical Higgs boson mass becomes constant for large c at about 100 GeV. One might worry that direct searches rule out a single Higgs boson with mass less than 115 GeV, and the model requires a large degree of fine tuning to reach such a mass. However, since r' is not the only TeV brane operator that affects the Higgs mass, we expect that there are $O(15\%)$ corrections to our calculation. For example, there can be additional quartic interactions, top Yukawa couplings and terms involving the F-fields of the matter multiplets, on the TeV brane. Although all of these terms are suppressed by the wavefunction overlaps of the various fields with the TeV brane and thus introduce only small corrections, they can give non-negligible effects on the physical Higgs boson mass; for example, a brane-localized quartic coupling is expected to introduce a $\lesssim 15\%$ correction in the physical Higgs mass in the limit of strong coupling. Therefore, taking note of these possible corrections, the model is not ruled out for a reasonably wide range of parameter space. Notice that for a given r', electroweak symmetry is not broken for all c. Below a certain c, the radiative corrections are unable to overcome the positive tree level mass squared and electroweak symmetry breaking does not occur. Thus the curves in Fig. 2 end at these points.

Taking $c > 1/2$ also leads to light Higgsinos. The lightest Higgsino mass is approximately

$$m_{\tilde{h}} \simeq 2\sqrt{c^2 - 1/4}\left(\frac{t}{k}\right)^{c-1/2} t. \tag{53}$$

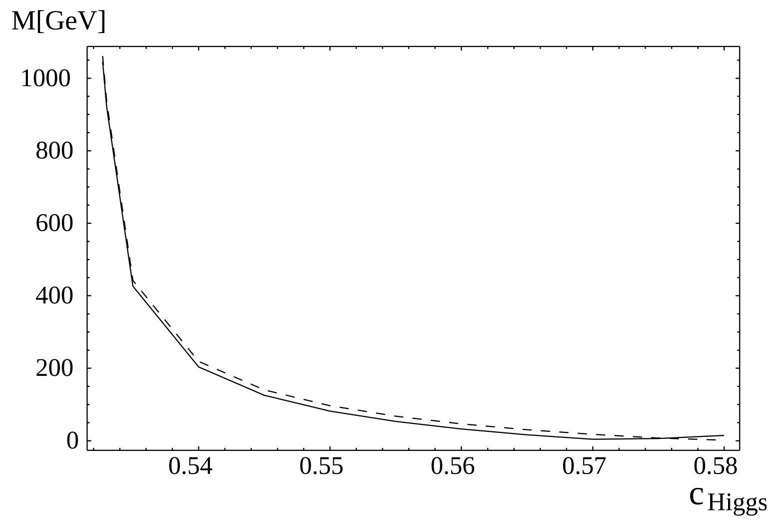

Figure 3: Lightest neutralino (solid) and lightest chargino (dashed) masses in GeV for $r' = -1$ and $z = -0.75$.

Notice that no brane mass term can be written for the Higgsinos because $\tilde{h}^1$ vanishes at $y = \pi R$. The masses of the lightest neutralino and lightest chargino therefore place a bound on c. However, this bound is weak because it depends on the size of the brane-localized kinetic term for $\tilde{h}^2$

$$S_4 = -\int d^4x (k/t)^{-1} Z \bar{\tilde{h}}^2 \partial_\mu \hat{\gamma}^\mu \tilde{h}^2 \left(\eta_{\tilde{h}^2}(\pi R)\right)^2, \tag{54}$$

where indices are raised and lowered with $\eta^{\mu\nu}$ and $\hat{\gamma}^\mu$ are the four dimensional Dirac gamma matrices. $\eta_{\tilde{h}^2}(y)$ is the wavefunction of the lightest right-handed Higgsino. If we consider the dimensionless combination $z = Z(k/t)^{-1}\left(\eta_{\tilde{h}^2}(\pi R)\right)^2$ we find that the four dimensional kinetic term for the lightest $\tilde{h}^2$ has a coefficient $\simeq 1 + z$ and that going to canonical normalization the lightest Higgsino mass is

$$m_{\text{canonical}} \simeq \frac{m_{\tilde{h}}}{\sqrt{1+z}}. \tag{55}$$

Notice that the strong peaking of the wavefunction enhances the effect due to the brane kinetic term, so that a correction to the Higgsino mass of order unity is expected. As an example, we show in Fig. 3 the lightest chargino and neutralino for the case that $z = -0.75$ and $r' = -1$, though there is only weak sensitivity to r'. In this case, the chargino mass is not ruled out by direct searches for $c \lesssim 0.55$.

By introducing a type II supersymmetry breaking defect on a warped background, we are able to construct a predictive theory of electroweak symmetry breaking with one Higgs doublet. The theory requires a moderate peaking of the Higgs boson on the $y = 0$ brane, which could be the origin of the m_t/m_b ratio. While the Higgs boson is expected to be close to its experimental limit

Matter			Higgs		
	$F_{++}(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$F_{-+}(\mathbf{3},\mathbf{1})_{\frac{1}{3}}$		$H_{++}(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$H_{-+}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$
$\mathcal{F}'$	$F^c_{--}(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$F^c_{+-}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$	$\mathcal{H}$	$H^c_{--}(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$H^c_{+-}(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$
	$F_{-+}(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$F_{++}(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$		$H_{++}(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$H_{-+}(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$
$\mathcal{F}$	$F^c_{+-}(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$F^c_{--}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$	$\bar{\mathcal{H}}$	$H^c_{--}(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$H^c_{+-}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$

Table 1: Superfields from matter and Higgs fields are listed by their quantum numbers and parity assignments before the supersymmetry breaking twist. The quantum numbers represent those under $SU(3)_C \times SU(2)_L \times U(1)_Y$. The far left column indicates what hypermultiplet the fields are contained in. $\mathcal{F}$ and $\mathcal{F}'$ are $\bar{\mathbf{5}}$'s of matter while $\mathcal{H}$ and $\bar{\mathcal{H}}$ are Higgs multiplets.

of 115 GeV, a precise prediction is not possible because of the degree of peaking of the Higgs boson wavefunction and the supersymmetry breaking interactions on the TeV brane. The lightest chargino and neutralino are also close to their experimental bounds.

4.2 Twisted warped grand unified theory

While the theory just described has logarithmic running of gauge couplings up to the mass scale of the Planck brane, it is not a theory of gauge coupling unification, since nothing in the theory requires the bulk gauge couplings to be unified. In order to construct such a theory we must consider a model in which the bulk Lagrangian is symmetric under some grand unified group. Here we consider an $SU(5)$ supersymmetric gauge theory on a slice of AdS$_5$. We take the warped supersymmetric grand unified theory of Ref. [15], with all matter and Higgs in the bulk, and break the $SU(5)$ symmetry by boundary conditions imposed at the Planck brane. Each generation contains $\mathcal{F} : \{F(\bar{\mathbf{5}}), F^c(\mathbf{5})\} + \mathcal{F}' : \{F'(\bar{\mathbf{5}}), F'^c(\mathbf{5})\}$ and $\mathcal{T} : \{T(\mathbf{10}), T^c(\bar{\mathbf{10}})\} + \mathcal{T}' : \{T'(\mathbf{10}), T'^c(\bar{\mathbf{10}})\}$ where $\Phi(R)$ represents a chiral supermultiplet in the R representation of $SU(5)$. There are, in addition, two Higgs hypermultiplets $\mathcal{H} : \{H(\mathbf{5}), H^c(\bar{\mathbf{5}})\}$ and $\bar{\mathcal{H}} : \{\bar{H}(\bar{\mathbf{5}}), \bar{H}^c(\mathbf{5})\}$. In this model, the boundary conditions are given such that each brane is a symmetry breaking defect of type I with respect to supersymmetry. The Planck brane is additionally a symmetry breaking defect of type I with respect to the gauge group, breaking $SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y$. The TeV brane respects the full $SU(5)$ group. (The boundary conditions for some of the bulk fields are given explicitly in Table 1.) The zero-mode particle content of the model is the same as in the MSSM, while the KK towers consist of $SU(5)$ symmetric particles of masses around TeV. It was shown in [15] that, if all bulk fields carry $c \geq 1/2$, only the zero modes contribute to the differential gauge coupling running, so that the model leads to the same beta functions as in the MSSM. Therefore, despite the drastic departure from the MSSM particle content at the TeV scale, the theory preserves logarithmic gauge coupling unification at a high scale.

With the boundary conditions described above, both branes are defects of type I respecting

Matter			Higgs		
	$\phi_{+-}(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\phi_{--}(\mathbf{3},\mathbf{1})_{\frac{1}{3}}$		$\phi_{++}(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$\phi_{-+}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$
$\mathcal{F}'$	$\psi_{++}(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\psi_{-+}(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$	$\mathcal{H}$	$\psi_{+-}(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$\psi_{--}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$
	$\phi^c_{-+}(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$\phi^c_{++}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$		$\phi^c_{--}(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\phi^c_{+-}(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$
	$\psi^c_{--}(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$\psi^c_{+-}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$		$\psi^c_{-+}(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\psi^c_{++}(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$
	$\phi_{--}(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\phi_{+-}(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$		$\phi_{++}(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\phi_{-+}(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$
$\mathcal{F}$	$\psi_{-+}(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\psi_{++}(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$	$\bar{\mathcal{H}}$	$\psi_{+-}(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\psi_{--}(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$
	$\phi^c_{++}(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$\phi^c_{-+}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$		$\phi^c_{--}(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$\phi^c_{+-}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$
	$\psi^c_{+-}(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$\psi^c_{--}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$		$\psi^c_{-+}(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$\psi^c_{++}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$

Table 2: Fields from $\bar{\mathbf{5}}$ matter and Higgs multiplets are listed by their quantum numbers and parity assignments after the supersymmetry breaking twist. ϕ and ϕ^c (ψ and ψ^c) represent complex scalar (Weyl fermion) fields in Φ and Φ^c superfields, respectively, where $\Phi = F, F', H, \bar{H}$.

the same 4D $N=1$ supersymmetry, so that there exists unbroken $N=1$ supersymmetry in the low-energy 4D theory. One way of breaking this remaining supersymmetry is to consider a supersymmetry breaking VEV located on the TeV brane. Instead, here we consider breaking the remaining supersymmetry by modifying the boundary conditions such that the brane at $y=\pi R$ becomes a supersymmetry breaking defect of type II, as in the previous sub-section. This is accomplished by introducing non-zero α' parameters in Eqs. (45, 46). Without loss of generality, we can take $\alpha'_1 = \alpha_{1,2} = 0$. We here choose the supersymmetry breaking parameter $\alpha'_2 = 1/2$. We also choose $P'_{\rm Higgs} = -1$ so that there are light Higgs scalars from the Higgs hypermultiplets.

We first consider the effect of the supersymmetry breaking twist, $\alpha'_2 \neq 0$, on the matter multiplets. Here, the net effect is that the parity under $y' \to -y'$, Z', changes sign for the $SU(2)_R$ doublet scalars. As a consequence, the MSSM sfermions no longer possess a zero mode while their first KK modes appear at O(TeV). However, a new scalar zero mode now appears. As shown in Tables 2 and 3, these scalars are related to the standard model fermions by the broken generators of $SU(5)$ and by the supersymmetry broken at the Planck brane. We will call these the $SU(5)$, $N=2$ partners of the standard model fermions. Notice that a full generation will possess both a fermion and a scalar with conjugate quantum numbers. Therefore, mere observation of quantum numbers and the mass spectrum (before electroweak symmetry breaking) could mimic the presence of an unbroken supersymmetry. However, the fermion and scalar originate from different hypermultiplets and thus there is no supersymmetry that relates the two. For example, the two fields will not be coupled by the gauginos. The extra bosonic fields can be made heavy by mass terms located on the $y=\pi R$ brane. In order for these fields to become sufficiently heavy their wavefunctions must have a sizable overlap with the TeV brane. This translates into the requirement that $c_{\rm matter} \geq 1/2$, in accordance with the requirement for gauge coupling unification

Standard Model Matter	$\psi(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$\psi(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}$	$\psi(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$	$\psi(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\psi(\mathbf{1},\mathbf{1})_{1}$
$SU(5), N=2$ Partners	$\phi^c(\bar{\mathbf{3}},\mathbf{2})_{-\frac{1}{6}}$	$\phi^c(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$	$\phi^c(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$	$\phi^c(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$\phi^c(\mathbf{1},\mathbf{1})_{-1}$

Table 3: Matter fields which have zero modes: standard model quarks and leptons and their $SU(5)$, $N=2$ partners.

and stability of the proton [15]. The resulting masses are naturally in the TeV region.[6]

In the Higgs sector, because of the change in sign of $P'_{\rm Higgs}$ in addition to the supersymmetry breaking twist, the Z' parities of the fermions change sign. As a result, the doublet Higgsino becomes heavy. However, as in the matter sector, the $SU(5)$, $N=2$ partners of the two Higgs bosons, a pair of color triplet fermions, now possess zero modes. Again, as in the matter sector, these unwanted fields can be made massive via a TeV brane localized mass term: a Dirac mass for the two colored Higgsinos. Again, to give sufficient mass for the undesired states, the wavefunction overlaps of these states with the TeV brane must be sizable, requiring $c_{\rm Higgs} \geq 1/2$. Note that larger values of $c_{\rm Higgs}$, $c_{\rm Higgs} > 1/2$, have the added benefit of minimizing the influence of the supersymmetry breaking brane on the Higgs bosons as discussed in the previous sub-section. We expect electroweak symmetry breaking to proceed much like in the previous sub-section including the issue of too light Higgsino doublets. To illustrate the effects of the supersymmetry breaking boundary conditions, both the zero and non-zero modes of the $\mathbf{5}$'s and $\bar{\mathbf{5}}$'s of the F,F' matter and Higgs fields are shown in Tables 1 and 2 both before and after the supersymmetry breaking twist.

We finally consider the gauge sector. This sector has a quite similar structure to the Higgs sector. The boundary condition twist at $y=\pi R$, $\alpha'_2 = 1/2$, acts on the $SU(2)_R$ doublets, so the Z' parities for the gauginos change sign. The MSSM gauginos become heavy while the $SU(5)$, $N=2$ partners of the gauge bosons are made light by the supersymmetry breaking twist. However, these undesired fields can gain masses via a mass term on the TeV brane. In the case of the gauge multiplets the bulk mass is required to be $c_{\rm gauge} = 1/2$ by gauge invariance, and thus the zero-mode fermions have conformally flat wavefunctions, insuring that they have sizable wavefunction overlaps with the TeV brane.

Now, we consider the effect of supersymmetry breaking twist, $\alpha'_2 \neq 0$, on the gauge coupling unification. A theory on the truncated AdS_5 space has a different description in terms of a 4D quasi-conformal field theory [29]. In this dual 4D picture, changing the boundary conditions at the TeV brane corresponds to changing the TeV physics, so that it can be viewed as an IR effect. Therefore, the boundary condition breaking twist at the TeV brane is expected not to change the differential running above the TeV scale, and the model is expected to preserve the successful

[6]The extra scalars can also obtain masses through radiative corrections from gaugino masses. In the case where these masses are sufficiently large, we do not necessarily satisfy the conditions $c_{\rm matter} \geq 1/2$ to make these fields heavy, and the scalar mass squareds are one-loop smaller than the supersymmetry breaking scale $\sim t^2$.

prediction for $\sin^2\theta_w$ [15]. This expectation can be confirmed by direct calculation of the beta functions with the twists, $\alpha'_2 = 1/2$ and $P'_{\rm Higgs} = -1$, using the formulae found in Ref. [14]. Notice, however, that there is no energy range in which our theory mimics the MSSM particle content. If the brane mass terms are smaller than the KK mass gap, then the first new particles to be created will include the $SU(5)$, $N=2$ partners of the standard model particles. These include colored Higgsinos and gauginos of the broken $SU(5)$ generators. If the brane mass terms are larger than the KK mass gap, then the first new particles to be created will be the first KK mode which contains many states in addition to those of the MSSM. The model makes the same prediction for $\sin^2\theta_w$ as the MSSM, despite these drastic departures from the MSSM particle content at the TeV scale.

We may also wish to maintain the feature of gauge coupling unification in the context of a theory with one Higgs doublet. In this case, it is easiest to return to the situation before the supersymmetry breaking twist was made. We can then imagine removing the $\{\bar{H}(\bar{\mathbf{5}}), \bar{H}^c(\mathbf{5})\}$, which contained the $\bar{H}(\mathbf{1},\mathbf{2})_{-1/2}$ Higgs doublet as a zero mode. This change in the zero-mode particle content will change the beta function; we therefore also remove the $\{F(\bar{\mathbf{5}}), F^c(\mathbf{5})\}$ that had contained the third generation $F(\bar{\mathbf{3}},\mathbf{1})_{1/3}$. The sum of these two zero-mode fields contribute to the differential running of the gauge couplings the same as a single $\bar{\mathbf{5}}$, and therefore their removal does not affect gauge unification. We will not worry about the missing b_R at this stage; it will reappear after we break the remaining $N=1$ supersymmetry.

Now let us consider the $\mathbf{5}$'s and $\bar{\mathbf{5}}$'s of the third generation matter and Higgs fields after breaking supersymmetry. Notice that the $SU(5), N=2$ partner of the remaining Higgs boson (*i.e.* the ψ^c_{++} component of the $\mathcal{H}$ hypermultiplet) is a fermion with the same quantum numbers as b_R. We may identify this field as the right-handed bottom quark thereby completing the third generation. This identification points out the fact that there no longer exists any distinction between Higgs-like and matter-like boundary conditions. Instead, we have unified matter and Higgs in the context of an $SU(5)$ model: from the 5D point of view the quantum numbers and boundary conditions for the $\mathcal{H}$ hypermultiplet and the two $\mathcal{F}$ hypermultiplets are the same. There are three potential Higgs bosons, each one an $SU(5), N=2$ partner of a right-handed down-type quark. However, since the $\mathcal{H}$ hypermultiplet has the bulk mass parameter such that the zero-mode doublet scalar is localized toward the Planck brane while the zero-mode scalars from the $\mathcal{F}$ hypermultiplets are localized to the TeV brane, it is natural that only two of these scalars receive $O(\mathrm{TeV})$ masses from the $y=\pi R$ brane, leaving one light Higgs doublet. This light Higgs field will develop an electroweak breaking expectation value through radiative corrections, as in the previous models. Therefore, we can naturally obtain the theory with logarithmic gauge coupling unification, which effectively has only a single Higgs boson. Note that the Yukawa couplings for the up-type quarks can be located both on the Planck and TeV branes but those for the down-type quarks (and charged leptons) can be located only on the TeV brane. Thus we naturally understand the origin of the m_t/m_b ratio in this theory through a moderate peaking of the Higgs wavefunction toward the Planck brane.

5 Conclusions

In this paper we have studied the properties of boundaries in higher dimensional theories that do not respect the full symmetries of the bulk. Such defects break symmetries explicitly, but since all the breakings are localized on boundaries, local counterterms in the bulk are restricted in exactly the same manner as they would be in the absence of the defects. In particular, the effects of explicit breaking are suppressed by the volume of the extra dimensions and/or small wavefunction overlaps in the low energy 4D theories. Therefore, symmetry breaking by point defects provides an interesting alternative to spontaneous symmetry breaking, in which we can systematically suppress the size of explicit breaking and can use the symmetry to control radiative corrections.

There are two different classes of symmetry breaking defects. A type I defect possesses the maximum symmetry allowed by the boundary conditions of the fields. Put another way, if the symmetry is made local the boundary conditions of the gauge parameters and gauge fields coincide. The two therefore have the same KK decomposition, and there exists a gauge transformation corresponding to each gauge field in the 4D theory. Such a theory can therefore be consistently gauged. If a theory possesses a defect of type II, then the symmetry may not be gauged. Type II defects arise by requiring the brane Lagrangian to be invariant under a smaller symmetry than that allowed by the boundary conditions. In order to enforce this, one must impose additional constraints on the symmetry transformation parameters at the boundaries. If one were to attempt to gauge such a theory, the additional boundary conditions on the gauge parameters, as compared to the gauge fields, would result in gauge fields without corresponding gauge transformations. This will therefore result in inconsistencies such as states with negative norm. These results apply to any theory in which a symmetry is broken by boundary conditions on extra dimensions.

In particular, these ideas may be applied to supersymmetry breaking on a slice of AdS_5. In this case, the Killing spinor equations result in a non-trivial constraint on the supersymmetry transformation parameters at the boundaries of the space. If only half of the supersymmetry is broken, reducing the four dimensional $N=2$ to $N=1$, then the Killing spinor condition coincides with the conditions required by the boundary conditions of the fields, and the boundaries become symmetry breaking defects of type I. However, if one attempts to break all of the supersymmetries, then the Killing spinor equations do in fact constitute an additional constraint and at least one boundary must be a type II defect. As a consequence, unlike flat space, breaking supersymmetry by boundary conditions on a warped background is not consistent with supergravity.

Despite this difficulty, we argue that a theory with supersymmetry breaking boundary conditions may approximate a theory that is consistent with gravity. We therefore presented two models that make use of this mechanism. First, we constructed a theory of electroweak symmetry breaking with a single Higgs doublet: a warped version of the constrained standard model. Supersymmetry greatly constrains the Higgs potential, while a bulk mass for the Higgs hypermultiplet reduces the sensitivity to the dynamics of the supersymmetry breaking brane. As a result, we have succeeded in constructing a predictive theory leading to a Higgs boson mass that may be close to its experimental lower bound.

In our second model we constructed a theory of gauge coupling unification in which both the grand unified group and supersymmetry are broken by boundary conditions on the same extra dimension. While gauge coupling unification occurs as in the MSSM, the low energy particle content may deviate drastically from that of the MSSM. Depending on the mass parameters of the supersymmetry breaking brane, it is possible that the lowest mass gaugino may be the super partner of the broken gauge bosons. It is also possible that the lightest Higgsinos are colored. We have also shown that it is possible to remove the Higgs hypermultiplets from this model and identify the Higgs boson as one of the $SU(5), N=2$ partners of the right handed down-type quarks and in this way unify the Higgs and matter in the context of an $SU(5)$ grand unified theory.

Acknowledgements

The work of L.H., T.O. and S.O. was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098, and in part by the National Science Foundation under grant PHY-00-98840. The work of S.O. was also supported by a National Science Foundation Graduate Research Fellowship.

References

[1] I. Antoniadis, S. Dimopoulos, A. Pomarol and M. Quiros, Nucl. Phys. B **544**, 503 (1999) [arXiv:hep-ph/9810410].

[2] R. Barbieri, L. J. Hall and Y. Nomura, Phys. Rev. D **63**, 105007 (2001) [arXiv:hep-ph/0011311].

[3] N. Arkani-Hamed, L. J. Hall, Y. Nomura, D. R. Smith and N. Weiner, Nucl. Phys. B **605**, 81 (2001) [arXiv:hep-ph/0102090].

[4] A. Pomarol and M. Quiros, Phys. Lett. B **438**, 255 (1998) [arXiv:hep-ph/9806263];
A. Delgado, A. Pomarol and M. Quiros, Phys. Rev. D **60**, 095008 (1999) [arXiv:hep-ph/9812489].

[5] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **429**, 263 (1998) [arXiv:hep-ph/9803315]; Phys. Rev. D **59**, 086004 (1999) [arXiv:hep-ph/9807344];
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **436**, 257 (1998) [arXiv:hep-ph/9804398].

[6] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [arXiv:hep-ph/9905221]; Phys. Rev. Lett. **83**, 4690 (1999) [arXiv:hep-th/9906064].

[7] R. Barbieri, L. J. Hall and Y. Nomura, Phys. Rev. D **66**, 045025 (2002) [arXiv:hep-ph/0106190].

[8] A. Pomarol, Phys. Rev. Lett. **85**, 4004 (2000) [arXiv:hep-ph/0005293].

[9] L. Randall and M. D. Schwartz, Phys. Rev. Lett. **88**, 081801 (2002) [arXiv:hep-th/0108115]; JHEP **0111**, 003 (2001) [arXiv:hep-th/0108114];

[10] W. D. Goldberger and I. Z. Rothstein, Phys. Rev. Lett. **89**, 131601 (2002) [arXiv:hep-th/0204160]; arXiv:hep-th/0208060.

[11] K. Agashe, A. Delgado and R. Sundrum, Nucl. Phys. B **643**, 172 (2002) [arXiv:hep-ph/0206099].

[12] K. w. Choi, H. D. Kim and Y. W. Kim, JHEP **0211**, 033 (2002) [arXiv:hep-ph/0202257]; arXiv:hep-ph/0207013.

[13] R. Contino, P. Creminelli and E. Trincherini, JHEP **0210**, 029 (2002) [arXiv:hep-th/0208002].

[14] K. w. Choi and I. W. Kim, arXiv:hep-th/0208071.

[15] W. D. Goldberger, Y. Nomura and D. R. Smith, arXiv:hep-ph/0209158.

[16] T. Gherghetta and A. Pomarol, Nucl. Phys. B **586**, 141 (2000) [arXiv:hep-ph/0003129].

[17] T. Gherghetta and A. Pomarol, Nucl. Phys. B **602**, 3 (2001) [arXiv:hep-ph/0012378].

[18] T. Gherghetta and A. Pomarol, Phys. Lett. B **536**, 277 (2002) [arXiv:hep-th/0203120].

[19] L. J. Hall and Y. Nomura, Phys. Rev. D **64**, 055003 (2001) [arXiv:hep-ph/0103125].

[20] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B **516**, 395 (2001) [arXiv:hep-th/0103135].

[21] L. J. Hall, H. Murayama and Y. Nomura, Nucl. Phys. B **645**, 85 (2002) [arXiv:hep-th/0107245].

[22] Y. Kawamura, Prog. Theor. Phys. **105**, 999 (2001) [arXiv:hep-ph/0012125].

[23] L. J. Hall and Y. Nomura, Phys. Rev. D **65**, 125012 (2002) [arXiv:hep-ph/0111068].

[24] Y. Nomura, D. R. Smith and N. Weiner, Nucl. Phys. B **613**, 147 (2001) [arXiv:hep-ph/0104041].

[25] E. Shuster, Nucl. Phys. B **554**, 198 (1999) [arXiv:hep-th/9902129].

[26] R. Altendorfer, J. Bagger and D. Nemeschansky, Phys. Rev. D **63**, 125025 (2001) [arXiv:hep-th/0003117];
A. Falkowski, Z. Lalak and S. Pokorski, Phys. Lett. B **491**, 172 (2000) [arXiv:hep-th/0004093]; Phys. Lett. B **509**, 337 (2001) [arXiv:hep-th/0009167];
E. Bergshoeff, R. Kallosh and A. Van Proeyen, JHEP **0010**, 033 (2000) [arXiv:hep-th/0007044];
M. Zucker, Phys. Rev. D **64**, 024024 (2001) [arXiv:hep-th/0009083];
T. Fujita, T. Kugo and K. Ohashi, Prog. Theor. Phys. **106**, 671 (2001) [arXiv:hep-th/0106051];
J. Bagger and D. V. Belyaev, Phys. Rev. D **67**, 025004 (2003) [arXiv:hep-th/0206024].

[27] T. Gherghetta and A. Pomarol, arXiv:hep-ph/0302001.

[28] D. Marti and A. Pomarol, Phys. Rev. D **66**, 125005 (2002) [arXiv:hep-ph/0205034];
R. Barbieri, G. Marandella and M. Papucci, Phys. Rev. D **66**, 095003 (2002) [arXiv:hep-ph/0205280];
R. Barbieri, L. J. Hall, G. Marandella, Y. Nomura, T. Okui, S. J. Oliver and M. Papucci, arXiv:hep-ph/0208153.

[29] N. Arkani-Hamed, M. Porrati and L. Randall, JHEP **0108**, 017 (2001) [arXiv:hep-th/0012148];
R. Rattazzi and A. Zaffaroni, JHEP **0104**, 021 (2001) [arXiv:hep-th/0012248];
M. Perez-Victoria, JHEP **0105**, 064 (2001) [arXiv:hep-th/0105048].