Fermilab FERMILAB-Conf-03/091-T May 2003

LITHIUM LENSES BASED MUON COOLING CHANNEL

V. Balbekov, FNAL, Batavia, IL 60510, USA
*

Abstract

A linear ionization cooling channel for neutrino factory or muon collider is considered. It includes short Li lenses, matching solenoids, and 201 MHz RF cavities. The basic challenge is a suppression of chromatic effects in a wide energy range typical for muon beams. A special lattice is proposed to reach this, and methodic of an optimization is developed to minimize the chromatic aberrations by suppression of several betatron resonances. The most engineering constraint is a high field of matching solenoids. A channel with less of 10 T field is considered in detail. It is capable to cool transverse emittance of a beam from 2-3 mm to 0.5 mm at the channel length of about 130 m. Because there is no emittance exchange, longitudinal emittance increases in the process from 10 to 20 mm at transmission of about 90%.

INTRODUCTION

Lithium lens is probably the most promising cooling device for a final stage of muon ionization cooling for a $\mu - \mu$ collider [1]. However, design of matching sections between the lenses is a big problem because of a huge energy spread of muon beam and a strong modulation of beta-function. In such conditions, chromaticity is a crucial point for performances of the channel. Two ways to mitigate this effect was discussed. First of them is to create a system with synchrotron phase advance as a multiple of 2π per cell, obtaining about the same average energy of all the particles. As is shown in Ref. [2], high accelerating frequency and gradient are required to reach this (e.g. 805 MHz and 36 MeV/m). However, even at such ultimate parameters, length of the cooling cell and Li lens should be very large: 10-12 m and 1.6-1.8 m correspondingly. At 201 MHz/15 MeV/m accelerating system accepted nowadays as a baseline for muon cooling system [3], the length would be 2-3 times more. Another method was proposed in Ref. [4], and is developed in this paper. The idea is (i) to use as small betatron phase advance per cell as possible to get a maximal energy space between linear stop-bands, and (ii) to suppress several of them by a special design of cooling cell. An additional optimization of the system with more realistic magnetic field is performed here.

LAYOUT AND PARAMETERS

The cooler is a linear periodical system consisting of cells shown in Fig.1. The cell includes Li lens, 3 solenoid coils, and 6 cavities. Parameters of the equipment are presented in Table 1.

* Work supported by the Universities Research Association, Inc., under contract DE-AC02-76CH03000 with the U. S. Department of Energy.

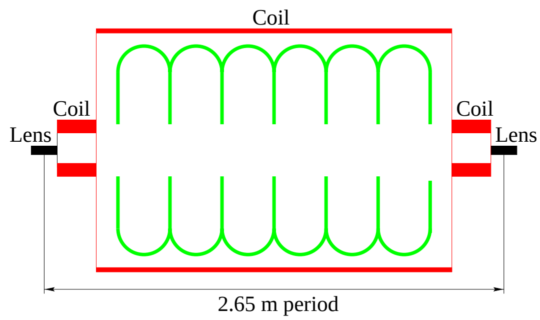

Figure 1: Schematic of a cooling channel with Li lenses, matching, and acceleration sections.

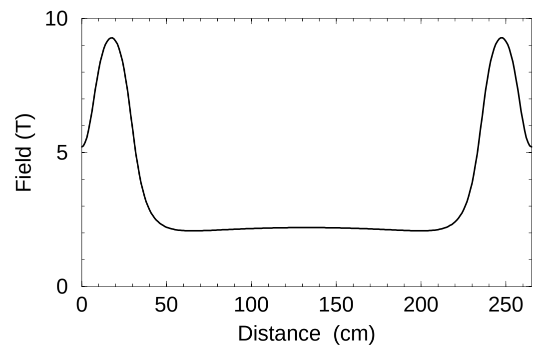

Figure 2: Axial magnetic field of the period .

Axial magnetic field of the cell is plotted in Fig.2. It penetrates into Li lens considerably increasing its focusing

Table 1: Parameters of the Cell

Period length	265 cm
Li lens length	13.46 cm
Li lens radius	3 cm
Li lens gradient	2.624 T/cm
Length of any short coil	22.45 cm
Inner radius of short coil	6 cm
Outer radius of short coil	14 cm
Current density	99.53 A/mm^2
Length of long coil	206.64 cm
Inner radius of long coil	69 cm
Outer radius of long coil	71 cm
Current density	96.09 A/mm^2
Radio frequency	201.25 MHz
Reference energy (total)	250 MeV
Accelerating gradient	12 MeV/m
Synchronous phase	30°

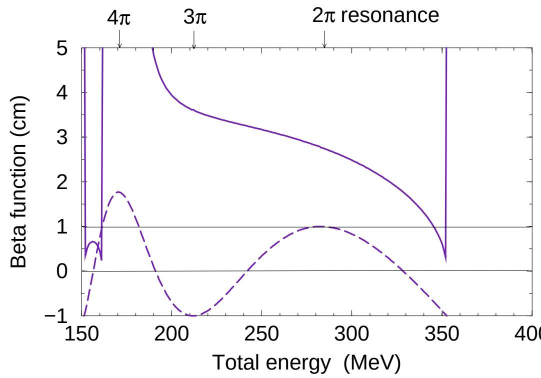

Figure 3: Solid line: beta-function in the center of Li lens *vs* total energy of muon, dashed - 1/2 of spur of transfer matrix ($\cos\mu$).

power. Proper transverse field of the lens is taken in hard edge approximation i.e. it drops sharply at the ends.

The cell is designed to maximize width of resonance-free energy interval by a suppression of linear resonances with phase advances 2π and 3π per period. Method of the tuning is thoroughly described in Ref. [4], and the result is presented in Fig.3, where beta-function in the center of Li lens is plotted *vs* total energy of muon. It is seen that the resonance-free region is significantly broader of 100 MeV what is certainly sufficient to accept and cool muon beam.

The parameters are chosen to get the stable region centered approximately at 250 MeV of total energy. However,

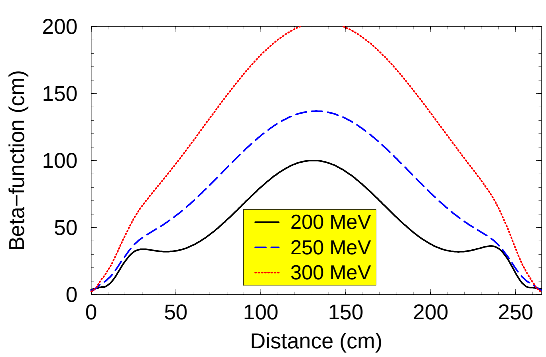

Figure 4: Beta-function of the period *vs* distance.

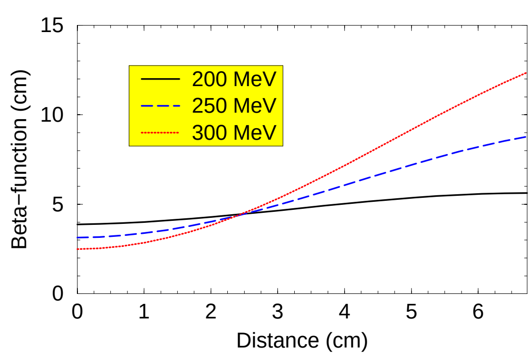

Figure 5: Beta-function in Li lens *vs* distance.

any other choice is also possible by scaling of the field proportionally to central momentum at the same geometry.

Dependence of beta-function of the cell on longitudinal coordinate is plotted in Fig.4 where total energy of muon is taken as a parameter. Maximum of beta-function does not exceed 2 m at $200 < E < 300$ MeV, and 1.3 m for the center of this interval. We are going to consider the cooling of a beam with initial unnormalized transverse r.m.s. emittance about 1000 mm-mrad. It is enough to have windows of radius about 10 cm in the high radius solenoids to accept this beam. Beta-function in Li is plotted in Fig.5 (only right hand part of this symmetric plot is shown). It does not exceed 12.3 cm what corresponds r.m.s. beam size about 11 mm, so that the lens radius 30 mm is sufficient. Average value of beta-function is 5 cm approximately, corresponding an equilibrium normalized beam emittance about 0.4 mm.

COOLING SIMULATION

Muon bunch with Gaussian distribution and initial parameters listed in Table 2 is taken for the simulation. A beam with such emittance would be provided e.g. by a ring cooler described in Ref. [5], We take an injection point at the center of Li lens where ratio of transverse size to momentum of the beam should correspond to beta-function of about 3 cm (see Fig.5). [1]

Table 2: Parameters of Injected Beam

Normalized transverse r.m.s. emittance	2.0 mm
Transverse r.m.s size in any direction	5.2 mm
Transverse r.m.s. momentum	40 MeV/c
Normalized longitudinal r.m.s. emittance	9.3 mm
R.m.s. bunch length (ct)	70 mm
R.m.s. energy spread	14 MeV
Normalized 6D emittance	37 mm^3

To take into account a dependence of longitudinal velocity on transverse momentum, the following correlation was applied after random generation:

$$E = E_{ref}\left[1 + \left(\frac{cp_t}{316\ \text{MeV}}\right)^2 + \left(\frac{r}{14\ \text{cm}}\right)^2\right]_{rn} + \Delta E_{rn}$$

Numerical coefficients are determined empirically to get maximal transmission of the channel. Note that this correlation is excluded by the inverse transformation at the calculation of emittance of the cooled beam. The only aperture restriction is applied by Li lens of radius 3 cm.

Evolution of the beam emittance and transmission is shown in Fig.6 Enough effective transverse and 6D cooling takes place approximately on 50 cooling cells that is 132.5 m long. After that, the beam obtains the parameters

[1] We take a periodical channel and do not consider a problem of matching of the beam in the beginning.

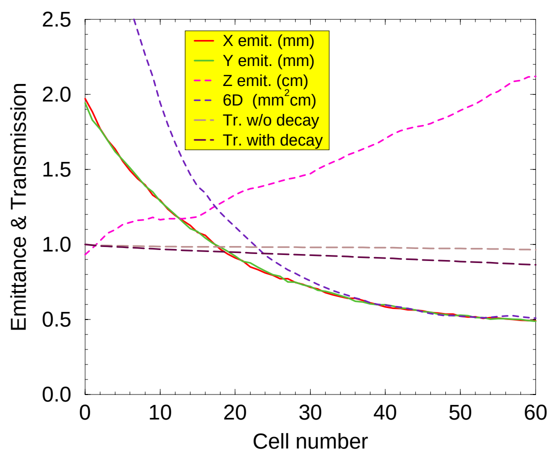

Figure 6: Evolution of the bean emittance and transmission (Gaussian distribution, 10000 muons).

listed in Table 3 where cooling factors are also given in a brackets.

Table 3: Beam Parameters after 50 Cooling Cells

Transverse r.m.s. emittance	0.52 mm	(3.8)
Transverse r.m.s size	2.7 mm	(1.9)
Transverse r.m.s. momentum	20 MeV/c	(2.0)
Longitudinal r.m.s. emittance	19 mm	(.49)
R.m.s. bunch length (ct)	100 mm	(.70)
R.m.s. energy spread	18 MeV	(.78)
Normalized 6D emittance	5.2 mm^3	(7.1)
Transmission with decay	0.89	
Transmission without decay	0.97	

Equilibrium transverse emittance of about 0.4 mm is really not achievable, first of all because of longitudinal heating resulting both from straggling and negative derivative of energy loss dE/dx in the used energy interval. The effect is readily illustrated by Fig.7 where the longitudinal phase space of the beam is shown before the cooling and after 60 cooling cells. There is a considerable lengthening of

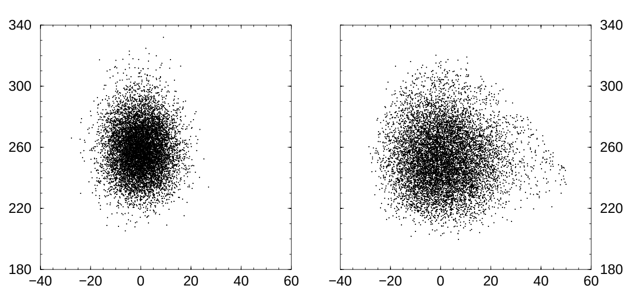

Figure 7: Longitudinal phase space of the beam at injection (left) and after 60 cooling cells (right). Horizontal axis – *ct* (cm), vertical – E (MeV).

the bunch resulting a violation of initial Gaussian distribution, and diffusion of some particles through the separatrix. An attempt to continue the cooling results an acceleration of the diffusion and fast growth of the particle loss.

CONCLUSION AND DISCUSSION

It is shown that a short periodical cooling channel with Li lens and specially tuned solenoids has enough large energy acceptance to be used for transverse cooling of muon beams. A channel of length 132.5 m provides a decrease of transverse emittance from 2 mm to 0.5 mm at transmission of about 89%. Emittance exchange should be introduced to overcome longitudinal heating and to continue the cooling without an additional particle loss. Another obstacle is a high field of matching (small radius) solenoid which reaches 9.3 T in this design. It is more then own (transverse) field on the surface of Li lens which is only 7.9 T. Solenoid field penetrating to the lens does not exceed 6.8 T, that is maximal total field is 10.4 T. A using for the matching of another Li lens with less gradient and more aperture is really impossible because of multiple Coulomb scattering. As is seen from Figs. 2 and 4, both length and beta-function of such lens should exceed corresponding parameters of the main lens by factor about 3. Therefore its contribution in the scattering and equilibrium emittance would be order of magnitude more that is inadmissible.

Distinguishing feature of the channel is a great quantity (~ 50) of very short (~ 13 cm) Li lenses. It results from an aspiration to get as large energy acceptance as possible. However, about 3 times longer lens (~ 40 cm) and the lattice period (6-7 m) could be used if desirable energy acceptance would as small as 40-50 MeV. It was shown recently that muon beam with similar energy spread can be obtained by means of so called RFOFO ring cooler [6].

REFERENCES

[1] Charles M. Ankenbrandt et al. ”Status of Muon Collider Research and Development and Future Plans”, Phys.Rev.ST Accel.Beams 2, 081001 (1999).

[2] V.Balbekov. ”Ionization Cooling of Muon Beam by Multistage System of Li Lenses with Matching Sections”, In Proc. of the 1999 PAC, V.5, p.3146.

[3] ”Feasibility Study-II of a Muon-Based Neutrino Source”, Editors S. Ozaki, R. Palmer, M. Zisman, and J. Gallardo, BNL-52623, June 2001; http://www.cap.bnl.gov/mumu/studyii/FS2-report.html.

[4] V.Balbekov. ”Achromatic Cooling Channel with Li Lenses”, FNAL MCNote 245, April 2002.

[5] V.Balbekov, S.Geer, et al. ”Muon Ring Cooler for the MUCOOL Experiment”, In Proc. of the 2001 PAC, V.5, p.3867.

[6] V.Balbekov. ”Simulation of RFOFO Ring Cooler with Tilted Solenoids”, FNAL MCNote 245, November 2002. R.C.Fernow, J.S.Berg, J.C.Gallardo, R.B.Palmer. ”Muon Cooling in the RFOFO Ring”, FNAL MCNote 273, April 2003.