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Simulations of Octupole Compensation of head-tail instability at the Tevatron

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Abstract

The proton lifetime in the Tevatron depends sensitively on chromaticities. Too low chromaticities can make the beam unstable due to the weak head-tail instability. One way to compensate this effect is to introduce octupoles to create a larger amplitude dependent betatron tune spread. However, the use of octupoles will also introduce additional side effects such as second order chromaticity, differential tune shifts and chromaticities on both proton and anti-proton helices. The non-linear effects may also reduce the dynamic aperture. There are 67 octupoles in 4 different circuits in the Tevatron which may be used for this purpose. We report on a simulation study to find the best combinations of polarities and strengths of the octupoles.

1 INTRODUCTION

In the Tevatron at injection energy, transverse coherent instability is identified as the weak head-tail phenomenon driven by the short-range wake-fields. The chromaticity has to be set at 8 units at present machine conditions(proton bunch intensity is about 2.2e11) without dampers. On the other hand, the beam lifetime at injection energy depends sensitively on chromaticity. In principle, lower the chromaticity would help to decrease the growth of higher order mode of instability, and improve the DA(Dynamic Aperture, consequently the beam lifetime. However, too low chromaticities can make the beam unstable [1]. Currently, the Tevatron is running with tranverse and logitudinal dampers. Better beam lifetime was observed when the chromaticity decreased from 8 units to 2 units. Another way to compensate the instability of the protons is to introduce octupoles to create larger amplitude dependent betatron tune spread. If the shifted coherent tunes due to wake fields is within the width of the incoherent betatron tune spread or spread in synchrotron tunes, all unstable higher order modes can be damped by landau damping.

There are 67 octupoles in 4 different circuits in the Tevatron. Table 1 lists the number and the average beta function and the dispersion of two octupole families we used for the simulations. The use of octupoles will also introduce additional side effects on both proton and pbar helices. These effects are second order chromaticity, addtional coupling and non-linearty. Octupoles will also produce differential tune shift and linear chromaticities on both proton and pbar helices.The purpose of this simulation is to find best combinations of polarities and strengths of the octupoles.

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Octupoles	Number	$<\beta_x>$, $<\beta_y>$[m]	$<D_x>$, $<D_y>$[m]
TOZD	24	93.6,30.2	3.67,-0.02
TOZF	12	30.5,92.5	2.07,-0.01

Table 1: Octupoles in the Tevatron used in this simulation

2 THE AMPLITUDE-DEPENDENT TUNE SHIFT DUE TO OCTUPOLES

The field due to an octupole in terms of the multipole components (b_3, a_3) is

$$B_y + iB_x = \frac{B_o(b_3 + ia_3)}{R^3_{ref}}[x^3 - 3xy^2 + i(3x^2y - y^3)] \quad (1)$$

The change in the Hamiltonian due to the octupole field is

$$\Delta H = \frac{R}{4!}k_3(\theta)[x^4 - 6x^2y^2 + y^4] \quad (2)$$

independent variable is θ, R is the machine radius, and the octupole strength parameter k_3 is defined as

$$k_3 = K_nL = \frac{n!}{(R^n_{ref}\rho)}b_nL = \frac{1}{(B\rho)}\frac{\partial^n B_y}{\partial x^n}.L \quad (3)$$

Introducing the action angle coordinates

$$x = \sqrt{2\beta_x J_x} cos\phi_x \quad (4)$$
$$y = \sqrt{2\beta_y J_y} cos\phi_y \quad (5)$$

and assume the phase and the beta functions are nearly constant over the length of a single octupole, we can found the action-dependent tune shift is as follows [2]

$$\Delta\nu_x = a_1 J_x + a_2 J_y \quad (6)$$
$$\Delta\nu_y = a_2 J_x + a_3 J_y \quad (7)$$
$$a_1 = \frac{1}{16\pi}\sum_n k_3(n)\beta^2_{x,n} \quad (8)$$
$$a_2 = -\frac{1}{8\pi}\sum_n k_3(n)\beta_{x,n}\beta_{y,n} \quad (9)$$
$$a_3 = \frac{1}{16\pi}\sum_n k_3(n)\beta^2_{y,n} \quad (10)$$

$J_{x,y}$ is the action,and related to the amplitude by

$$J_{x,y} = \frac{(a_{x,y})^2}{2}\cdot\epsilon_{nx,y} \quad (11)$$

here $a_{x,y}$ is the amplitidue im units of beam size σ, $\epsilon_{nx,y}$ is the normalized emittance (1σ)in x or y plane.

Let $k_3(F)$ be TOZF's strength, $k_3(D)$ be TOZD's strength. At Tevatron injection, the action-dependent tune-shifts due to octupoles are obtained as follows:

$$\begin{aligned}\Delta\nu_x &= 2091.5251([k_3(F) + 0.2124k_3(D)]J_x \\ &\quad -0.6444[k_3(F) + 1.9961k_3(D)]J_y) \quad (12)\\ \Delta\nu_y &= 1349.6594(-[k_3(F) + 1.9961k_3(D)]J_x \\ &\quad +3.0269[0.0533k_3(F) + k_3(D)]J_y) \quad (13)\end{aligned}$$

If we consider the particles along the diagonal in x-y physical space, then $J_x = J_y$. At present machine conditions, the proton emittance is 25πmm- mrad(95%). For the particle with initial ampplidute of 5σ, we can get the expression for the octupole's strengths which are needed to compensate the decrease of tune spread due to lowering the chromaticities as follows:

$$\begin{aligned}k_3(F) &= -4.5726E3[0.6194 \times \Delta\nu_x + \Delta\nu_y] \quad (14)\\ k_3(D) &= -2.3044E3[\Delta\nu_x + 0.6570 \times \Delta\nu_y] \quad (15)\end{aligned}$$

3 QUANTITIVE CALCULATION FOR THE OCTUPOLE STRENGTH

The coherent stability condition [1]

$$\Delta\nu_s = \frac{\Delta\Omega_s}{\omega_o} \quad (16)$$

$$\Delta\Omega_s > \Lambda[l_s] \quad (17)$$

where $\Lambda[l_s]$ is the growth rate of the head-tail modes,$\omega_o = 2\pi f_0$, f_0 is the RF frequency.

The growth rate $\Lambda^{[l_o]}$(χ=-1.5,$\chi = \xi \cdot l_s\omega_o/\eta$) measured at 150 GeV is about 110/sec, and the coherent tune shift of mode 0 is about $-1. \times 10^{-3}$ [3] .On the other hand, the synchrotron tune spread at 150 GeV is calculated as follows:

$$\Delta\nu_s \approx \nu_{s0} \cdot \frac{1}{16}\left(\langle\phi\rangle\right)^2 \approx 2.2 \times 10^{-4} \quad (18)$$

$\langle\phi\rangle \approx 1.4rad, \nu_{s0} = 1.8 \times 10^{-3}$ at injection. The incoherent linear tune shift due to the space charge for the particle near the center of the proton bunch with 3-D Gaussian density distribution is

$$\begin{aligned}(\Delta\nu_x)_{SC} &= -0.36 \times 10^{-3} \quad (19)\\ (\Delta\nu_y)_{SC} &= -0.70 \times 10^{-3} \quad (20)\end{aligned}$$

Therefore, in order for the landau damping to work, the betatron tune spread should be

$$\begin{aligned}(\Delta\nu_x)_\beta &= [\Delta\nu_{coh} - (\Delta\nu_x)_{SC} - \Delta\nu_s]\\ &= -0.42e-3 \quad (21)\\ (\Delta\nu_y)_\beta &= [\Delta\nu_{coh} - (\Delta\nu_y)_{SC} - \Delta\nu_s]\\ &= -0.08e-3 \quad (22)\end{aligned}$$

Then, the octupole strengths calculated from Eq. 15 and Eq. 15 for TOZF and TOZD are

$$\begin{aligned}k3(F) &= 1.55 \quad (23)\\ k3(D) &= 1.09 \quad (24)\end{aligned}$$

At tevatron injection,

$$k3 = K_3L = 1.2267506/m^3/Amps \times I \quad (25)$$

I is the magnet currents of octupoles.

4 DYNAMIC APERTURE TRACKING WITH AND WITHOUT OCTUPOLES

Before the octupoles were introduced into the lattice, we calculated the dynamic aperture at different chromaticities. Fig. 1 shows DAs of protons and pbars in function of momentum deviation at different chromaticities. The particles were tracked 100,000 turns by code MAD. We can see that the DAs of pbars at chromaticity of 2 units are 0.5σ larger than those of 8 units, although the DAs of protons seems not much improved. Chaotic border is defined as

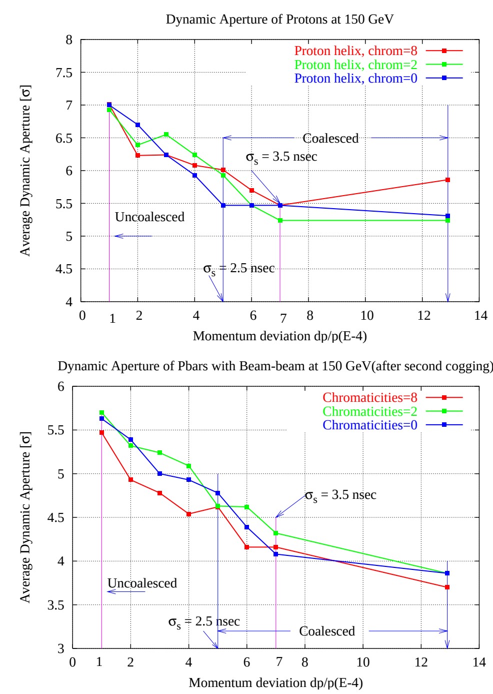

Figure 1: DA of protons and anti-protons vs. momentun deviations at different chromaticities

the border between regular and chaotic motion, it allows to predict the long term Dynamic Aperture. Table 2 lists DAs and chaotic borders at different chromaticities, calculated using code SIXTRACK. The momentum deviation in this calculation is dp/p=4.3e-4. Small chromaticities help to improve long term dynamic aperture since chromaticity sextupole strength are reduced, and synchro-betatron resonances are less.

In the machine, when we reduce the chromaticities from 8 units to 2 units, pbar lifetime was observed significately increased(from 2hrs to 12hrs), while proton lifetime of protons increased not so much.

A pair of tentative values of two octupole familaies, $k_3(F)$=2, $k_3(D)$=1, which are very closed to the estimated

chromaticities	(ξ_x=2, ξ_y=2)				(ξ_x=2,ξ_y=2)	(ξ_x=8,ξ_y=8)
octupole strength (k_3(F),k_3(D))	(+2,+1)	(+2,-1)	(-2,+1)	(-2,-1)	(0,0)	(0,0)
$a_1 = \frac{\partial\nu_x}{\partial(a_x^2)}$	-0.0001538	-0.0000815	- 0.0000429	0.0001060	-0.0002321	-0.0000429
$a_3 = \frac{\partial\nu_y}{\partial(a_y^2)}$	0.00006336	-0.0000504	- 0.0000405	-0.0001491	0.0000282	-0.0000529
$a_2 = \frac{\partial\nu_x}{\partial(a_y^2)}$ or $\frac{\partial\nu_y}{\partial(a_x^2)}$	0.0000500	-0.00000034	-0.0000529	-0.0001236	0.0000669	-0.0000405
Average DA	7.0	7.1	6.6	6.2	6.8	6.4
minimum DA	6.2	6.3	5.9	5.4	6.3	6.0
Chaotic border	5.8	5.5	4.5	4.4	5.2	4.7

Table 3: Dynamic Aperture and chotic boder with/without octupoles

Table 2: Dynamic aperture and chaotic border of protons(p) and anti-protons ($\overline{p}$) at injection

	Chromaticities	(8,8)	(2,8)	(2,2)	(0,0)
p	Chaotic border(σ)	4.8	5.0	5.2	5.4
	DA (σ)	6.4	6.9	6.8	6.9
$\overline{p}$	Chaotic border(σ)	2.9	4.5	4.6	4.7
	DA (σ)	5.1	5.4	5.4	5.3

values, and their combinations of different polarities, have been used to see the effects of the compensation on the Dynamic aperture. The results are listed in Table 3. From Table 3 we can see that if TOZF is positive, the dynamic aperture and chaotic border are larger than the other combinatons. We also analyzied the coefficents a_1,a_2,a_3 in Eq. 7 and Eq. 8 for each combination. We expect that a_1 and a_3 must be much larger than a_2, so that the tune spread will be less dependent on coupling term. On the other hand, the a_1 and a_3 must get larger with octupoles. It was found that both a_1 and a_3 are increasing but a_2 is decreasing if TOZD is also positive, it means that the tune spread will be increased more for the larger amplitude particles, while the coupling effects will be decreased more.

With $k_3(F)$=2, $k_3(D)$=1, and chromaticity set at (ξ_x=ξ_y=2), we calculated dynamic aperture and chaotic border of the anti-protons. It was found that the average DA is 5.6σ, the chaotic boeder is 5.1σ.

The study of octupoles in the machine has been done by P. Ivanov in Tevatron department. So far, the attempt was successful to inject beam (280-300e9/bunch) with small(zero) chromaticity with octupoles TOZF=+2A, TOZD=+5A($k_3(F)$=1.6, $k_3(D)$=4.1). Nevertheless, beam loss occured while going from injection bump to open helix - probably because of significant tune changes and strong coupling due to orbit motion. Further study in the machine is in progress.

5 CONCLUSION

The optimized combination is found by Dynamic Aperture tracking that both octupole families need to be posotive.

Stronger octupole streangths have been tried and it's found that they are not better for the compensation. Besides, the tune shifts feeddown from octupoles on the helices are larger, and it gets difficult to make feeddown sextupoles to work. It suggested that the octupole strengths used need to be as weak as possible if the compesation is strong enough.

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7 REFERENCES

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