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Determining the phases α and γ from time-dependent CP violation in B^0 decays to $\rho(\omega) +$ pseudoscalar

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Abstract:

A method is proposed for the determination of the unitarity angle α through tree penguin interference. The modes needed would be of the form $B^0/\bar{B}^0 \rightarrow \rho^0 \mathcal{M}$ and $B^0/\bar{B}^0 \rightarrow \omega \mathcal{M}$ where \mathcal{M} is spin-0 $u\bar{u}/d\bar{d}$ meson, for instance $\mathcal{M} = \pi^0, \eta, \eta', a_0$ or f_0 . An analogous method can also determine γ using $\mathcal{M} = K_S$ or K_L . The validity of the theoretical approximations used may be tested by over determining α with several modes. If two or more modes are used, the determination has a four-fold ambiguity but additional information from pure penguin decays or theoretical estimates may be used to reduce the ambiguity to $\alpha, \alpha + \pi$. The method as applied to determining γ is probably less promising.

The early indications of CP violation [1, 2, 3] in the neutral B system is an important development in our understanding of this phenomenon. It is expected that in the near future the angle β of the unitarity triangle will be determined with considerable accuracy. This is a crucial first step in the program of verifying that the Cabibbo-Kobayashi-Maskawa (CKM) matrix [4] of the Standard Model is the origin of CP-violation in the B and K systems. To complete this task, the more difficult angles α and γ also need to be determined. Information about these angles, when combined with the information concerning the sides of the CKM triangle [5], will either provide

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an impressive verification for the CKM paradigm or convincing evidence for the presence for new physics.

In this Letter we propose a method to extract α through the interference of the $b \rightarrow u\bar{u}d$ tree with the $b \rightarrow d$ penguin; an analogous method may also be used for γ through the interference of the $b \rightarrow u\bar{u}s$ tree with the $b \rightarrow s$ penguin. In particular, we will require time dependent observations of $B^0/\bar{B}^0 \rightarrow \rho\mathcal{M}$ and $B^0/\bar{B}^0 \rightarrow \omega\mathcal{M}$ where \mathcal{M} is any self-conjugate spin-0 state with a suitable quark content. For instance to obtain α we can use $\mathcal{M} = \pi^0, \eta, \eta', a_0$ or f_0 as well as related excited states, while to obtain γ we may use $\mathcal{M} = K_S, K_L$ or related spin 0 kaonic resonances. Comparison of $\rho\mathcal{M}$ and $\omega\mathcal{M}$ data for each (pseudo)-scalar \mathcal{M} gives up to a 4-fold degeneracy in the determination of $\sin 2\alpha$ (or $\sin 2\gamma$) and when information from two or more modes is combined, a single value of this quantity should emerge. The remaining four fold degeneracy in α (γ) can then be reduced to two-fold by either using theoretical information concerning the tree or penguin amplitudes or by measuring a pure penguin mode related by $SU(3)$.

There have been several methods proposed to extract these angles. For α one can consider oscillation effects in $B^0 \rightarrow \pi^+\pi^-$ although one must account for the penguin through isospin analysis [6] by observing $B^0 \rightarrow \pi^0\pi^0$. Since the branching ratio to $\pi^0\pi^0$ is expected to be small and hard to observe, it may be preferable to consider three π final states. In [7], α is determined through isospin analysis of $B^\pm \rightarrow \pi^\pm\pi^+\pi^-$, $B^\pm \rightarrow \pi^\pm\pi^0\pi^0$ and $B^0/\bar{B}^0 \rightarrow \pi^+\pi^-\pi^0$. In this approach one can take advantage of resonance effects in the Dalitz plot; however, there may be problems in precise modeling of the resonance structure. Another method for extracting α from the interference of u -penguins with t -penguins in $B^0 \rightarrow K^{(*)}K^{(*)}$ may overcome the disadvantages of the 2π and 3π final states [8] although the analogous B_s decays are required for the analysis. The angle γ may be extracted via direct CP violation through the interference of $b \rightarrow u\bar{c}s$ and $b \rightarrow c\bar{u}s$ [9] and via time-dependent studies of CP violation in the B_s system [10].

In the method discussed here, we rely on the following two approximations:

- (1) The contribution of the electro-weak penguin (EWP) is small.
- (2) The $q\bar{q}$ pair which arises in a strong penguin does not form the ω in the final state.

Recall that the EWP are also assumed to be small for some of the other proposed methods [6, 7] mentioned in the preceding paragraph for extracting α since they rely on isospin or SU(3). Our second assumption is true at lowest order in perturbation theory by color conservation and, as discussed in [11], using renormalization group improved perturbation theory one can show that it is valid to a few percent in general.

Let us first consider the case of determining α through the interference of the $b \rightarrow d$ penguin with the $b \rightarrow u\bar{u}d$ tree. For a given final state of this form f let us define A to be the amplitude for $B^0 \rightarrow f$ and \bar{A} the amplitude for $\bar{B}^0 \rightarrow f$. The SM amplitude for $B^0 \rightarrow f$ can be written as:

$$A_f = T(f)v_u + P_u(f)v_u + P_c(f)v_c + P_t(f)v_t \equiv \hat{T}_f v_u + \hat{P}_f v_t \quad (1)$$

where $T(f)$ is the tree contribution to the amplitude, $P_i(f)$ is the penguin contribution due to the diagram with an internal quark of type u_i and $v_i = V_{ib}^* V_{id}$. Unitarity of the CKM matrix allows us to express the amplitudes in terms of $\hat{T}_f = T(f) + P_u(f) - P_c(f)$ and $\hat{P}_f = P_t(f) - P_c(f)$.

If we denote $t_f = |\hat{T}_f v_u|$ and $p_f = |\hat{P}_f v_t|$ then, using the phase convention of [12], we can write:

$$\begin{aligned} A_f &= t_f e^{+i\gamma} + p_f e^{-i\beta} \\ \bar{A}_f &= t_f e^{-i\gamma} + p_f e^{+i\beta} \end{aligned} \quad (2)$$

In the limit that $\Delta\Gamma/\Gamma$ is small, which is a good approximation for the B^0 meson, the time dependent decay rate is:

$$\frac{1}{\Gamma_{B^0}} \frac{d\Gamma(f)}{d\tau} = \frac{1}{2} e^{-|\tau|} \left(X_f + b Y_f \cos x_b \tau - b Z_f^I \sin x_b \tau \right) \quad (3)$$

where $b = +1$ (-1) for B^0 (\bar{B}^0)-mesons, $\tau = \Gamma_B t$ and the coefficients X_f , Y_f and Z_f^I are related to the amplitudes in eq. (2) by:

$$\begin{aligned} X_f &= (|A_f|^2 + |\bar{A}_f|^2)/2 \\ Y_f &= (|A_f|^2 - |\bar{A}_f|^2)/2 \\ Z_f^I &= \text{Im}(e^{-2i\beta} A_f^* \bar{A}_f) \end{aligned} \quad (4)$$

From these, we can also determine, up to a two fold ambiguity, the quantity:

$$Z_f^R = \text{Re}(e^{-2i\beta} A_f^* \bar{A}_f) \quad (5)$$

since $(Z_f^I)^2 + (Z_f^R)^2 = X_f^2 - Y_f^2$. We can expand X and Z in terms of the tree and penguin amplitudes as:

$$\begin{aligned} X_f &= |p_f|^2 + |t_f|^2 - 2r_f \cos \alpha \\ Z_f^R &= |p_f|^2 + |t_f|^2 \cos 2\alpha - 2r_f \cos \alpha \\ Z_f^I &= |t_f|^2 \sin 2\alpha - 2r_f \sin \alpha \end{aligned} \quad (6)$$

where $r_f = \text{Re}(t_f p_f^*)$.

If we eliminate $|t_f|^2$ and r_f from eq. (6) and denote $Z_f = Z_f^R + iZ_f^I = e^{-2i\beta} A_f^* \bar{A}_f$, we obtain:

$$X_f - Z_f^R \cos 2\alpha - Z_f^I \sin 2\alpha \equiv X_f - \text{Re}(Z_f e^{-2i\alpha}) = (1 - \cos 2\alpha) |p_f|^2 \quad (7)$$

Likewise, if we eliminate $|p_f|^2$ and r_f we obtain:

$$X_f - Z_f^R = (1 - \cos 2\alpha) |t_f|^2 \quad (8)$$

Our primary approach to obtaining α is to use eqn. (7) to relate two modes by matching $B^0 \rightarrow \rho^0 \mathcal{M}$ and $B^0 \rightarrow \omega \mathcal{M}$, $\mathcal{M} =$ any spin-0 $u\bar{u}/d\bar{d}$ meson. The fact that the magnitude of the penguin is the same for these two modes in principle provides enough equations to cleanly determine α . This, however, will normally result in discrete ambiguities, so it is important to supplement this method in such a way as to over determine α and thus reduce these ambiguities. This may be accomplished in a number of ways:

- (a) Use the same procedure with a number of different candidates for \mathcal{M} , indeed $\mathcal{M} = \{\pi^0, \eta, \eta', a_0, f_0\}$ and higher excited states may all be viable. In addition a meson with non-zero spin may be used for \mathcal{M} if one analyzes the angular distributions of the final state to determine the time dependent magnitude of the specific helicity amplitudes. Of particular interest in this category are $f = \rho^0 \rho^0, \rho^0 \omega$ and $\omega \omega$ all three of which share a common $|p_f|^2$.

- (b) Use SU(3) to determine the magnitude of the penguin amplitude from a pure penguin process. As we shall see, even if there are considerable theoretical errors in the application of SU(3), this additional information will often resolve the ambiguity between α and $\pi - \alpha$ which cannot be resolved purely from the above $\rho - \omega$ matching.
- (c) Other theoretical bounds such as the ratio between the tree and the penguin, again even with the presence of appreciable uncertainty, may be used to constrain the results providing a means to distinguish between the different ambiguous solutions.
- (d) Finally, this method may be used in conjunction with other methods for obtaining α . In the Standard Model all such methods should agree on a single value for α and so all independent methods to determine α should be used in parallel.

We now illustrate our approach to extracting α from a comparison of $\rho^0\mathcal{M}$ with $\omega\mathcal{M}$ for the specific case when $\mathcal{M} = \pi^0$.

It follows from approximations (1) and (2) then that the penguin contributes to these final states only via the $d\bar{d}d\bar{d}$ channel, therefore $p_{\rho^0\pi^0} = -p_{\omega\pi^0}$ and thus from eqn. (7):

$$X_{\rho^0\pi^0} - X_{\omega\pi^0} = \text{Re}(e^{-2i\alpha}(Z_{\rho^0\pi^0} - Z_{\omega\pi^0})) \quad (9)$$

If we then define

$$\xi = X_{\rho^0\pi^0} - X_{\omega\pi^0}; \quad \zeta e^{i\theta} = Z_{\rho^0\pi^0} - Z_{\omega\pi^0} \quad (10)$$

we obtain

$$\alpha = (\theta \pm \cos^{-1} \frac{\xi}{\zeta})/2 + \begin{cases} 0 \\ \text{or} \\ \pi \end{cases} \quad (11)$$

In general, this will give a 16-fold ambiguity in α caused by the four fold ambiguity in eqn. (11) together with the sign ambiguities in $Z_{\rho^0\pi^0}^R$ and $Z_{\omega\pi^0}^R$.

However, if $|\xi/\zeta| > 1$ for some choices of the signs for Z_f^R , there will be no corresponding solutions for α and in such cases the degeneracy will be smaller. In addition, notice that if one takes the incorrect sign choice for both $Z_{\rho^0\pi^0}^R$ and $Z_{\omega\pi^0}^R$ and also takes the incorrect sign choice for $\cos^{-1}(\xi/\zeta)$, one will obtain $\pi/2 - \alpha$. There will therefore always be a four fold ambiguity in this method with $\alpha, \alpha + \pi, \pi/2 - \alpha$ and $3\pi/2 - \alpha$ being possible solutions. Note that all these solutions have the same value of $\sin 2\alpha$ so, in effect, this method determines the value of $\sin 2\alpha$ with up to a 4-fold ambiguity.

The $\rho - \omega$ comparison is clean in that no theoretical assumptions are required beyond (1) and (2) so the determination of $\sin 2\alpha$ is limited only by statistics. Furthermore it is self-contained in that only information from the B^0 is needed, therefore it may be used at asymmetric B-factories or at hadronic B facilities although the ability to detect π^0 is required in the modes we consider.

As suggested above, the first step to resolve the ambiguity is to apply eqn. (11) to as many final states as possible. By combining the results from several values for \mathcal{M} , one should be able to extract a unique value for $\sin 2\alpha$ (hence a four-fold ambiguity for α).

One way to further reduce this ambiguity to 2-fold is to use SU(3) to estimate the magnitude of p_f from a related pure penguin $b \rightarrow s$ transition. In the case of the $\rho^0\pi^0/\omega\pi^0$ final states, the appropriate pure penguin decay is $B_s \rightarrow K_S K_S^*$ or $K_L K_L^*$ where $K_{S,L}^*$ means a neutral K^* meson which decays to a final state $\pi^0 K_{S,L}$. Note that in this mode the amplitude for the spectator to form the pseudo-scalar and the amplitude for the spectator to form the vector states are combined in the same way as they are in the $B^0 \rightarrow \rho^0\pi^0$ case. This decay is a pure penguin assuming that the rescattering contribution of tree diagrams is negligible and so its decay rate allows us to estimate $|p_f|$. Generally the penguin amplitude obtained by $\rho - \omega$ comparison will be different for α and $\pi/2 - \alpha$.

Bearing in mind the SU(3) assumption used to determine $|p_f|$ from $B_s \rightarrow K_S K_S^*$ or $K_L K_L^*$, this pure-penguin method could also be used as a ‘‘stand-alone’’ method for determining α by rearranging eqn. (7) into:

$$X_f - |p_f|^2 = \text{Re}((Z_f - |p_f|^2)e^{-2i\alpha}) \Rightarrow \alpha = (\mu_f \pm \cos^{-1} \frac{X_f - |p_f|^2}{R_f})/2 + \begin{cases} 0 \\ \text{or} \\ \pi \end{cases} \quad (12)$$

where R_f and μ_f are defined by $Z_f - |p_f|^2 = R_f e^{i\mu_f}$. This gives up to an 8-fold ambiguity: 2-fold from the sign of Z_f^R , two fold from the sign of \cos^{-1} , and two fold from the *mod* π ambiguity. In this context both $f = \rho\pi^0$ and $f = \omega\pi^0$ may be used so the ambiguity of this stand-alone method may, in principle, be reduced to the 2-fold *mod* π ambiguity.

Theoretical input can also be used to reduce the ambiguities and refine the determination of α . Estimates of the penguin or tree amplitudes may serve this purpose. In this context it is useful to note that the ratio of eqn. (7) with eqn. (8) gives us the tree-penguin ratio as a function of α :

$$\left| \frac{t_f}{p_f} \right|^2 = \frac{X_f - Z_f^R}{X_f - Z_f^R \cos 2\alpha - Z_f^I \sin 2\alpha} \quad (13)$$

A theoretical range for this ratio will therefore translate directly into a range of values for α given the experimental inputs for each final state f separately.

The degree of precision which may be achieved depends very much how the different ambiguous solutions happen to align up for various modes and in turn on the phases for the various amplitudes involved. In order to illustrate the situation, let us construct a toy model based on related observed branching ratios taking $\mathcal{M} = \pi^0$ and η . In the case of $\mathcal{M} = \eta$, the analogous $b \rightarrow s$ penguin modes: $B^- \rightarrow K^{*-}\eta$ and $B^0 \rightarrow K^{*0}\eta$ have been observed at CLEO [13, 14] with an average branching ratio of $\sim 2 \times 10^{-5}$. We thus expect that $|p_{\rho\eta}|^2 \approx |V_{td}/V_{ts}|^2 Br(K^*\eta)$, (using amplitudes in units that square to branching ratio), so taking $|V_{td}/V_{ts}| \approx 0.2$ we obtain $|p_{\rho\eta}|^2 = 0.8 \times 10^{-6}$. In order to estimate the penguin rate for $\rho^0\pi^0$ let us consider the analogous pure penguin mode (ϕK) which has been observed at BaBar [18] at a rate of $Br(B \rightarrow \phi K) \approx 10^{-5}$. As before we estimate $|p_{\rho^0\pi^0}|^2 \approx 0.4 \times 10^{-6}$.

The tree process in this channel which have been observed at CLEO [19] are $\pi^-\rho^0$ and $\pi^0\rho^-$ both of which have a branching ratio $\sim 10^{-5}$. These are color allowed while the processes we are interested in are color suppressed, hence we will assume that $|t_f|^2 \sim O(10^{-6})$.

For the purpose of generating specific illustrative examples, we will assume that for each \mathcal{M} , we will parameterize the amplitudes in the following general form:

$$t_{\rho\mathcal{M}} = A_0^{\mathcal{M}} \quad t_{\omega\mathcal{M}} = a_{\omega}^{\mathcal{M}} e^{i\psi_{\omega}^{\mathcal{M}}} A_0^{\mathcal{M}} \quad p_{\rho\mathcal{M}} = -p_{\omega\mathcal{M}} = a_p^{\mathcal{M}} e^{i\psi_p^{\mathcal{M}}} A_0^{\mathcal{M}} \quad (14)$$

where we will specifically look at the parameter set:

$$\begin{aligned} A_0^\pi = A_0^\eta = 10^{-3}, \quad a_\omega^\pi = 1.1 = a_\omega^\eta = 1.1, \quad a_p^\pi = 0.65, \quad a_p^\eta = 0.90, \\ \psi_\omega^\pi = -90^\circ, \quad \psi_\omega^\eta = 15^\circ, \quad \psi_p^\pi = 20^\circ, \quad \psi_p^\eta = 10^\circ, \end{aligned} \quad (15)$$

which are consistent with the estimated rates above. Further, for the purpose of this illustration, we will also assume that the true value of $\alpha = 75^\circ$.

Given perfect experimental data, the set of solutions for α in the case of $\mathcal{M} = \pi^0$ are $\{52.4^\circ, 75^\circ, 75.5^\circ, 114.1^\circ\}$ together with angles related to these by $\alpha \rightarrow \pi/2 - \alpha$; $\alpha \rightarrow 3\pi/2 - \alpha$ and $\alpha \rightarrow \pi + \alpha$. In the $\mathcal{M} = \eta$ case, the corresponding solution set is $\{75^\circ, 78.9^\circ, 82.6^\circ, 86.9^\circ\}$ and the related angles. Only the true solution and the three other related angles (i.e. 15° , 195° and 255°) are common to both sets.

To get an idea of how well we can do with the statistical errors from a finite amount of data, let us define:

$$\tilde{N}_0 = [(\text{number of } B^0) + (\text{number of } \bar{B}^0)] \cdot (\text{acceptance}) \quad (16)$$

Let us now derive a chi-squared function, χ^2 , using statistical errors of the input quantities X_f , Y_f and Z_f^I . If we define $N_f = (Br(B^0 \rightarrow f) + Br(\bar{B}^0 \rightarrow f))\tilde{N}_0/2$ then:

$$(\Delta X_f)^2 = X_f^2/N_f \quad (17)$$

If we want to determine Y_f and Z_f^I from the time dependent distributions in eq. (3), we can use the expectation value of time dependent operators which are proportional to these quantities. Using the optimal observable as defined in [20] we obtain:

$$\begin{aligned} g_Y = b \left(\frac{1+4x_b^2}{1+2x_b^2} \right) \cos(x_b\tau); \quad g_{Z^I} = b \left(\frac{1+4x_b^2}{2x_b^2} \right) \sin(x_b\tau); \\ \langle g_Y \rangle = Y_f/X_f; \quad \langle g_{Z^I} \rangle = Z_f^I/X_f \end{aligned} \quad (18)$$

Assuming a tagging efficiency T , the statistical errors in these observables are:

$$\begin{aligned}
\langle (\Delta g_y)^2 \rangle &= \frac{1}{TN_f} \left(\frac{1 + 4x_b^2}{1 + 2x_b^2} - \frac{Y^2}{X^2} \right) \\
\langle (\Delta g_{Z_I})^2 \rangle &= \frac{1}{TN_f} \left(\frac{1 + 4x_b^2}{2x_b^2} - \frac{Z_I^2}{X^2} \right)
\end{aligned} \tag{19}$$

In Fig. 1 we show the minimum χ^2 as a function of α for these two cases given $\tilde{N}_0 = 10^9$ and tagging efficiency $T = 0.5$ assuming that the central values are those given by the parameters in eq. (15). The $\mathcal{M} = \pi^0$ case is indicated by the dashed curve, the $\mathcal{M} = \eta$ case by the dotted curve and the sum by the solid curve. By construction, in each of the cases the curve must hit 0 for possible solutions. This is clearly true in the case of $\mathcal{M} = \pi^0$ while for $\mathcal{M} = \eta$, the solutions occur in tight clumps so, for instance, in the range $75^\circ - 86.9^\circ$, χ^2 is close to 0 but rises rapidly outside of this range. Looking at the sum, 1σ uncertainty range about 75° is $75_{-3.5}^{+7.4}$ (as well as the corresponding range around 15°). It is asymmetric due to the $\mathcal{M} = \eta$ curve. If we lower \tilde{N}_0 to 2×10^8 (i.e. scale χ^2 down by a factor of 5), then the range becomes $75_{-6.8}^{+15.6}$ where most of the bound now depends on the $\mathcal{M} = \eta$ case. Likewise for $\tilde{N}_0 = 10^8$ the range is $75_{-8.8}^{+18.9}$. Clearly if a single mode has a tight clump of solutions as exemplified by the $\mathcal{M} = \eta$ case, early data can put relatively tight bounds on the solution but it is difficult to distinguish between the members of the clump. To refine the solution with more statistics a mode with more widely scattered solutions such as $\mathcal{M} = \pi^0$ is helpful.

Fig. 1 also shows how a pure penguin mode may be helpful in distinguishing between the solution at 75° and the solution at 15° . The magnitude of the penguin amplitude in the $\mathcal{M} = \pi^0$ case for the minimum χ^2 solution at each α is shown with the dash-dotted curve. This is clearly very different in the two regions illustrating how pure penguin data from $B_s \rightarrow K_S K_S^*$ may solve this ambiguity. Of course the entire graph repeats itself in the range $180^\circ - 360^\circ$ leaving a *mod* π ambiguity.

Fig. 2 plots the same quantities as Fig. 1 where we have changed ψ_p^η to 190° to illustrate a somewhat different behavior for the $\mathcal{M} = \eta$ case. In this case, the solution set is $\{75^\circ, 80.1^\circ, 101.3^\circ, 106.2^\circ\}$ and we have somewhat better determination of the true solution since the $\mathcal{M} = \eta$ mode now only has one false solution near to the true one. The two false solutions for $\mathcal{M} = \eta$

near $\sim 105^\circ$ are eliminated by the $\mathcal{M} = \pi^0$ data although there is a local minimum of χ^2 in that vicinity since the latter case has a false solution at 114.1° . In particular, for $\tilde{N}_0 = 10^9$, the $1\text{-}\sigma$ range about the true solution is $75_{-2.5}^{+6.5}$ while if $\tilde{N}_0 = 2 \times 10^8$ then the range is $75_{-4.8}^{+9.3}$ but the other minimum near 108° begins to become a viable solution.

The information from this method can and should be combined with various other methods for determining α . In particular the method of [7] uses one of the same modes ($\rho^0\pi^0$) so jointly fitting α , $t_{\rho\pi^0}$ and $p_{\rho^0\pi^0}$ between the two data sets will produce more constrained results. In addition, since that method uses cross-channel interference, the ambiguity between α and $\alpha + \pi$ may be resolved. The interference between $B^\pm \rightarrow \rho^\pm \rho^0$ and $B^\pm \rightarrow \rho^\pm \omega$ in the $\omega \rightarrow \pi^+\pi^-$ channel as discussed in [21] may also be used to address this ambiguity.

The comparison between α as determined here and the value determined by the method of [8] is particularly interesting since in that case \hat{T} is purely the u-penguin; deviation between the two values of α could therefore be an indication of physics beyond the Standard Model. This relation between the u-penguin phase and the tree phase may also be tested in the method of [7] through the comparison of charged and neutral B decays. Furthermore, the methods in [11] depend only on direct CP violation and so a discrepancy with the methods involving time-dependent CP violation could indicate new physics in $B\bar{B}$ oscillation.

To obtain the angle γ we now consider modes which are sensitive to the interference of the $b \rightarrow s$ penguins with the $b \rightarrow su\bar{u}$ tree. The spin-0 particle which recoils against the ρ/ω should therefore be a K_S , K_L or any other spin-0 kaonic resonance that is self conjugate by decaying to a K_S or K_L [22] in the final state. For this analysis we need to assume that an accurate knowledge of β is available, which will likely be the case.

Let us denote such final states as g . In this case, the weak phase of the tree is still $+\gamma$ but the $b \rightarrow s$ penguin phase is ≈ 0 in the Standard Model. The decomposition analogous to eqn. (2) thus gives:

$$\begin{aligned} A_g &= \tilde{t}_g e^{+i\gamma} + \tilde{p}_g \\ \bar{A}_g &= \tilde{t}_g e^{-i\gamma} + \tilde{p}_g \end{aligned} \quad (20)$$

where $\tilde{t}_g = |\hat{T}_g V_{ub}^* V_{ud}|$ and $\tilde{p}_g = |\hat{P}_g V_{tb}^* V_{td}|$. Using the observables previously introduced in eqns. (4,5) we can define

$$\tilde{Z}_g = Z_g e^{+2i\beta}; \quad \tilde{Z}_g^R = \text{Re}(\tilde{Z}_g); \quad \text{and} \quad \tilde{Z}_g^I = \text{Im}(\tilde{Z}_g). \quad (21)$$

where we need to know β in order to obtain \tilde{Z} from Z . As in the case of α we find that:

$$X_g - \text{Re}(\tilde{Z}_g e^{+2i\gamma}) = (1 - \cos \gamma) |p_g|^2 \quad (22)$$

We can use this equation in the same way as eqn. (7) above since here too the penguin produces ρ and ω via the $d\bar{d}$ channel so we can equate $|p_g|^2$ between ωK_s and $\rho^0 K_s$. If we now define:

$$\tilde{\zeta} e^{i\tilde{\theta}} = \tilde{Z}_{\rho^0 K_s} - \tilde{Z}_{\omega K_s} \quad (23)$$

then

$$\gamma = -(\tilde{\theta} \pm \cos^{-1} \frac{\xi}{\tilde{\zeta}})/2 \quad \text{or} \quad \pi - (\tilde{\theta} \pm \cos^{-1} \frac{\xi}{\tilde{\zeta}})/2 \quad (24)$$

In this case, however we are somewhat restricted in the number of modes g which are experimentally accessible since spin-0 kaonic resonances are required and, aside from K^0 , these particles are close in mass to higher spin resonances with similar decay modes. One could, however, gain additional modes by time dependent angular analysis of $g = \rho/\omega K^{*0}$ and treating each of the three helicity states as a separate mode.

As in the α case, one can also resolve the ambiguities by comparing this to a pure penguin amplitude. In this case the pure penguin which is appropriate is $B^\pm \rightarrow K^0 \rho^\pm$. This has the advantage that it will also be produced at asymmetric B-factories.

Using these methods for the determination of γ , however is likely to be less promising than the α determination. Although the branching ratio to such modes is relatively large ($O(10^{-5})$), the color suppressed tree amplitude is only about 4% of the penguin so that the interferences effects required to solve for γ will be $\lesssim 4\%$.

To see how the method works in this case, it is useful to consider the limit that $t_g \ll p_g$, we can then use the following approximate relations:

$$\tan \gamma = \frac{\tilde{Z}_g^I}{|p_g|^2 - X_g} + O(|t_g/p_g|^2) \quad (25)$$

$$\tan \gamma = -\frac{\tilde{Z}_{\omega K}^I - \tilde{Z}_{\rho^0 K}^I}{X_{\omega K} - X_{\rho K}} + O(|t_g/p_g|^2) \quad (26)$$

In the first relation, if $|p_g|$ is known from the analogous pure penguin mode, this gives γ up to a 4-fold ambiguity (2-fold for $Z_R \rightarrow -Z_R$ and 2-fold for $\gamma \rightarrow \gamma + \pi$).

In order to estimate the number of B -mesons required we note that the key measurement would be the determination of \tilde{Z}_I which would in turn rely on an accurate determination of Z_I . Assuming that $Z_I/X \approx 0.04$, we see from eqn. (19) that to get a 3-sigma determination of this quantity requires $N_g \approx (X_g/\Delta Z_g^I)^2(1 + 4x_b^2)/(2Tx_b^2)$ so that if we take $X_g/\Delta Z_g^I = 3/0.04$ we obtain $N_g = 3 \times 10^4$. If the branching ratio for $B^0 \rightarrow \omega K^0$ is roughly the same as the measured branching ratio [23] $B^+ \rightarrow \omega K^+$ of $1.5_{-0.6}^{+0.7} \times 10^{-5}$, then $\tilde{N}_0 \approx 2 \times 10^9$. With this number of B -mesons, determination of X_g and $|p_g|^2$ to a precision of $< 1\%$ which is also required should be statistically possible. However very tight control over the backgrounds would be necessary to accurately determine the denominator of eqn. (25). Eqn. (26) can give γ up to an 8-fold ambiguity and the experimental requirements are similar.

In summary, our primary method for determining α involves observing the time dependent decays $B^0/\bar{B}^0 \rightarrow \rho^0 \mathcal{M}$ and $B^0/\bar{B}^0 \rightarrow \omega^0 \mathcal{M}$ where \mathcal{M} is a $u\bar{u}/d\bar{d}$ spin-0 meson. If we assume that electro-weak penguins are negligible and that the gluon fragmentation to an ω is also negligible, the penguin contribution to these two modes has the same magnitude. This allows us to solve for α with up to 16-fold discrete ambiguity. These ambiguities may be reduced to 4-fold by combining two or more such modes. Using other theoretical input or estimating the magnitude of the penguin by SU(3) related pure penguin modes one can reduce this to a two fold ambiguity. The precision that can be achieved depends on the various amplitudes and strong phases involved and it was found that reasonable bounds might begin to be achieved with $\tilde{N}_0 \sim 10^8$ while with $\tilde{N}_0 = 10^9$ relatively tight bounds on α appear possible. In the case of γ it is possible to use the analog of this method for the

decays $B^0/\bar{B}^0 \rightarrow \rho^0 K_S$ and $B^0/\bar{B}^0 \rightarrow \omega K_S$ and in principle it can be carried out with $\tilde{N}_0 \sim 10^9$. In this case however the relevant interference effects are small so that systematic errors in the measurements must be controlled to $\lesssim 1\%$.

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Figure 1: The χ^2 function for the minimum χ^2 solution at various values of α is shown for the inputs given in eqn. (15) where the true value of $\alpha = 75^\circ$. The $B^0 \rightarrow \rho/\omega \pi^0$ results are shown as a dashed curve, the $B^0 \rightarrow \rho/\omega \eta$ results are shown as a dotted curve while the sum is shown as a solid curve. The magnitude of the penguin for the minimum χ^2 solution is shown in units of 10^{-3} by the dot-dashed curve.

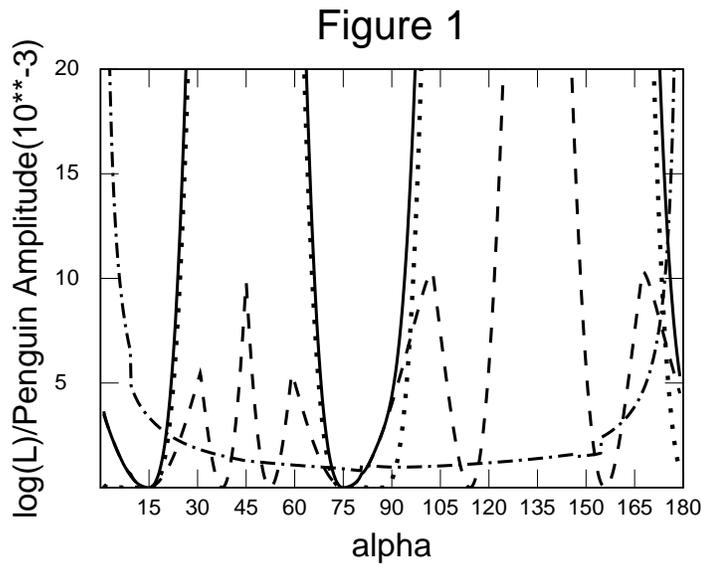


Figure 2: The χ^2 function and penguin amplitude as in Fig. 1 with $\alpha = 75^\circ$ and the inputs as in eqn. (15) except with $\psi_p^\eta = 190^\circ$.

