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Do SNe Ia Provide Direct Evidence for Past Deceleration of the Universe?

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ABSTRACT

Observations of SN 1997ff at $z \sim 1.7$ favor the accelerating Universe interpretation of the high-redshift type Ia supernova data over simple models of intergalactic dust or SN luminosity evolution. Taken at face-value, they provide direct lines of evidence that the Universe was decelerating in the past, an expected but untested feature of the current cosmological model. We show that the strength of this conclusion depends upon the nature of the dark energy causing the present acceleration. Only for a cosmological constant is the SNe evidence definitive. Using a new test which is independent of the contents of the Universe, we show that the SN data favor recent acceleration ($z < 0.5$) and past deceleration ($z > 0.5$).

1 Introduction

The discovery in 1998 that the Universe is speeding up and not slowing down (Riess et al, 1998; Perlmutter et al, 1999) was a startling "u-turn" in the quest to finally pin down the second of Sandage's two numbers (H_0 and q_0; Sandage 1961, 1988): The deceleration parameter, q_0, is negative. Observations of SN 1997ff, a type Ia supernova (SN Ia) at $z \sim 1.7$ (Riess et al 2001; Gilliland, Nugent, & Phillips 1999), and one or two more at $z \sim 1.2$ (Tonry et al. 2001; Aldering et al. 2001) disfavor the two simple alternates to an accelerating Universe – intergalactic dust and SN luminosity evolution (see e.g., Drell, Loredo and Wasserman 2000; Aguire 1999a,b; Riess 2000) – and bolster the case for the accelerating Universe hypothesis. CMB measurements that indicate the Universe is flat (Jaffe et al, 2001; Pryke et al, 2001; Netterfield et al, 2001) and the failure of the matter density (by a factor of three) to account for the critical density provides supporting evidence for an additional component of energy density with repulsive gravity (see e.g., Turner, 2000a).

There are very good theoretical reasons to believe that accelerated expansion is a recent phenomenon. A long matter-dominated phase is needed for the observed structure to develop from the small density inhomogeneities revealed by COBE and other anisotropy measurements made since (Turner & White, 1997). Further, the success of big-bang nucleosynthesis in predicting the light-element abundances (see e.g., Burles et al, 2001), and most recently the stunning confirmation of the BBN baryon density by CMB anisotropy measurements (Pryke et al, 2001; Netterfield et al, 2001), is strong evidence that the Universe was radiation dominated when it was seconds old. The gravity of both matter and radiation is attractive, leading to a strong theoretical prejudice for an early decelerating phase, lasting from at least as early as 1 sec to until a few billion years ago. However, this required feature of the current cosmological model remains largely untested.

The purpose of our *Letter* is to assess the strength of the *direct* empirical evidence from type Ia SNe for slowing expansion in the past.

2 Preliminaries

With good precedent (Robertson 1955; Hoyle & Sandage 1956), we introduce a generalized, epoch-dependent deceleration parameter[1]

$$q(z) \equiv (-\ddot{R}/R)/H^2(z) = \frac{dH^{-1}(z)}{dt} - 1 \tag{1}$$

where $R(t)$ is the cosmic-scale factor (normalized to be unity today), $H(z) = \dot{R}/R$ is the expansion rate and $q_0 = q(z=0)$. Just like q_0, for a matter-dominated, flat Universe, $q(z) = 0.5$ and for a vacuum-energy dominated, flat Universe $q(z) = -1.0$.

A useful measurement of the change in the current expansion rate during the span of a human time interval is far beyond the precision attainable by known cosmological probes

[1] As can be seen from this equation, accelerated growth of the cosmic-scale factor, i.e., $\ddot{R} > 0$, corresponds to $q < 0$; accelerated expansion rate, $\dot{H} > 0$, corresponds to $q < -1$.

(but see Loeb 1998). However, measurements of distant supernovae can probe the expansion *history* by determining luminosity distances, which in turn are related to the integral of the inverse of the preceding expansion rate. For a flat Universe

$$d_L(z) = c(1+z)\int_0^z \frac{du}{H(u)}\,, \tag{2}$$

and more generally,

$$d_L(z) = c(1+z)|1-\Omega_0|^{-1/2}H_0^{-1}\mathrm{S}\left[|1-\Omega_0|^{1/2}H_0\int_0^z dz/H(z)\right] \tag{3}$$

where $\mathrm{S}(x) = \sin(x)$ $(\Omega_0 > 1)$, $\sinh(x)$ $(\Omega_0 < 1)$, and x $(\Omega_0 = 1)$. The quantities H_0 and Ω_0 refer to the current $(z=0)$ Hubble constant and the sum of today's energy densities in units of the critical density $(\rho_{\rm crit} = 3H_0^2/8\pi G)$, respectively. The comoving distance to an object at redshift z is always $r(z) = d_L/(1+z)$.

Equation 2 can be rewritten in terms of the epoch-dependent deceleration parameter of Eq (1):

$$d_L = c(1+z)\int_0^z \frac{du}{H(u)} = c(1+z)H_0^{-1}\int_0^z du\exp\left[-\int_0^u [1+q(v)]d\ln(1+v)\right] \tag{4}$$

again for a flat Universe, though easily generalized as above. It is worth noting that only the assumption of the Robertson – Walker metric underlies Eqs 1 - 4. Said another way, deceleration/acceleration can be probed without assuming the validity of general relativity or without providing a manifest of the contents of the Universe. In the absence of the Friedmann equation of general relativity to relate the curvature radius to the matter/energy content, $cH_0^{-1}/|1-\Omega_0|^{1/2}$ is replaced by the spatial curvature radius.

Since supernovae measurements determine luminosity distances, they cannot directly measure the instantaneous expansion rate or deceleration rate. (Number counts of standard objects, which depend upon $r^2(z)/H(z)$, together with SNe, could in principle determine $H(z)$ directly; see Huterer & Turner, 2000.) To use SNe to probe the expansion history, one must make assumptions about the evolution of $H(z)$ or $q(z)$; in turn, we shall take both approaches.

3 Simple dark-energy models: Λ and const w_X

While surprising, accelerated expansion can be accommodated within the framework of the standard FRW cosmological model. According to general relativity, the source of gravity is proportional to $(\rho + 3p)$, stress-energy with large, negative pressure, $p_X < -\rho_X/3$, has repulsive gravity. In the absence of an established cause for cosmic acceleration, the causative agent has been referred to as "dark energy." (In relativity theory, any substance with pressure comparable in magnitude to its energy density is relativistic – more energy-like than matter-like, and hence the name dark energy.)

The simplest possibility for dark energy is the energy of the quantum vacuum (mathematically equivalent to a cosmological constant), for which $p_{\rm vac} = -\rho_{\rm vac}$. However, the natural scale for vacuum energy is at least 55 orders-of-magnitude too large to allow the formation of structure by gravitational instabilities in the early Universe (see e.g., Weinberg, 1989 or Carroll, 2000). The implausibility of reducing this by precisely 54 orders-of-magnitude, suggests to some the existence of an unrecognized symmetry that requires the energy of the quantum vacuum to be precisely zero. If this is so, then some other source for the accelerated expansion is required. Theorists have put forth a plethora of examples, from a rolling scalar field (a mini episode of inflation, often called quintessence; see e.g., Peebles & Ratra 1988 or Caldwell, Dave, & Steinhardt 1998) to the influence of hidden additional space dimensions (Deffayet, Dvali, & Gabadadze 2001; for reviews see Carroll, 2000; Turner, 2000b; or Sahni and Starobinskii, 2001).

For most purposes, the dark energy can be considered to be a smooth component characterized by its equation-of-state, $w_X \equiv p_X/\rho_X$, which may be a function of time (Turner & White, 1997). Doing so, and allowing for the fact that w_X may vary with time, the Friedmann equation for the expansion rate can be written as

$$\begin{aligned} H^2(z) &= H_0^2[\Omega_M(1+z)^3 + \Omega_X \exp[-3\int_0^z (1+w_X(u)) d\ln(1+u)] \\ &\quad + \Omega_R(1+z)^4 + (1-\Omega_0)(1+z)^2] \end{aligned} \tag{5}$$

where Ω_i refers to the present fraction of critical density in matter ($i = M$), in dark energy ($i = X$), and in radiation ($i = R$). The final term (curvature term) vanishes for a flat Universe; the radiation term, $\Omega_R \sim 10^{-4}$, is negligible for $1+z \ll 10^3$. Neglecting radiation, the generalized deceleration parameter of Eq (1) can then be written as

$$q(z) = \frac{\Omega_0}{2} + \frac{3}{2} w_X(z)\Omega_X(z) \tag{6}$$

Specializing to a flat Universe, as indicated by recent CMB anisotropy measurements which determine $\Omega_0 = 1 \pm 0.04$ (Jaffe et al, 2000; Pryke et al, 2001; Netterfield et al, 2001), and constant w_X, these expressions become

$$H^2(z) = H_0^2[\Omega_M(1+z)^3 + \Omega_X(1+z)^{3(1+w_X)}] \tag{7}$$

$$q(z) = \frac{1}{2}\left[\frac{1 + (\Omega_X/\Omega_M)(1+3w_X)(1+z)^{3w_X}}{1 + (\Omega_X/\Omega_M)(1+z)^{3w_X}}\right] \tag{8}$$

From this it follows that the redshift of transition from deceleration to acceleration ($\equiv z_{q=0}$) is

$$\begin{aligned} 1 + z_{q=0} &= [(1+3w_X)(\Omega_M - 1)/\Omega_M]^{-1/3w_X} \\ &= [2\Omega_\Lambda/\Omega_M]^{1/3} \end{aligned} \tag{9}$$

where the second equation is for vacuum energy (i.e., $w_X = -1$).

To begin, let us assume that the dark energy is simply the energy of the quantum vacuum ($w_X = -1$). If this is the case, the Universe must have been decelerating in the past, for $z > z_{q=0}$ (provided that $\Omega_M > 0$). Still, we may ask, have we seen direct evidence of that deceleration (yet)?

The current SN Ia sample (see Riess et al. 1998; Perlmutter et al. 1999; Tonry et al. 2001) provides measurements of the luminosity distance out to a redshift of $z \sim 1.7$ with the extreme redshift provided by SN 1997ff (Riess et al. 2001). From Eq (11) it follows that accelerated expansion throughout the interval sampled by the SNe is equivalent to $\Omega_M < 0.09$. Using the data employed by Riess et al. (2001), we have constructed the a posteriori probability density for Ω_M. The null hypothesis (i.e., $\Omega_M < 0.09$) is rejected with greater than 99.9% confidence. To be specific, the 99% confidence interval is

$$0.14 < \Omega_M < 0.60 \tag{10}$$

Further, SN 1997ff alone is inconsistent with $\Omega_M < 0.09$ at about the 99% confidence level.

Without SN 1997ff there is little direct evidence for past deceleration. The next highest redshift supernova in the sample used by Riess et al. (2001) was at $z \simeq 1$ (SN 1997ck; Garnavich et al. 1998a). In order to conclude that this supernova had directly probed the deceleration, would require constraining $\Omega_M > 0.2$, cf. Eq (11). The 99% confidence interval for the SNe used in Riess et al (2001), excluding SN 1997ff, is $0.11 < \Omega_M < 0.58$.

Now, consider dark-energy models with constant equation-of-state w_X (or approximately constant for $z < 1.7$). Assuming once again a flat Universe, this leaves two cosmological parameters: w_X and Ω_M. For constant w_X models, the Universe always has a decelerating phase at high-redshift, cf. Eq (10). Provided the matter density is sufficient, the epoch of transition from acceleration to deceleration occurs at $z_{q=0} < 1.7$; specifically, if

$$\Omega_M > \frac{1}{1 - (2.7)^{-3w_X}/(1 + 3w_X)} \, . \tag{11}$$

The region in the $w_X - \Omega_M$ plane where the transition from acceleration to deceleration occurs at $z < 1.7$ is shown in Fig. 1.

Also shown in Fig. 1 are confidence contours for the data employed by Riess et al (2001), computed with and without SN1997ff (see also Garnavich et al. 1998b; Perlmutter et al. 1999). As can be seen, SN1997ff significantly increases confidence that the transition from acceleration to deceleration occurred within the redshift interval sampled by the supernovae. Even so, only for w_X near -1, is $z_{q=0}$ constrained to less than 1.7 with high confidence.

For $w_X > -0.9$, SNe alone do not provide significant, direct evidence for past deceleration. However, with an external and reasonable constraint based upon dynamical measurements of $\Omega_M > 0.12$ (e.g., see Primack, 2000; or Turner, 2001), deceleration is guaranteed for $z < 1.7$.

SNe Ia at the next highest redshift known, $z = 1.2$, (e.g., SN 1999fv; Tonry et al. 2001, SN 1998eq; Aldering et al. 1999) or $z = 1.0$ (e.g., SN 1997ck; Garnavich et al. 1998a; SN 1999fk; Tonry et al. 2001) are likely too near to provide a direct requirement for deceleration at the time of their explosion due to the reduced leverage at these redshifts (see Figure 1). Even indirect evidence of deceleration is marginal at these redshifts requiring $\Omega_M > 0.2$.

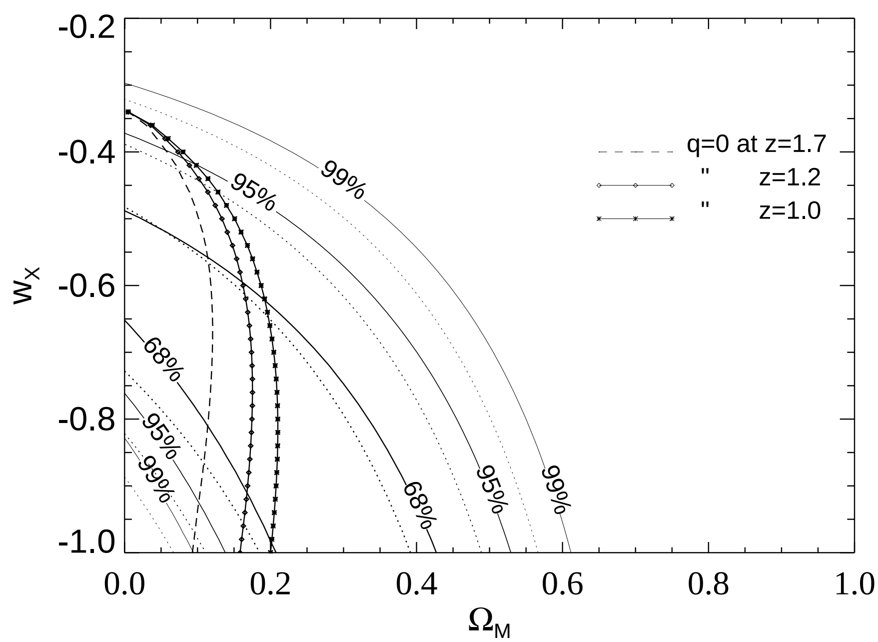

Figure 1: The solid contours show the regions of probability indicated; the dotted contours show the same, but without SN 1997ff. The three horizon curves delineate the mass density required so that the transition from acceleration to deceleration occurs at $z = 1.7, 1.2, 1.0$ (see text). Note: while indicative of the $w_X = -1$ case considered previously, the contours cannot be directly used to infer the confidence range for Ω_M for $w_X = -1$.

4 A new, model-independent test for deceleration

As discussed in §2, luminosity distance can be written in terms of the epoch-dependent deceleration parameter, $q(z)$, with only the assumption of the Robertson-Walker metric, cf. Eq 4. Proceeding from this equation, one can test for past deceleration in the most general way.

As a null hypothesis, suppose that the Universe never decelerated across the redshift interval sampled by the current set of supernovae: that is, $q \leq 0$ for $0 < z < 1.7$. Using the fact that $-1 < -(1+q)$ if $q \leq 0$, it then follows that

$$d_L(z) \geq cH_0^{-1}(1+z)\ln(1+z) \tag{12}$$

The logic of the inequality is clear: in a universe that is always coasting or accelerating, objects of a given redshift are farther away (and fainter) than in a universe that at some time has decelerated. (Note: the equality applies for a flat, eternally coasting universe, which can be achieved with $\Omega_X = 1$ and $w_X = -1/3$.)

Using the measurements for SN1997ff, this inequality reads

$$c^{-1}H_0 d_L(z = 1.7) = 2.4 \pm 0.4 > 2.7$$

The null hypothesis (Universe has never decelerated between $z = 0$ and $z = 1.7$) is violated, though with little significance. However general and simple, this analysis does not take into account the evidence for recent acceleration and thus dilutes the possible evidence for past deceleration.

We can increase our resolution to past episodes of deceleration by considering a sharper, two-epoch model that allows for the possibility of a change in the deceleration parameter

$$\begin{aligned} q(z) &= q_1 \qquad \text{for } z < z_1 \\ &= q_2 \qquad \text{for } z > z_1 \end{aligned} \tag{13}$$

The motivation for this ansatz is to test for what theory and data suggest: early deceleration (i.e., $q_2 > 0$) followed by recent acceleration (i.e., $q_1 < 0$), without specializing to a particular dark-energy model, or even assuming that the Friedmann equation describes $H(z)$. Physically, the parameters q_1 and q_2 correspond to average deceleration parameters for redshifts less than z_1 and greater than z_1 respectively.

For this two-parameter model, it is straightforward to obtain the luminosity distance:

$$\begin{aligned} d_L &= (c/H_0)(1+z)\frac{[1-(1+z)^{-q_1}]}{q_1} \qquad z < z_1 \\ &= (c/H_0)(1+z)\left[\frac{[1-(1+z_1)^{-q_1}]}{q_1} + \frac{(1+z_1)^{q_2-q_1}\left[(1+z_1)^{-q_2}-(z+z)^{-q_2}\right]}{q_2}\right] \qquad z > z_1 \end{aligned} \tag{14}$$

The transition redshift z_1 is arbitrary. However, due to the limited sampling of the redshift interval provided by the SNe and our interest in resolving the behavior in *both* regions, we selected values of z_1 near $z_1 \sim 0.5$.

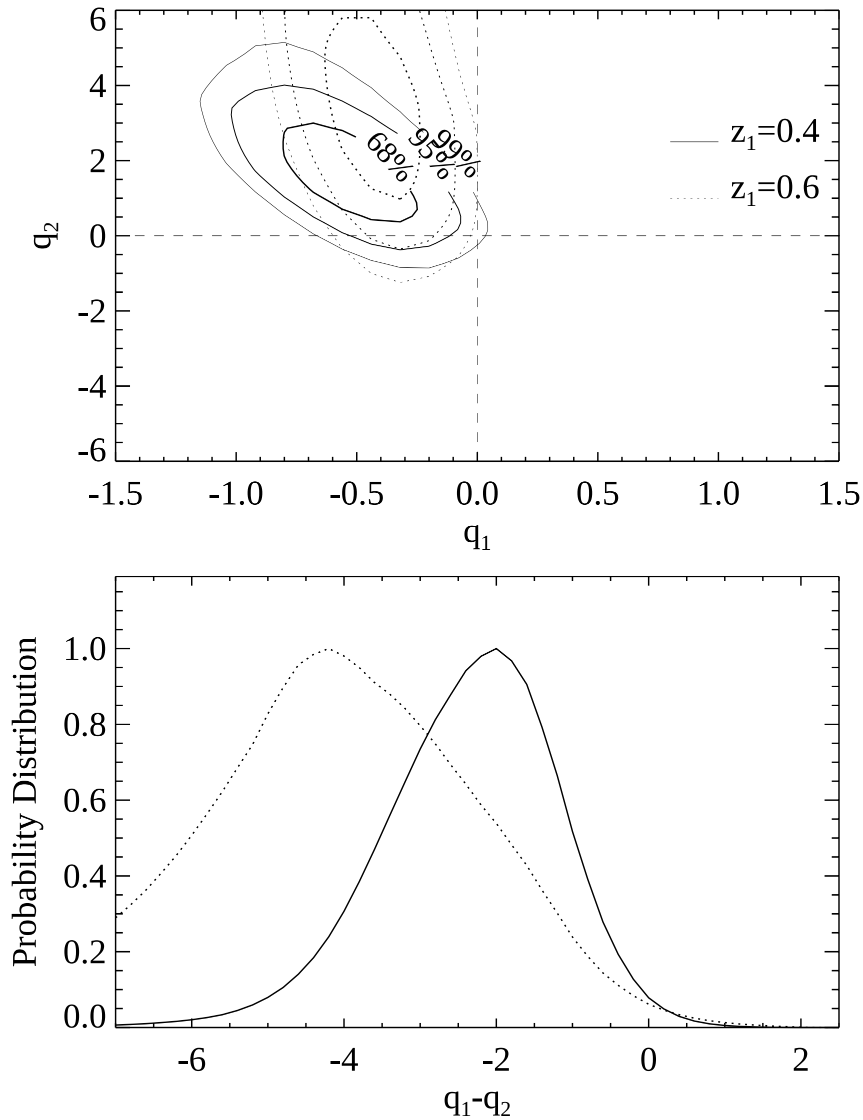

Figure 2: (a) Probability contours in the $q_1 - q_2$ plane; solid curves are for $z_1 = 0.4$ and dotted for $z_1 = 0.6$. The four quadrants correspond to the four different acceleration histories. With better than 90% confidence the SN data prefer recent acceleration ($q_1 < 0$) and past deceleration ($q_2 > 0$). (b) Probability distribution for $q_1 - q_2$; solid curve for $z_1 = 0.4$ and dotted curve for $z_1 = 0.6$. The SN data strongly indicate lessening acceleration with redshift – the assertion of attractive gravity at around $z \sim 0.5$.

In panel (a) of Fig. 2 we display confidence contours in the $q_1 - q_2$ plane, for $z_1 = 0.4$ and $z_1 = 0.6$ using the current SN sample. The four quadrants of the plot represent the four histories for cosmic expansion; past and present acceleration or deceleration, or a transition in the sign of q_i across z_1.

Not surprisingly, it is possible to reject both right quadrants, for which $q_1 > 0$, with very high confidence. Recent acceleration is a robust feature of the SN data. For the remaining two quadrants, the upper left quadrant ($q_2 > 0$) indicating past deceleration is preferred by the data, but we cannot exclude past acceleration ($q_2 < 0$) with great confidence ($\sim 90\%$ confidence).

We have constructed the contours for the current SN Ia sample excluding SN 1997ff; while the evidence for recent acceleration remains equally significant, the evidence for past deceleration is much less significant. Although SN 1997ff provides the greatest leverage for any single SN for this test, more SNe Ia at $z > 1$ are needed to sharpen this model-independent, direct test for past deceleration.

Finally, in panel (b) of Fig. 2 we show the a posteriori probability distribution for $q_1 - q_2$. The SNe data strongly prefer an increase in $q(z)$ with increasing redshift. This is a strong, model independent indication for a change in the deceleration rate with time in the sense of moving from recent acceleration to past deceleration. Said another way, we see direct evidence for the assertion of attractive gravity in the past.

5 Concluding remarks

The absence of an early, decelerating phase would be a much bigger surprise than the discovery that the Universe is accelerating today. It would be essentially impossible to reconcile with the standard hot big-bang cosmology. In addition to providing strong support for the accelerating Universe interpretation of high-redshift SNe Ia, SN 1997ff at $z \sim 1.7$ provides direct evidence for an early phase of slowing expansion *if* the dark energy is a cosmological constant (Riess et al. 2001). However, because supernova observations do not directly measure changes in the expansion rate, a model for $H(z)$ or $q(z)$ is needed to perform a more robust test for past deceleration. The former requires assumptions (or a deeper understanding) about the nature of dark energy responsible for the recent speed up while the latter requires more SNe Ia at $z > 1$.

In our analysis we have employed the measurement of SN 1997ff by Riess et al. (2001) at “face-value”. Riess et al. (2001) discuss a number of possible contaminants to this measurement which, if manifested, could significantly reduce the cosmological utility of this supernova. For example, host extinction could make the SN appear dimmer or foreground lensing (Lewis & Ibata 2001; Moertsell, Gunnarsson, & Goobar 2001) could make the SN brighter. However, a face-value treatment of this SN is plausible as significant contaminantion, while possible, appears to be unlikely. The star formation history of the red, elliptical host of SN 1997ff suggests that substantial foreground extinction of the supernova is not likely. Likewise, due to the lack of apparent shear of the host galaxy, the simplest interpretation is that the SN is not greatly magnified by the nearest foregrounds (as opposed to more complex scenarios in which the SN is highly magnified *and* a corresponding tangential shear of the host counteracts an intrinsic, radial elongation of the host; Riess et al. 2001).

In summary, we have shown that for a flat Universe the current supernova data:

1. provide strong, direct evidence of past deceleration *if* it is assumed that the dark energy is vacuum energy (cosmological constant).

2. alone do not provide direct evidence of past deceleration unless the dark-energy equation-of-state w_X is close to -1. However, using dynamical measurements of the amount of matter, deceleration can be indirectly inferred for the redshift range of the SN sample (if $\Omega_M > 0.12$).

3. without recourse to a specific model of the contents of the Universe, favor deceleration at $z > 0.5$ with $\sim$90% confidence. An even stronger statement is that the SN data favor increasing $q(z)$ with increasing redshift, a sign of the assertion of attractive gravity in the past.

What then can make the SN Ia evidence for a decelerating phase in the past stronger? Additional high-redshift ($z > 1$) supernovae would strengthen both the model-independent and the $w_X - \Omega_M$ analysis. Interestingly enough, very-low redshift supernovae also have significant leverage by reducing the uncertainty in the contemporary expansion rate (although uncertainty in the zeropoint calibration of SNe Ia does not affect the analysis). Specifically, if we had fixed the Hubble constant, the q_1-q_2 analysis would have implied a past decelerating phase with greater than 95% confidence and the 68% confidence contours in the $w_X - \Omega_M$ plane would have closed at $\Omega_M > 0$. Fortunately, systematic programs are underway to garner many more SNe Ia at both low and high redshifts.

High-redshift SNe Ia fill a unique niche in the toolbox of observational cosmology. As demonstrated here, they can provide a direct test of past deceleration, a salient and testable prediction of our current cosmological paradigm. In addition, they have great potential to unlock the mystery of the nature of the dark energy.

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References

[1] Aguirre, A. 1999a, ApJ, 512, 19

[2] Aguirre, A. 1999b, ApJ, 525, 583

[3] Aldering, G., et al. 1998, IAU Circ. 7046

[4] Burles, S., Nollett, K. & Turner, M.S. 2001, ApJ 552, L1

[5] Caldwell, R. R., Davé, R., & Steinhardt, P. J. 1998, Ap&SS, 261, 303

[6] Carroll, S.M. 2000, http://www.livingreviews.org/Articles/Volume4/2001-1carroll

[7] Deffayet, C., Dvali, G., & Gabadadze, G., 2001, astro-ph/0105068

[8] Drell, P. S., Loredo, T. J., & Wasserman, I. 2000, ApJ, 530, 593

[9] Garnavich, P., et al. 1998a, ApJ, 493, 53

[10] Garnavich, P., et al. 1998b, ApJ, 509, 74

[11] Gilliland, R. L., Nugent, P. E., & Phillips, M. M. 1999, ApJ, 521, 30 (GNP99)

[12] Hoyle, F. & Sandage, A. 1956, PASP 68, 301

[13] Huterer, D. & Turner, M.S. 2000, astro-ph/0012510 (Phys Rev D in press)

[14] Jaffe et al 2001, Phys Rev Lett 86, 3475

[15] Lewis, G. F., & Ibata, R. A. 2001, MNRAS, submitted (astro-ph/0104254)

[16] Moertsell, E., Gunnarsson, C., & Goobar, A., 2001, (astro-ph/0105355)

[17] Netterfield, C.B. et al 2001, astro-ph/0104460

[18] Peebles, P. J. E., & Ratra, B. 1988, ApJ, 325, L17

[19] Perlmutter, S., et al. 1999, ApJ, 517, 565

[20] Primack, J. 2000, in Sources and Detection of Dark Matter and Dark Energy in the Universe, ed. D. Cline (Springer-Verlag, Berlin), p. 3 (astro-ph/0007187)

[21] Pryke, C. et al 2001, astro-ph/01044490

[22] Riess, A. G., et al. 2001, ApJ, in press (astro-ph/0104455)

[23] Riess, A. G. 2000, PASP, 112, 1284

[24] Riess, A. G., et al. 1998, AJ, 116, 1009

[25] Robertson, H. P., 1955, PASP, 67, 82

[26] Sahni, V. & Starobinsky, A., 2000, Int. Journ. Mod. Phys. D., 9, 373

[27] Sandage, A., 1961, ApJ, 133, 355

[28] Sandage, A., 1988, ARAA, 26, 561

[29] Tonry, J. L., et al. 2001, in Proceedings of Astrophysical Ages and Timescales, Gemini, (astro-ph/0105413)

[30] Turner, M.S. & White, M. 1997, Phys Rev D 56, R4439

[31] Turner, M.S. 2000a, in Proceedings of Type Ia Supernovae: Theory and Cosmology (Chicago, 29 - 31 October 1998), edited by J.C. Niemeyer and J.W. Truran (Cambridge Univ. Press, Cambridge, UK, 2000), p. 101 (astro-ph/9904049)

[32] Turner, M.S. 2000b, Phys Rep 334, 619

[33] Turner, M.S. 2001, astro-ph/0106035

[34] Weinberg, S. 1989, Rev Mod Phys 61, 1