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Bimaximal Mixing in an $SO(10)$ Minimal Higgs Model

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Abstract

An $SO(10)$ SUSY GUT model was previously presented based on a minimal set of Higgs fields. The quark and lepton mass matrices derived fitted the data extremely well and led to large $\nu_\mu - \nu_\tau$ mixing in agreement with the atmospheric neutrino data and to the small-angle MSW solution for the solar neutrinos. Here we show how a slight modification leading to a non-zero up quark mass can result in bimaximal mixing for the atmospheric and solar neutrinos. The “just-so” vacuum solution is slightly favored over the large-angle MSW solution on the basis of the hierarchy required for the right-handed Majorana matrix and the more nearly-maximal mixing angles obtained.

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Atmospheric neutrino data [?] reveal a large and, in fact, nearly maximal mixing between ν_μ and some other neutrino, which most plausibly is assumed to be ν_τ , though it may also be a sterile neutrino. The solar neutrino problem [?], on the other hand, can be solved either by a small mixing ($\sin^2 2\theta_{e\mu} \sim 6 \times 10^{-3}$) or by a nearly maximal mixing of ν_e with ν_μ . In the former case one has the small-angle MSW solution [?], while in the latter case one has either the large-angle MSW solution [?] or the vacuum oscillation solution [?]. Cases where both the atmospheric and solar neutrino anomalies are solved by nearly maximal mixing [?] are often called “bimaximal.”

There have been several interesting suggestions about how bimaximal mixing might arise theoretically [?]. Here we suggest a scheme that has certain novel features. The basic idea, which at first sounds artificial but which actually can emerge quite simply and naturally as will be seen, is that the large mixing of ν_μ and ν_τ originates from transformation of the *charged lepton* mass matrix, whereas the large mixing of ν_e and ν_μ originates in the *neutrino* mass matrix itself. (Of course, it only makes sense to draw this distinction if there is a preferred basis of families, which is here provided by some underlying theory of flavor.) The idea that the large $\nu_\mu - \nu_\tau$ mixing arises from the mass matrix of the charged leptons was proposed in [?], where it emerged as part of a complete model for the quark and lepton masses and mixings. One of the virtues of this idea is that it allows a simple resolution of the supposed paradox that the mixing of the second and third families is small for the quarks, $V_{cb} \cong 0.04$, and large for the leptons, $V_{\mu 3} \simeq 0.7$.

The model developed in [?] was only a model for the heaviest two families with the first family being approximated as massless. In [?] this model was extended to the first family in a very economical way that gave several additional predictions, among which were that the $\nu_e - \nu_\mu$ mixing angle is small, and in fact precisely in the presently allowed range for the small-angle MSW solution. This small $\nu_e - \nu_\mu$ angle, like the large $\nu_\mu - \nu_\tau$ angle, arose from diagonalization of the *charged* lepton mass matrix.

In the present paper we show that a slight alteration of that model leads to an equally predictive scheme that has bimaximal mixing. All the predictions for quark and charged lepton masses, for the CKM parameters, and for the $\nu_\mu - \nu_\tau$ mixing are left essentially unaffected; however, the $\nu_e - \nu_\mu$ mixing moves from the small-angle MSW value to become maximal. We will first very briefly review the model of [?] and [?], noting the features relevant to lepton mixing, and then proceed to show how bimaximal mixing can arise in it.

The model is based on supersymmetric $SO(10)$ and leads to the following matrices: U^0 for up-type quarks, D^0 for down-type quarks, L^0 for the charged leptons, and N^0 for the Dirac neutrino masses, where the superscript 0 refers to the matrices at the unification scale.

$$\begin{aligned}
U^0 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} M_U, & D^0 &= \begin{pmatrix} 0 & \delta & \delta' \\ \delta & 0 & \sigma + \epsilon/3 \\ \delta' & -\epsilon/3 & 1 \end{pmatrix} M_D, \\
N^0 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} M_U, & L^0 &= \begin{pmatrix} 0 & \delta & \delta' \\ \delta & 0 & -\epsilon \\ \delta' & \sigma + \epsilon & 1 \end{pmatrix} M_D.
\end{aligned} \tag{1}$$

These matrices arise from simple diagrams in the $SO(10)$ unified model, which involve only five effective Yukawa terms that have the forms $(\mathbf{16}_3 \mathbf{16}_3) \mathbf{10}_H$, $(\mathbf{16}_2 \mathbf{16}_3) \mathbf{10}_H \mathbf{45}_H$, $[\mathbf{16}_2 \mathbf{16}_H]$ $[\mathbf{16}_3 \mathbf{16}'_H]$, $[\mathbf{16}_1 \mathbf{16}_2][\mathbf{16}_H \mathbf{16}'_H]$, and $[\mathbf{16}_1 \mathbf{16}_3][\mathbf{16}_H \mathbf{16}'_H]$. These lead, respectively, to the entries in Eq. (1) that are denoted 1, ϵ , σ , δ , and δ' . The numerical subscripts are family indices, while the subscript H denotes a Higgs multiplet. The notation $[\mathbf{16} \mathbf{16}]$ implies the spinors are contracted into an $SO(10)$ vector. The VEV, $\langle \mathbf{16}_H \rangle \sim M_G$, lies in the $SU(5)$ singlet direction and helps break $SO(10)$ down to the Standard Model, while $\langle \mathbf{16}'_H \rangle \sim M_W$ lies in the weak doublet direction and helps break the electroweak interactions. The expectation value of the adjoint ($\langle \mathbf{45}_H \rangle$) is proportional to the $SO(10)$ generator $B - L$, as required for the Dimopoulos-Wilczek mechanism to provide the doublet-triplet splitting [?]. The foregoing information is sufficient to derive the matrices in Eq. (1).

The family hierarchy results from the smallness of the parameters $\epsilon \cong 0.14$, $\delta \cong 0.008$, and $|\delta'| \cong 0.008$. The parameter $\sigma \cong 1.8$ is not small, however, and is the key to understanding many of the qualitative and quantitative features of the quark and lepton spectrum, including the fact that the second and third families of leptons have a large mixing while that of the quarks is small. The point is that $SU(5)$ relates left-handed (right-handed) down quarks to right-handed (left-handed) charged leptons, and consequently relates the ij element of D^0 to the ji element of L^0 . That is why in Eq. (1) the large parameter σ appears in L^0_{32} where it leads to large $\mu^- - \tau^-$ mixing and hence large $\sin^2 2\theta_{\mu\tau}$, whereas it does not appear in D^0_{32} where it would give large V_{cb} , but rather appears in D^0_{23} where it affects only the unobservable mixing of right-handed quarks. The great difference in magnitude between V_{cb} and $\sin^2 2\theta_{\mu\tau}$ is thus a consequence of D^0 and L^0 being highly asymmetric and the peculiarities of $SU(5)$ invariance.

As shown in [?] and [?], the matrices in Eq. (1) give a remarkably good fit to all the known quark and lepton masses and mixings, in particular, when the small-angle MSW solution [?] is relevant for the solar neutrino problem. We now review the lepton results and explore the possibility of large-angle $\nu_e - \nu_\mu$ mixing in this $SO(10)$ unified model.

The lepton mixing matrix is given by

$$V_{lepton} = U_L^\dagger U_\nu, \quad (2)$$

where U_L is the unitary transformation of the left-handed charged leptons required to diagonalize L^0 , and U_ν is the complex orthogonal transformation of the left-handed neutrinos required to diagonalize the light-neutrino mass matrix, $M_\nu = -N^T M_R^{-1} N$. In $SO(10)$ the Dirac neutrino mass matrix N^0 and up quark mass matrix U^0 are related, and as given in Eq. (1) have vanishing first rows and first columns. This is to be regarded as only an approximation to the real world, but as we shall see later, it is a very good approximation.

With this particular texture for N^0 , no matter what form M_R assumes, the light-neutrino mass matrix M_ν will also have vanishing first row and column, and we can write

$$U_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix}, \quad (3)$$

where the parameters s and c are complex, in general, with $c^2 + s^2 = 1$. From the form of N^0 it is easy to see that, formally speaking, $|s| = O(\epsilon)$. If the unknown matrix M_R is parametrized by $(M_R^{-1})_{ij} = a_{ij} \Lambda_R^{-1}$, then, in units of (M_U^2/Λ_R) , one has $(M_\nu)_{1j} = (M_\nu)_{j1} = 0$, $(M_\nu)_{22} = \epsilon^2 a_{33}$, $(M_\nu)_{23} = (M_\nu)_{32} = \epsilon a_{33} - \epsilon^2 a_{23}$, and $(M_\nu)_{33} = a_{33} - 2\epsilon a_{23} + \epsilon^2 a_{22}$. This gives

$$\tan 2\theta_{23}^\nu \equiv 2sc/(c^2 - s^2) = 2\epsilon \left(\frac{a_{33} - \epsilon a_{23}}{a_{33} - 2\epsilon a_{23} + \epsilon^2 a_{22} - \epsilon^2 a_{33}} \right), \quad (4)$$

and thus $\theta_{23}^\nu \sim \epsilon$, unless the parameters a_{ij} are fine tuned to have a special relationship to each other.

We see then, that the contributions to the leptonic mixings coming from U_ν are either zero or small. The leptonic mixings thus arise from U_L . This is good news, since L^0 in Eq. (1) is known in this model. With the values of the parameters ϵ , σ , δ , and δ' given earlier as determined by fitting known quantities, one finds that

$$U_L^\dagger = \begin{pmatrix} c_{12} & -s_{12}c_{23} & s_{12}s_{23} \\ s_{12} & c_{12}c_{23} & -c_{12}s_{23} \\ 0 & s_{23} & c_{23} \end{pmatrix}, \quad (5)$$

where

$$\begin{aligned}\tan \theta_{23}^L &= s_{23}/c_{23} \cong \sigma + \epsilon \cong 1.9, \\ \sin \theta_{12}^L &= s_{12} \cong \delta \sqrt{\sigma^2 + 1}/\epsilon \sigma \cong 0.07.\end{aligned}\tag{6}$$

Note that the mixing θ_{23}^L is large and the mixing θ_{12}^L is small. As can be seen from Eq. (1), the smallness of the angle θ_{12}^L is related to the smallness of the corresponding angle for the quarks. The largeness of θ_{23}^L , on the other hand, is due to the large “lopsided” entry σ . There is no similar entry for the mixing of the first family. One could imagine introducing one, but one would find that doing so would make it hard to fit many known quantities such as V_{us} , V_{ub} , m_e/m_μ , and m_d/m_s . It would seem, then, that this model must give large $\nu_\mu - \nu_\tau$ mixing and small $\nu_e - \nu_\mu$ mixing. However, as we shall now show, the same model actually can give bimaximal mixing.

We will suppose now that the up quark matrix element U_{11}^0 is not exactly zero, but is given by ηU_{33}^0 . If $m_u(1 \text{ GeV}) \approx 4 \text{ MeV}$, then $\eta \approx 6 \times 10^{-6}$, which is a thousand times smaller than the smallest of the other model parameters, δ and δ' . It is in this sense that the vanishing of the first row and column of U^0 is an excellent approximation. In $SO(10)$ the simplest possibility is that N_{11}^0 is also given by ηM_U . Thus, we assume for N^0 the form

$$N^0 = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} M_U.\tag{7}$$

This tiny modification of Eq. (1) allows interesting effects for certain forms of M_R . What we find is that if the off-diagonal elements in the first row and first column of M_R are small or zero, then the small-angle MSW solution results as in [?], whereas if these elements are important, the large-angle solution of either the “just-so” vacuum or large-angle MSW oscillation type can result. Instead of looking at the most general forms for M_R , we will illustrate this in two representative cases:

$$(I) \quad M_R^I = \begin{pmatrix} B & 0 & 0 \\ 0 & 0 & A \\ 0 & A & 1 \end{pmatrix} \Lambda_R, \quad (II) \quad M_R^{II} = \begin{pmatrix} 0 & A & 0 \\ A & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Lambda_R.\tag{8}$$

Small-angle MSW Solution

Form (I):

$$M_\nu^I = -N^{0T}(M_R^I)^{-1}N^0 = - \begin{pmatrix} \eta^2/B & 0 & 0 \\ 0 & 0 & -\epsilon^2/A \\ 0 & -\epsilon^2/A & -2\epsilon/A - \epsilon^2/A^2 \end{pmatrix} \frac{M_U^2}{\Lambda_R}. \quad (9)$$

This light neutrino mass matrix is diagonalized by an orthogonal transformation of the type given in Eq. (3) for which Eq. (4) applies with $\theta_{23}^\nu \sim \epsilon$. The presence of the new η contribution to the 11 element of N^0 thus does not modify the small-angle MSW solution obtained earlier, if M_R is of type (I). The ratio of the mass differences required for this solution, $\delta m_{23}^2 \cong 3.5 \times 10^{-3} \text{ eV}^2$, $\delta m_{12}^2 \cong 7 \times 10^{-6} \text{ eV}^2$, can easily be obtained with $A \simeq -\epsilon$, while the absolute mass scale follows with $M_U \sim 100 \text{ GeV}$ and $\Lambda_R \sim 1.5 \times 10^{14} \text{ GeV}$. Moreover, the mixing results, $\sin^2 2\theta_{\mu\tau} \cong 1.0$ and $\sin^2 2\theta_{e\mu} \cong 0.008$, lie in the desired ranges for the atmospheric and small-angle MSW solutions [?, ?].

Vacuum Oscillation Solution

Form (II):

$$M_\nu^{II} = -N^{0T}(M_R^{II})^{-1}N^0 = - \begin{pmatrix} 0 & 0 & -(\eta/A)\epsilon \\ 0 & \epsilon^2 & \epsilon \\ -(\eta/A)\epsilon & \epsilon & 1 \end{pmatrix} \frac{M_U^2}{\Lambda_R}. \quad (10)$$

A rotation in the 2-3 plane by an angle $\tan \theta_{23}^\nu = \epsilon$ will eliminate the 22, 23, and 32 entries and induce 12 and 21 entries that are equal to $(\eta/A)\epsilon^2$ (neglecting terms higher order in ϵ^2). Following this with a rotation in the 1-3 plane by an angle $\theta_{13}^\nu \cong (\eta/A)\epsilon$ brings the matrix to the form

$$M_\nu^{III} \cong - \begin{pmatrix} -(\eta/A)^2\epsilon^2 & (\eta/A)\epsilon^2 & 0 \\ (\eta/A)\epsilon^2 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{M_U^2}{\Lambda_R}. \quad (11)$$

For $|\eta/A| \ll 1$, a pseudo-Dirac form for the mass matrix of ν_e and ν_μ obtains with nearly-degenerate neutrinos. One finds

$$\begin{aligned} m_{\nu_e} &\cong m_{\nu_\mu} \cong |\eta/A| \epsilon^2 (M_U^2/\Lambda_R), \\ \delta m_{12}^2 &\cong 2 |\eta/A|^3 \epsilon^4 (M_U^2/\Lambda_R)^2, \\ \delta m_{23}^2 &\cong m_{\nu_\tau}^2 \cong (M_U^2/\Lambda_R)^2 \end{aligned} \quad (12)$$

If one takes $\delta m_{12}^2 \simeq 4 \times 10^{-10} eV^2$, corresponding to the vacuum oscillation solution of the solar neutrino problem [?], and $\delta m_{23}^2 \simeq 3.5 \times 10^{-3} eV^2$ [?], then $|\eta/A| \simeq 0.05$ and the pseudo-Dirac condition is satisfied. This means that $|\theta'_{13}| \simeq 7 \times 10^{-3}$, which we shall ignore, and $|\theta'_{12} - \pi/4| \simeq 0.05$. Thus, to a good approximation we can write $\theta'_{23} = \epsilon$, $\theta'_{13} = 0$, and $\theta'_{12} = \pi/4$, or

$$U_\nu = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & \epsilon \\ -\epsilon/\sqrt{2} & -\epsilon/\sqrt{2} & 1 \end{pmatrix}. \quad (13)$$

With the use of Eqs. (2) and (5), this gives for the lepton mixing matrix

$$V_{lepton} \cong \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & -s_{12}s'_{23} \\ c'_{23}/\sqrt{2} & c'_{23}/\sqrt{2} & -s'_{23} \\ s'_{23}/\sqrt{2} & -s'_{23}/\sqrt{2} & c'_{23} \end{pmatrix}. \quad (14)$$

with $s'_{23} \equiv \sin \theta_{\mu\tau}$, where $\theta_{\mu\tau} \cong \theta_{23}^L - \epsilon \cong 63^\circ - 8^\circ = 55^\circ$; hence $\sin^2 2\theta_{\mu\tau} \simeq 0.9$ while $\sin^2 2\theta_{e\mu} \simeq 1.0$, which are consistent with the experimental limits.

It is interesting that the very small value of δm_{12}^2 needed for the vacuum oscillation solution to the solar neutrino problem has a natural explanation in this approach. From Eq. (12) it is apparent that $m_{\nu_\mu} \cong m_{\nu_e} \propto \eta$. In other words, the very tiny parameter η required to fit m_u/m_t — a quantity pertaining to the *first* family — actually ends up controlling the masses of both the muon- and electron-neutrino which are nearly degenerate.

Large-angle MSW Solution

The question arises whether form (II) can also give the large-angle MSW solution to the solar neutrino problem [?]. The answer is yes, though in this case $\sin^2 2\theta_{e\mu}$ departs significantly from maximality. For this solution one needs to have $\Delta m_{12}^2 \simeq 2 \times 10^{-5} eV^2$ along with $\Delta m_{23}^2 \simeq 3.5 \times 10^{-3} eV^2$. Although Eq. (11) still applies, the condition for pseudo-Dirac neutrinos is no longer satisfied as one finds $\eta/A \sim 1.8$. Three Majorana neutrinos emerge for which

$$\begin{aligned} m_3 &\cong M_U^2/\Lambda_R, \\ m_2 &\cong \frac{1}{2}(\eta/A)\epsilon^2 \left[\eta/A + \sqrt{4 + (\eta/A)^2} \right] M_U^2/\Lambda_R, \\ m_1 &\cong \frac{1}{2}(\eta/A)\epsilon^2 \left[-\eta/A + \sqrt{4 + (\eta/A)^2} \right] M_U^2/\Lambda_R \end{aligned} \quad (15)$$

With $M_U \sim 100$ GeV and $\Lambda_R \sim 1.7 \times 10^{14}$ GeV, the three neutrino masses are given numerically by 5.9×10^{-2} eV, 4.7×10^{-3} eV and 9.3×10^{-4} eV. Making use of the above value for η/A , we can write down the equivalent of Eq. (13) for the neutrino mixing matrix

$$U_\nu = \begin{pmatrix} 0.389 & -0.878 & -1.73\epsilon \\ 0.922 & 0.392 & 0.961\epsilon \\ -0.18\epsilon & -2\epsilon & 0.961 \end{pmatrix}. \quad (16)$$

The lepton mixing matrix is then found numerically to be

$$V_{lepton} \cong \begin{pmatrix} 0.360 & -0.908 & -0.186 \\ 0.469 & 0.367 & -0.812 \\ 0.811 & 0.222 & 0.556 \end{pmatrix}. \quad (17)$$

from which one obtains $\sin^2 2\theta_{\mu\tau} = 0.82$ and $\sin^2 2\theta_{e\mu} = 0.43$. These values for the mixing angles are on the low side of the allowed experimental region for this large-angle MSW type of solution. Moreover, the $A \simeq 3 \times 10^{-6}$ parameter required is some thirty times smaller than that found earlier with a pair of pseudo-Dirac neutrinos; thus a considerably larger hierarchy is required in the right-handed Majorana neutrino matrix to reproduce the large-angle MSW mixing than for the vacuum solution. These features have also appeared in other forms we have assumed for M_R .

In summary, we have shown that, through the introduction of a small correction which gives mass to the up quark and hence also modifies the related Dirac neutrino matrix, a bimaximal solution to the solar and atmospheric neutrino oscillations can be achieved, provided the right-handed Majorana neutrino matrix mixes the first family with the other two. The large-angle vacuum solution is somewhat preferred over the large-angle MSW solution for the solar neutrino problem, since a smaller hierarchy is required in the Majorana matrix and the mixing angles are more nearly maximal as suggested by present experimental data for those type of solutions. But as presently understood, the model does not suggest a preference for the large-angle solutions over the small-angle MSW solution.

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