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# A short look at $\epsilon'/\epsilon$

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## Abstract

We analyze the theoretical implications of the new KTeV measurement of direct CP-violation in  $K \rightarrow \pi\pi$  decays. The result is found consistent with the Standard Model for low values of the strange quark mass  $m_s$ . If the hadronic parameters  $B_6^{(1/2)}$  and  $B_8^{(3/2)}$  satisfy  $2B_6^{(1/2)} \Leftrightarrow B_8^{(3/2)} \leq 2$ , as suggested by lattice and  $1/N_c$  calculations, we find an upper bound of 110 MeV for  $m_s(2 \text{ GeV})$ . We parametrize potential new physics contributions to  $\epsilon'/\epsilon$  and illustrate their correlation with upper bounds on  $m_s$ . Finally we discuss a non-perturbative mechanism, which is not contained in the existing calculations of  $B_6^{(1/2)}$ . This mechanism enhances  $B_6^{(1/2)}$  and thereby leads to a better understanding of the  $\Delta I = 1/2$  rule and the high measured value of  $\text{Re}(\epsilon'/\epsilon)$ .

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Recently the KTeV collaboration at Fermilab has precisely determined the measure of direct CP-violation in  $K \rightarrow \pi\pi$  decays [1]:

$$\text{Re}(\epsilon'/\epsilon) = (28 \pm 4) \cdot 10^{-4}. \quad (1)$$

This measurement is consistent with the result of the CERN experiment NA31, which has also found a non-vanishing value for  $\text{Re}(\epsilon'/\epsilon)$  [2]. Within the last two decades a tremendous effort has been made to calculate the short distance QCD effects with next-to-leading order accuracy [3] and to obtain the relevant hadronic matrix elements using various non-perturbative methods

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[4–6]. Yet while the Standard Model predicts  $\epsilon'/\epsilon$  to be non-zero, the theoretical prediction of its precise value is plagued by large uncertainties due to an unfortunate cancellation between two hadronic quantities. Nevertheless for the ballpark of popular input parameters  $\text{Re}(\epsilon'/\epsilon)$  results in 2–16 times  $10^{-4}$  [7], so that the large value in (1) came as a surprise to many experts. Here a key role is played by the strange quark mass, whose size is not precisely known at present. In the Standard Model  $\text{Re}(\epsilon'/\epsilon)$  can be summarized in the handy formula [7, 9]:

$$\text{Re}(\epsilon'/\epsilon) = \text{Im} \lambda_t \left[ \Leftrightarrow 1.35 + R_s \left( 1.1 \left| r_Z^{(8)} \right| B_6^{(1/2)} + (1.0 \Leftrightarrow 0.67 \left| r_Z^{(8)} \right|) B_8^{(3/2)} \right) \right]. \quad (2)$$

Here  $\lambda_t = V_{td}V_{ts}^*$  is the CKM factor and  $r_Z^{(8)}$  comprises the short distance physics. Including next-to-leading order QCD corrections the short distance factor is in the range [3, 7]

$$6.5 \leq \left| r_Z^{(8)} \right| \leq 8.5 \quad (3)$$

for  $0.113 \leq \overline{\alpha_s^{\overline{\text{MS}}}}(M_Z) \leq 0.123$ . Relevant contributions to  $\epsilon'/\epsilon$  stem from the  $\Delta I = 1/2$  matrix element of the operator  $Q_6$  and the  $\Delta I = 3/2$  matrix element of  $Q_8$  (see [3, 7, 8] for their precise definition). These hadronic matrix elements are parametrized by  $B_6^{(1/2)}$  and  $B_8^{(3/2)}$ . Finally the dependence on the strange quark mass is comprised in

$$R_s = \left( \frac{150 \text{ MeV}}{m_s(m_c)} \right)^2.$$

From standard analyses of the unitarity triangle [10, 11] one finds

$$1.0 \cdot 10^{-4} \leq \text{Im} \lambda_t \leq 1.7 \cdot 10^{-4}. \quad (4)$$

Lattice calculations [4] and the  $1/N_c$  expansion [5] predict

$$0.8 \leq B_6^{(1/2)} \leq 1.3, \quad 0.6 \leq B_8^{(3/2)} \leq 1.0. \quad (5)$$

The maximal possible  $\text{Re}(\epsilon'/\epsilon)$  for the quoted ranges of the input parameters is plotted vs.  $m_s$  in Fig. 1. Hence if the Standard Model is the only source of direct CP-violation in  $K \rightarrow \pi\pi$  decays, the  $2\sigma$  bound from (1),  $\text{Re}(\epsilon'/\epsilon) \geq 20 \cdot 10^{-4}$ , implies

$$m_s(m_c) \leq 126 \text{ MeV} \quad \Leftrightarrow \quad m_s(2 \text{ GeV}) \leq 110 \text{ MeV}. \quad (6)$$

in the  $\overline{\text{MS}}$  scheme. The upper bounds in (6) correspond to the maximal values for  $\text{Im} \lambda_t$ ,  $2 B_6^{(1/2)} \Leftrightarrow B_8^{(3/2)}$  and  $\left| r_Z^{(8)} \right|$ . In the chiral quark model [6, 8]  $B_6^{(1/2)}$  can exceed the range in (5) and  $2 B_6^{(1/2)} \Leftrightarrow B_8^{(3/2)}$  can be as large as 2.9 relaxing the bound in (6) to  $m_s(m_c) \leq 151 \text{ MeV}$ . In [8] it has been argued that this feature of the chiral quark model prediction should also be present in other approaches, once certain effects (final state interactions,  $\mathcal{O}(p^2)$  corrections to the electromagnetic terms in the chiral lagrangian) are consistently included. Hence the result in (1) is perfectly

consistent with values for  $m_s$  obtained in quenched lattice calculations favouring  $m_s(2 \text{ GeV}) = (110 \pm 30) \text{ MeV}$  [12]. From unquenched calculations one expects even smaller values [13]. It is also consistent with recent sum rule estimates [14]. However the preliminary ALEPH result for the determination of  $m_s$  from  $\tau$  decays,  $m_s(m_\tau) = (172 \pm 31) \text{ MeV}$  [15], violates the bound in (6). The compatibility of the ranges in (4) and (5) with large values of order  $\mathcal{O}(2 \cdot 10^{-3})$  for  $\text{Re}(\epsilon'/\epsilon)$  has been pointed out earlier in [16]. Here instead we aim at the most conservative upper bound on  $m_s$  from (3), (4), (5) and the experimental result in (1), as quoted in (6).

With the present uncertainty in (5) and in the lattice calculations of  $m_s$  one cannot improve the range for  $\text{Im} \lambda_t$  in (4). Hence at present  $\epsilon'/\epsilon$  is not useful for the construction of the unitarity triangle.

While we do not claim the necessity for new physics in  $\epsilon'/\epsilon$ , there is certainly plenty of room for it in  $\epsilon'/\epsilon$  and other observables in the Kaon system such as  $\epsilon_K$  or  $\Delta M_K$  [17] or rare K decays [18]. Now (1) correlates non-standard contributions to  $\epsilon'/\epsilon$  with upper bounds on  $m_s$ , which might become weaker or stronger compared to (6) depending on the sign of the new physics contribution. We want to stress that this feature is very useful to constrain new physics effects in other  $s \rightarrow d$  transitions: Most extension of the Standard Model involve new helicity-flipping operators, for example  $\epsilon_K$  can receive contributions from the  $\Delta S = 2$  operator  $Q_S = \bar{s}_L d_R \bar{s}_L d_R$ , which is absent in the Standard Model. Yet the matrix elements of operators like  $Q_S$ , which involves two (pseudo-)scalar couplings, are proportional to  $1/m_s^2$ . Hence upper bounds on  $m_s$  imply *lower* bounds on the matrix elements of  $Q_S$  and similar operators multiplying the new physics contributions of interest. To exploit this feature one must, of course, first explore the potential impact of the considered new model on  $\epsilon'/\epsilon$ . Recently Buras and Silvestrini [9] have pointed out that  $\epsilon'/\epsilon$  is sensitive to new physics contributions in the effective  $\bar{s}dZ$ -vertex. This vertex can be substantially enhanced in generic supersymmetric models, as discovered by Colangelo and Isidori [19]. By contrast supersymmetric contributions to the gluonic penguins entering  $\epsilon'/\epsilon$  are small [20]. We want to parametrize the new physics in a model independent way and write

$$\text{Re}(\epsilon'/\epsilon)|_{new} = \text{Im} Z_{ds}^{new} \left[ 1.2 \Leftrightarrow R_S |r_Z^{(8)}| B_8^{(3/2)} \right] + \text{Im} C_{ds}^{new} \cdot 0.24 + 15 \cdot 10^{-4} R_s B_6^{(1/2)} R_6 \quad (7)$$

Here  $Z_{ds}^{new}$  is the new physics contribution to the effective  $\bar{s}dZ$ -vertex  $Z_{ds}$  defined in [9].  $C_{ds}$  is the effective chromomagnetic  $\bar{s}dg$ -vertex defined by

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} C_{ds} \cdot Q_{11}(M_W), \quad Q_{11} = \frac{g_s}{16\pi^2} m_s \bar{s} \sigma^{\mu\nu} T^a (1 \Leftrightarrow \gamma_5) d G_{\mu\nu}^a. \quad (8)$$

The impact of the chromomagnetic operator  $Q_{11}$  has been analyzed in [21]. In the Standard Model one has  $C_{ds} = \Leftrightarrow 0.19 \lambda_t$  with negligible impact on  $\epsilon'/\epsilon$ . In extensions of the Standard Model, however,  $C_{ds}$  can be larger by an order of magnitude or more, because the factor of  $m_s$  in (8) accompanying the helicity flip of the  $s$ -quark may be replaced by the mass  $M$  of some new heavy particle appearing in the one-loop  $\bar{s}dg$ -vertex [22,23]. There are no constraints on  $\text{Im} C_{ds}^{new}$  from  $\epsilon_K$  or  $\Delta m_K$ . In the  $\bar{b}sg$ -vertex the corresponding enhancement factor is smaller by a factor

of  $m_s/m_b$ . The numerical factor of 0.24 in (7) incorporates the renormalization group evolution from  $M_W$  down to 1 GeV and the hadronic matrix element calculated in [21]. Finally new physics could enter the Wilson coefficient  $y_6(1 \text{ GeV}) \approx \Leftrightarrow 0.1$  multiplying  $B_6^{(1/2)}$  (and hidden in  $|r_Z^{(8)}|$  in (2)). (For definitions and numerical values of the Wilson coefficients see [7,8,11].) The parameter  $R_6$  in (7) is defined as

$$R_6 = \frac{\text{Im} [\lambda_t y_6^{new}(1 \text{ GeV})]}{\Leftrightarrow 0.17 \cdot 10^{-4}}. \quad (9)$$

Hence  $R_6 = 1$  means that the new physics contribution to  $y_6(1 \text{ GeV})$  is approximately equal to the Standard Model contribution. There is no simple relation between  $y_6(1 \text{ GeV})$  and the new physics amplitude at  $\mu = M_W$ , because the initial values of all QCD operators contribute to  $y_6(1 \text{ GeV})$  due to operator mixing. In a given model one has to calculate these initial coefficients and to perform the renormalization group evolution down to  $\mu = 1 \text{ GeV}$ . In  $R_6$  no order-of-magnitude enhancement like in  $C_{ds}$  is possible. Only small effects have been found in [8], because  $y_6(1 \text{ GeV})$  is largely an admixture of the tree-level coefficient  $y_2(M_W)$ , which is unaffected by new physics. While still  $R_6$  can be more important than  $C_{ds}$  due to the larger coefficient in (7), it will be less relevant than  $Z_{ds}^{new}$ . In Fig. 2 we have plotted the correlation between  $\text{Im} Z_{ds}^{new}$  and  $m_s$  for  $C_{ds} = R_6 = 0$ . We have used the range in (4). The upper bound on  $\Leftrightarrow \text{Im} Z_{ds}^{new}$  is related to the lower bound on  $\text{Im} \lambda_t$ , which can be invalidated, if new physics contributes to  $\epsilon_K$ . The more interesting lower bound, however, corresponds to the upper limit in (4) stemming from tree-level semileptonic  $B$  decays, which are insensitive to new physics.

Maybe future determinations of  $m_s$  and more precise measurements of  $\text{Re}(\epsilon'/\epsilon)$  will eventually be in conflict with (6). Before then discussing the possibility of new physics it is worthwhile to consider, if  $B_6^{(1/2)}$  can be increased over the maximal value quoted in (5) by some strong interaction dynamics. In [24] it has been pointed out that the existence of  $f_0(400 \Leftrightarrow 1200)$ , a  $\pi\pi$  S-wave  $I = 0$  resonance, introduces a pole in the  $\Delta I = 1/2$  matrix element of  $Q_6$ . This mechanism is not contained in standard chiral perturbation theory and therefore not included in the calculations leading to (5). It can lead to a factor of 2-4 enhancement of  $B_6^{(1/2)}$  allowing to relax the upper limit in (6).  $B_6^{(1/2)}$  also enters the real part of  $\Delta I = 1/2$  amplitude  $A_0$ , whose large size is an yet unexplained puzzle of low energy strong dynamics ( $\Delta I = 1/2$  rule).<sup>4</sup> Now with the large measured value for  $\text{Re}(\epsilon'/\epsilon)$  an enhancement of  $B_6^{(1/2)}$  becomes phenomenologically viable. We have extracted the maximum value of  $B_6^{(1/2)}$  compatible with (1), (3) and (4). The result is plotted vs.  $m_s$  in Fig. 3. Subsequently we have inserted the extracted result for  $(B_6^{(1/2)}, m_s)$  together with the  $1/N_c$  predictions for  $B_1^{(1/2)}$  and  $B_2^{(1/2)}$  [5,7] into the theoretical prediction for  $\text{Re} A_0$  and solved for the Wilson coefficient  $z_6(\mu \approx 1 \text{ GeV})$ .  $B_6^{(1/2)}$  depends only weakly on  $\mu$  [3,7,8]. The numerical value of  $z_6(\mu)$  suffers from severe scheme and scale dependences [3,7,11]. We could fit the measured result for  $\text{Re} A_0$  with a value for  $z_6(\mu)$ , which exceeds the value of  $z_6(1 \text{ GeV})$

<sup>4</sup>We consider an explanation in terms of a new physics enhancement of the chromomagnetic vertex  $C_{ds}$  as proposed in [23] to be unlikely in view of the small matrix element found in [21].

in the NDR scheme by less than a factor of 2. The extracted value depends only weakly on  $m_s$  and slightly decreases with increasing  $m_s(m_c)$ . Considering the large uncertainty in  $z_6(1 \text{ GeV})$  and the fact that the true scale of the hadronic interaction is probably well below  $1 \text{ GeV}$  ( $|z_6|$  increases with decreasing  $\mu_s$ , but cannot reliably be predicted for too low scales) we conclude that the mechanism proposed in [24] leads to a semiquantitative understanding of the  $\Delta I = 1/2$  rule while simultaneously being consistent with the measurement of  $\text{Re}(\epsilon'/\epsilon)$  in (1). This conclusion would not have been possible with the old low result of the E731 experiment [25]. Also the upper bound on  $m_s$  in (6) becomes invalid, so that one can even accommodate for the high value of  $m_s$  measured by ALEPH [15] and quoted after (6).

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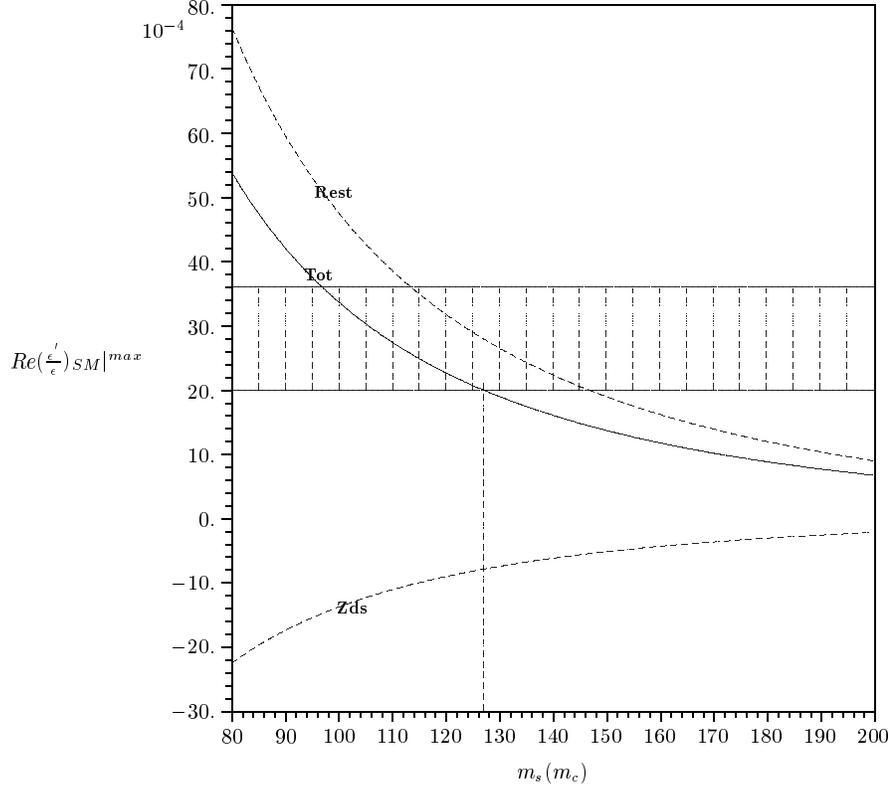


Figure 1: The maximal  $\text{Re}(\epsilon'/\epsilon)$  vs.  $m_s(m_c)$ .  $\text{Re}(\epsilon'/\epsilon)|_{max}$  corresponds to  $\text{Im} \lambda_t = 1.7 \cdot 10^{-4}$ ,  $2 B_6^{(1/2)} \Leftrightarrow B_8^{(3/2)} = 2.0$  and  $|r_Z^{(8)}| = 8.5$ . The plotted relation for different values of  $2 B_6^{(1/2)} \Leftrightarrow B_8^{(3/2)}$  can be obtained by replacing  $m_s(m_c)$  with  $m_s(m_c) \cdot \left[ (2 B_6^{(1/2)} \Leftrightarrow B_8^{(3/2)}) / 2.0 \right]^{-1/2}$ . The contribution of the  $\bar{s}dZ$ -vertex is shown separately. The hatched area corresponds to the  $2\sigma$  range of the measured value in (1).

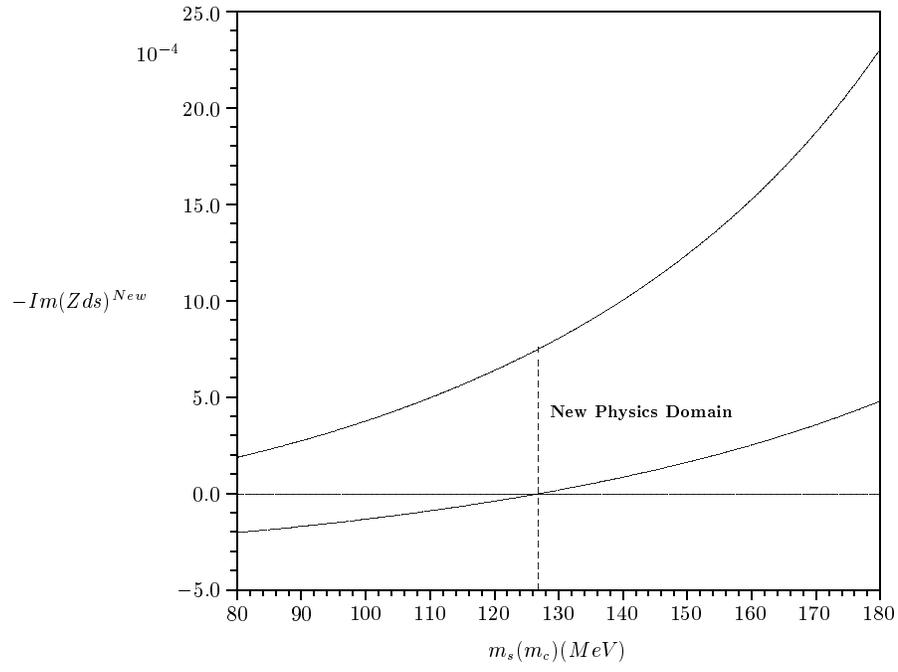


Figure 2: Correlation between new physics contributions to  $\text{Im } Z_{ds}$  and  $m_s$  for the ranges in (3), (4) and (5). To relax the range in (5) for the hadronic B-factors see the caption of Fig. 1.

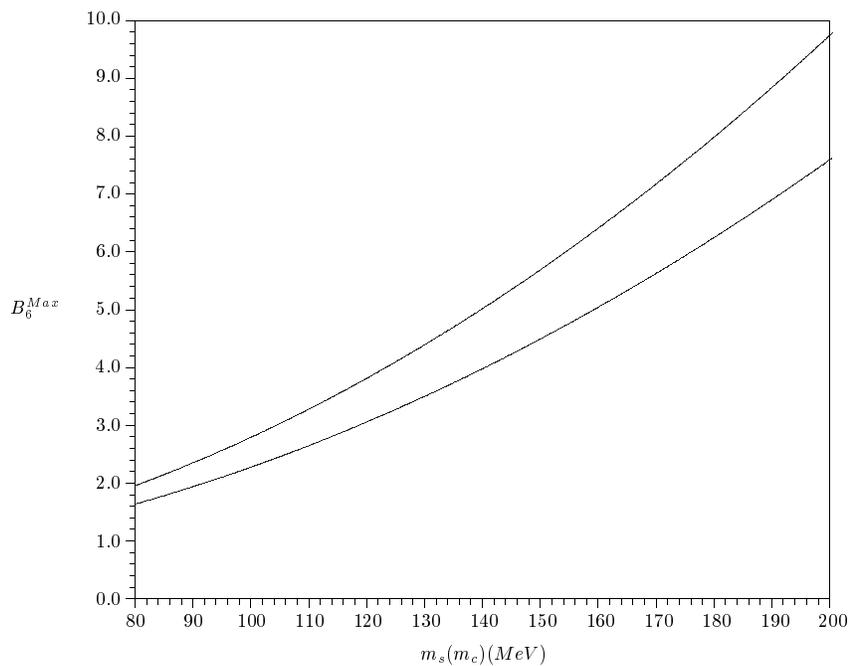


Figure 3: Maximal value of  $B_6^{(1/2)}$  vs.  $m_s$  extracted from (1) for  $\text{Im } \lambda_t$  and  $B_8^{(3/2)}$  in the ranges in (4) and (5). The lower (upper) curve corresponds to  $|r_Z^{(8)}| = 8.5$  ( $|r_Z^{(8)}| = 6.5$ ).